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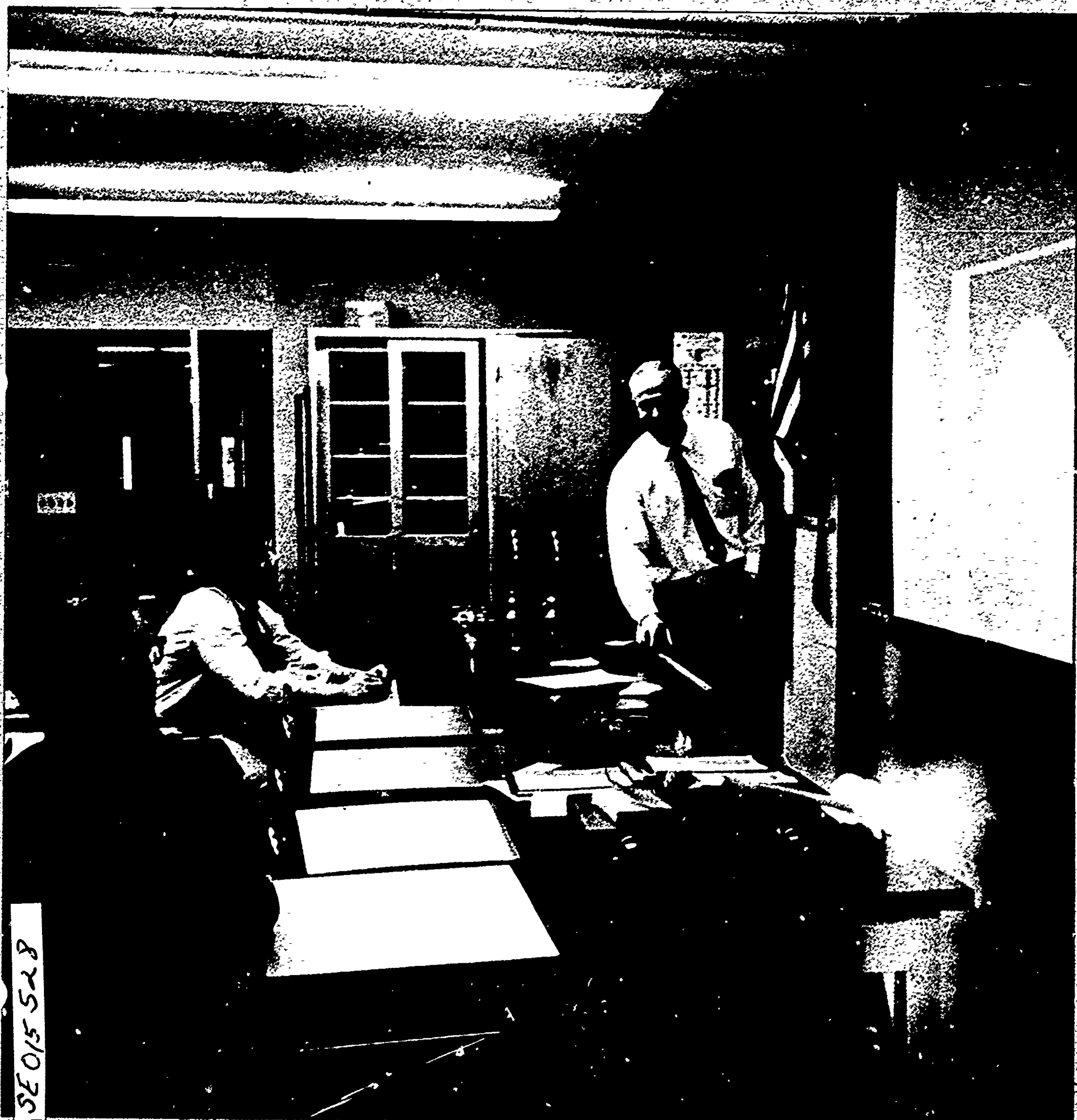
ABSTRACT

Teaching procedures of Project Physics Unit 2 are presented to help teachers make effective use of learning materials. The unit contents are discussed in connection with teaching aid perspectives, multi-media schedules, schedule blocks, and resource charts. Analyses are made for transparencies, 16mm films, and reader articles. Included is information about the background and development of each unit chapter, demonstration methods, apparatus operations, notes on the student handbook, and explanation of film loops. Additional articles deal with calendars, armillary sphere, epicycles, "chase problem," atmospheric refraction, relations in an ellipse, moon's irregular motion, initial and gravitational mass, G measuring, and "true" scale of sun, moon, and earth. A redesigned epicycle machine is analyzed, and a bibliography of reference texts and periodicals is given. Solutions to study guide problems in the text and review problems in the handbook are provided in detail along with suggested answers to test items. The second unit of the text, with marginal notes on each section, is also compiled in this guide. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

Project Physics **Teacher Guide**

An Introduction to Physics

Motion in the Heavens



Project Physics **Text**

An Introduction to Physics 2 Motion in the Heavens

Authorized Interim Version 1968-69

Distributed by Holt, Rinehart and Winston, Inc. New York Toronto

The Project Physics course has been developed through the creative assistance of many colleagues. The following is a partial list of those contributors (the affiliations indicated are those just prior to or during their association with the Project).

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Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Harvard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

An Introduction to Physics **2** **Motion in the Heavens**

The Aztec Calendar, carved over 100 years before our calendar was adopted, divides the year into eighteen months of twenty days each.

Prologue . Curiously, the oldest science, astronomy, deals with objects which are now known to be the most distant. Yet to you and me, as to the earliest observers of the skies, the sun, moon, planets and stars do not seem to be far away. On a clear night they seem so close that we can almost reach out and touch them.

This section is meant to create interest. Treat as a reading assignment.

The lives of the ancient people, and indeed of nearly all people who lived before electric lighting, were dominated by heavenly events. The working day began when the sun rose and ended when the sun set. Human activity was dominated by the presence or absence of daylight and the sun's warmth, changing season by season.

Even in modern times outdoorsmen use the sun by day and the stars by night as a clock. Directions are indicated by the sun at rising and setting time, and true south can be determined from the position of the sun at noon. The pole star gives a bearing on true north after dark. The sun is also a crude calendar, its noontime altitude varying with the seasons.

Over the centuries our clock has been devised so we can subdivide the days, and the calendar developed so we can record the passage of days into years. Of all the units used in regular life, "one day" is probably the most basic and surely the most ancient. For counting longer intervals, a "moon" or month was an obvious unit. But the moon is unsatisfactory as a timekeeper for establishing the agricultural year.

The lunar month is not the same as a calendar month.

When, some 10,000 years ago, the nomadic tribes settled down to live in towns, they became dependent upon agriculture for their food. They needed a calendar for planning their plowing and sowing. Indeed, throughout recorded history most of the world's population has been involved in agriculture in the spring. If the crops were planted too early they might be killed by a frost, or the seeds rot in the ground. But if they were planted too late, the crops might not ripen before winter came. Therefore, a knowledge of the times for planting and harvesting had high survival value. A calendar for the agricultural year was very important. The making and improvement of the calendar were often the tasks of priests, who also set the dates for the religious festivals. Hence, the priests became the first astronomers.

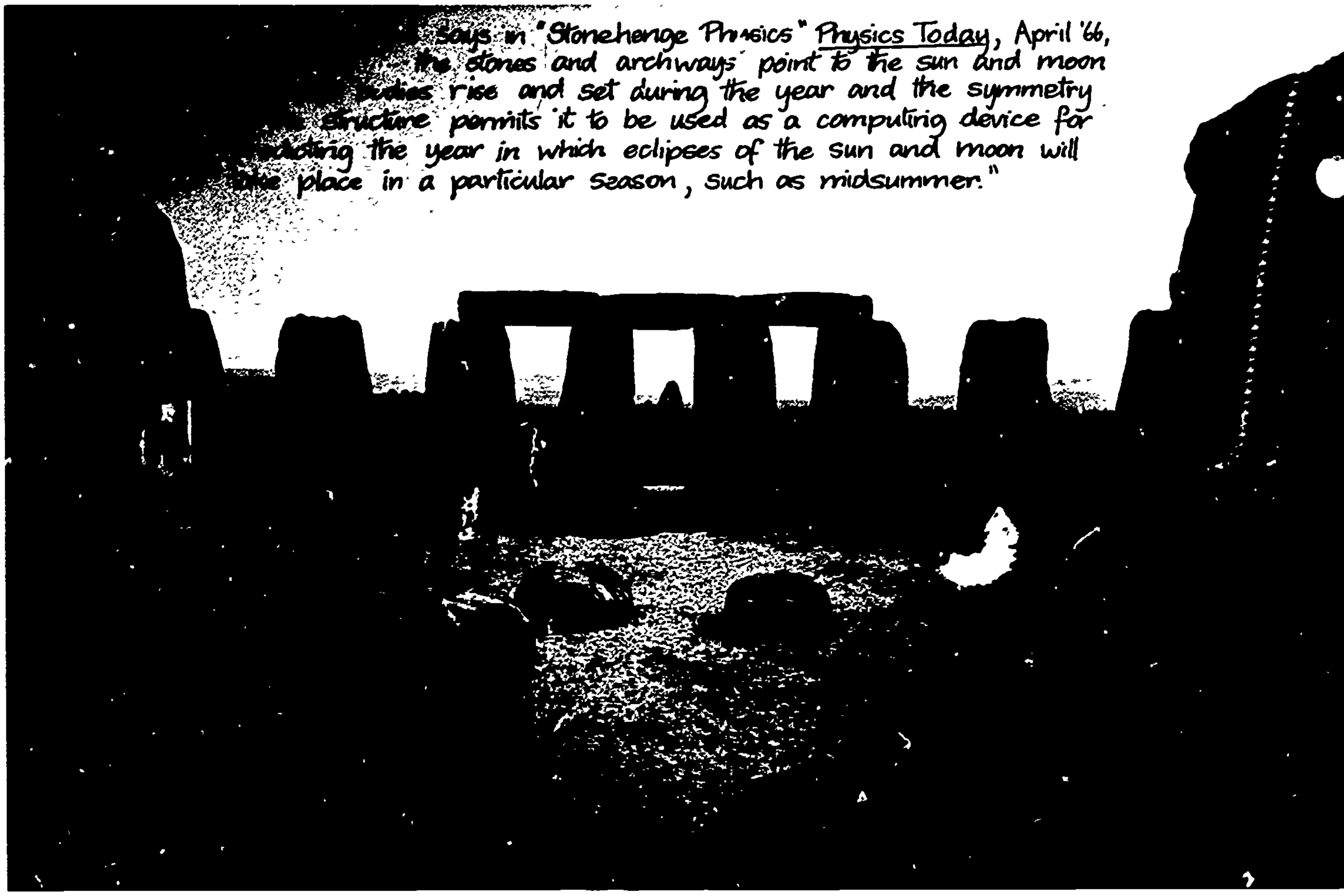
F6 : Universe

F7: Mystery of Stonehenge, two parts, 25 min each.

Many of the great buildings of ancient times were constructed with careful astronomical orientation. The great pyramids of Egypt, tombs of the Pharaohs, have sides that run due north-south, and east-west. The impressive, almost frightening, circles of giant stones at Stonehenge in England appear to have been arranged about 2000 B.C. to permit accurate astronomical observations of the positions of the sun and moon. The Mayans and the Incas in America, as well as the Chinese, put enormous effort into buildings from which they could measure the changes in the positions of the

If you have never watched the night sky, start observations now. Chapters 5 and 6 are based on some simple observations of the sky.

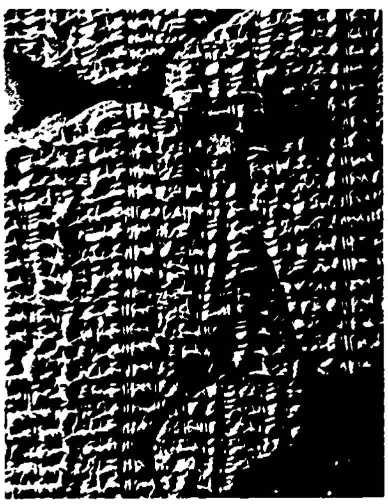
See : Stonehenge, Jaquetta Hawkes, Scientific American, June 1955.



Stonehenge, England, apparently a prehistoric observatory.

sun, moon and planets. Thus we know that for thousands of years men have carefully observed the motions of the heavenly bodies.

At least as early as 1000 B.C. the Babylonians and Egyptians had developed considerable ability in timekeeping. Their recorded observations are now being slowly unearthed. But the Egyptians, like the Mayans and others, were interested in practical forecasts and date-setting. To the best of our knowledge they did not try to explain their observations other than by tales and myths. Our western culture owes much to the efforts of the Egyptians and Babylonians who carefully recorded their observations of heavenly cycles. But our debt is greatest to the Greeks who began trying to explain what was seen.



Section of Babylonian clay tablet, now in the British Museum, records the positions of Jupiter from 132 B.C. to 60 B.C.

The observations to be explained were begun out of idle curiosity, perhaps by shepherds to pass away the time, and later took on practical importance in calendar making. These simple observations became, during following centuries, the basis for one of the greatest of scientific triumphs: the work of Isaac Newton. The present unit tells how he united the study of motions on the earth with the study of motions in the heavens to produce a single universal science of motion.

The Greeks recognized the contrast between forced and temporary motions on the earth and the unending cycles of motions in the heavens. About 600 B.C. they began to ask a new question: how can we explain these cyclic events in the sky in a simple way? What order and sense can we make of the heavenly happenings? The Greeks' answers, which are discussed in Chapter 5, had an important effect on science. For example, as we shall see, the writings of Aristotle, about 330 B.C., became widely studied and accepted in western Europe after 1200 A.D., and were important in the scientific revolution that followed.

After the conquests of Alexander the Great, the center of Greek thought and science shifted to Egypt at the new city of Alexandria, founded in 332 B.C. There a great museum, actually similar to a modern research institute, was created about 290 B.C. and flourished for many centuries. But as the Greek civilization gradually declined, the practical-minded Romans captured Egypt, and interest in science died out. In 640 A.D. Alexandria was taken by the Moslems as they swept along the southern shore of the Mediterranean Sea and moved northward through Spain to the Pyrenees. Along the way they seized and preserved many libraries of Greek documents, some of which were later translated into Arabic and seriously studied. During the following centuries the Moslem scientists made new and better observations of the heavens, although they did not make major changes in the explanations or theories of the Greeks.

During this time, following the invasions by warring tribes from northern and central Europe, civilization in western Europe fell to a low level, and the works of the Greeks were forgotten. Eventually they were rediscovered by Europeans through Arabic translations found in Spain when the Moslems were forced out. By 1130 A.D. complete manuscripts of one of Aristotle's books were known in Italy and in Paris. After the founding of the University of Paris in 1170, many other writings of Aristotle were acquired and studied both there and at the new English Universities, Oxford and Cambridge.

During the next century, Thomas Aquinas (1225-1274) blended major elements of Greek thought and Christian theology into a single philosophy. This blend, known as Thomism, was widely accepted in western Europe for several centuries. In achieving this commanding and largely successful synthesis, Aquinas accepted the physics and astronomy of Aristotle. Because the science was blended with theology, any questioning of the

A: Cosmogony

science seemed also to be a questioning of the theology. Thus for a time there was little criticism of the Aristotelian science.

The Renaissance movement, which spread out across Europe from Italy, brought new art and new music. Also, it brought new ideas about the universe and man's place in it. Curiosity and a questioning attitude became acceptable, even prized. Men acquired a new confidence in their ability to learn about the world. Among those whose work introduced the new age were Columbus and Vasco da Gama, Gutenberg and da Vinci, Michelangelo and Dürer, Erasmus, Vesalius, and Agricola, Luther, and Henry VIII. The chart opposite page 29 shows their life spans. Within this emerging Renaissance culture lived Copernicus, whose reexamination of astronomical theories is discussed in Chapter 6.

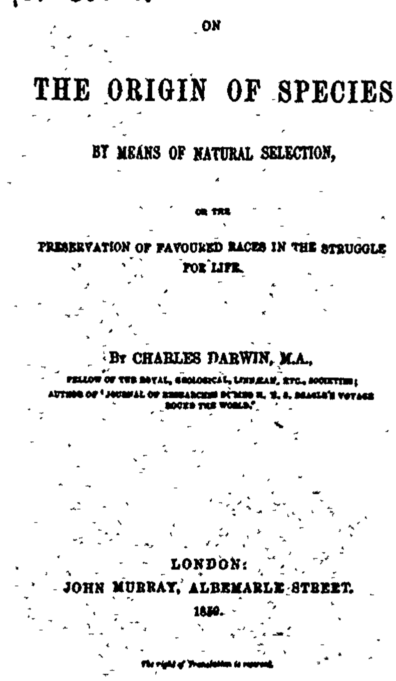
Further changes in astronomical theory were made by Kepler through his mathematics and by Galileo through his observations and writings, which are discussed in Chapter 7. In Chapter 8 we shall see that Newton's work in the second half of the seventeenth century extended the ideas about earthly motions so that they could also explain the motions observed in the heavens—a magnificent synthesis of terrestrial and celestial dynamics.

Louis XIV visiting the French Academy of Sciences, which he founded in the middle of the seventeenth century. Seen through the right-hand window is the Paris Observatory, then under construction.

Great scientific advances can affect ideas outside science. For example, Newton's impressive work helped to bring a new sense of confidence. Man now seemed capable of understanding all things in heaven and earth.• This great change in attitude was a characteristic of the Age of Reason in the eighteenth century. To a degree, what we think today and how we run our affairs are still based on these events of three centuries ago.

•This was also the Greek ideal, but they had a "two universe" model which has separate mechanics for heaven and for earth.

The decisive changes in thought that develope[illegible] in start of the Renaissance within a period of a century can be compared to changes during the past hundred years. This period might extend from the publication in 1859 of Darwin's <u>Origin of Species</u> to the first large-scale release of atomic energy in 1945. Within this recent interval lived such scientists as Mendel and Pasteur, Planck and Einstein, Rutherford and Bohr. The ideas they and others introduced into science during the last century have had increasing importance. These scientific ideas are just as much a part of our time as the ideas and works of such persons as Roosevelt, Ghandi and Pope John XXIII; Marx and Lenin; Freud and Dewey; Picasso and Stravinsky; or Shaw and Joyce. If, therefore, we understand the way in which science influenced the men of past centuries, we shall be better prepared to understand how science influences our thought and lives today.


ON

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

BY CHARLES DARWIN, M.A.,

LONDON:
JOHN MURRAY, ALBEMARLE STREET.
1859.

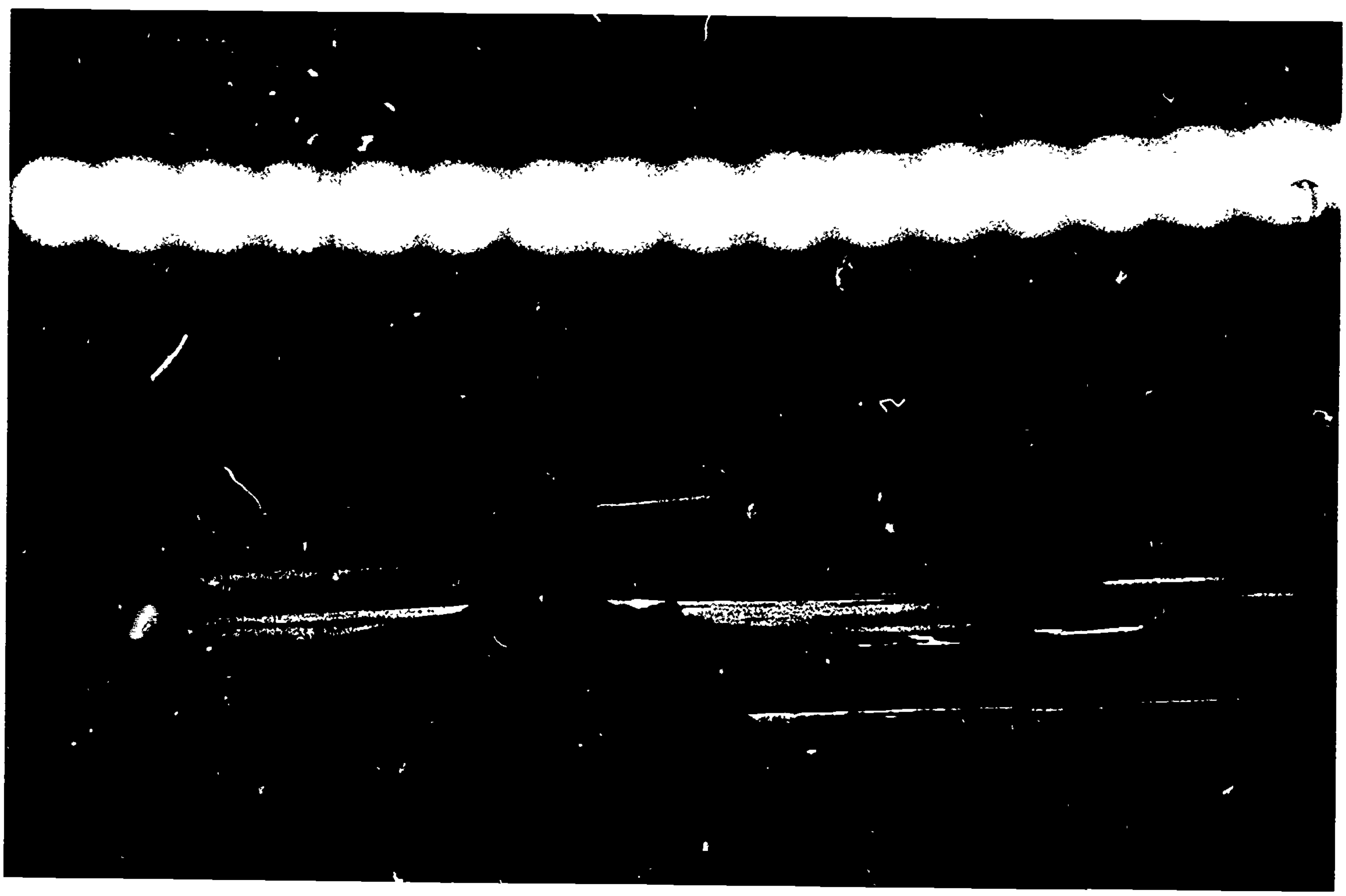
The development of so-called modern physics starts with Röntgen about 1895 and is treated in Unit 5.

See "The Black Cloud" in <u>Project Physics Reader 2</u>.

Chapter 5 Where is the Earth? — The Greeks' Answers

Thales (c 600 BC) is usually credited as the founder of Greek science and philosophy. He assumed that the whole universe can be explained by [illegible] knowledge and reason.

In chapter 5 students become more familiar with the motions of objects in the sky. They will consider the central question raised by the ancients: What accounts for the movement of the heavenly bodies? The heliocentric (sun centered) model of Aristarchus and the geocentric (earth centered) system of Ptolemy are discussed in their historical and philosophical context.



T13 will aid in clarifying students' observations of the sky.

E13: The size of the earth

E1: Naked-Eye Astronomy (cont.)

5.1 Motions of the sun and stars. The facts of everyday astronomy, the heavenly happenings themselves, are the same now as in the times of the Greeks. You can observe with the unaided eye most of what they saw and recorded. You can discover some of the long-known cycles and rhythms, such as the seasonal changes of the sun's height at noon and the monthly phases of the moon. If our purpose were only to make accurate forecasts of eclipses and planetary positions, we could, like the Babylonians and Egyptians, focus our attention on the cycles and rhythms. If, however, like the Greeks, we wished to explain these cycles, we must imagine some sort of relatively simple model or theory with which we can predict the observed variations. But before we can understand the several theories proposed in the past, we must review the major observations which the theories were to explain: the motions of the sun, moon, planets and stars.

Each day the sun rises above your horizon on the eastern side of the sky and sets on the western side. At noon, halfway between sunrise and sunset, the sun is highest above your horizon. Thus, a record of your observations of the sun would be similar to that shown in Fig. 5.1 (a). Every day the same type of pattern occurs from sunrise to sunset. Thus the sun, and indeed all the objects in the sky, show a daily motion. They rise on the eastern horizon, pass a highest point, and set on the western horizon. This is called the daily motion.

Day by day from July through November, the noon height of the sun above the horizon becomes less. Near December 22, the sun's height at noon (as seen from the northern hemisphere) is least and the number of hours of daylight is smallest. During the next six months, from January into June, the sun's height at noon slowly increases. About June 21 it is greatest and the daylight is longest. Then the sun's gradual southward motion begins again, as Fig. 5.1 (b) indicates.

This year-long change is the basis for the seasonal or solar year. Apparently the ancient Egyptians thought that the year had 360 days, but they later added five feast days to have a year of 365 days that fitted better with their observations of the seasons. Now we know that the solar year is 365.24220 days long. The fraction of a day, 0.24220, raises a problem for the calendar maker, who works with whole days. If you used a calendar of just 365 days, after four years New Year's Day would come early by one day. In a century you would be in error by almost a month. In a few centuries

Summary 5.1

1. Our Gregorian calendar was devised to keep the months and days (e.g., March 21) consistent with the sun's seasonal position in the sky (N-S motion).

2. The stars have a single, steady, daily motion westward; they appear to be on a great sphere turning around the observer.

3. Day after day the stars set earlier or appear to be moving westward, overtaking the sun. Conversely, the sun moves eastward through the star field. If we use the stars as a frame of reference, the sun makes one cycle each year eastward through the star field. (about 1° per day). The sun's path among the stars is called the ecliptic.

See "Roll Call" in Project Project Physics Reader 2.

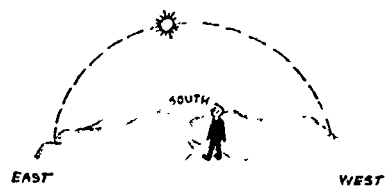
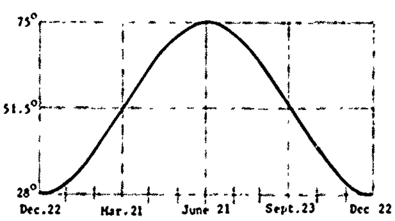


Fig. 5.1
a) Daily path of the sun through the sky.

A: Time zones



b) Noon altitude of the sun as seen from St. Louis, Missouri throughout the year.

Fig. 5.2 Midnight sun photographed at 5-minute intervals over the Ross Sea in Antarctica. The sun appears to move from right to left.

Background material on the calendar. See Article Section.

A: Photograph the night sky
Additional Activity: Calendars
Background on Calendars, see Article Section.

SG 5.1

the date called January first would come in the summertime! In ancient times extra days or even whole months were inserted from time to time to keep a calendar of 365 days and the seasons in fair agreement.

Such a makeshift arrangement, is, however, hardly satisfactory. In 45 B.C. Julius Caesar decreed a new calendar which averaged 365 1/4 days per year (the Julian calendar), with one whole day (a leap day) being inserted each fourth year. This calendar was used for centuries during which the small difference between the decimal parts 0.25 and 0.24220 accumulated to a number of days. Finally, in 1582 A.D., under Pope Gregory, a new calendar (the Gregorian calendar) was announced. This had only 97 leap days in 400 years. To reduce the number of leap days from 100 to 97 in 400 years, century years not divisible by 400 were omitted as leap years. Thus the year 1900 was not a leap year, but the year 2000 will be a leap year (try comparing 97/400 to 0.24220).

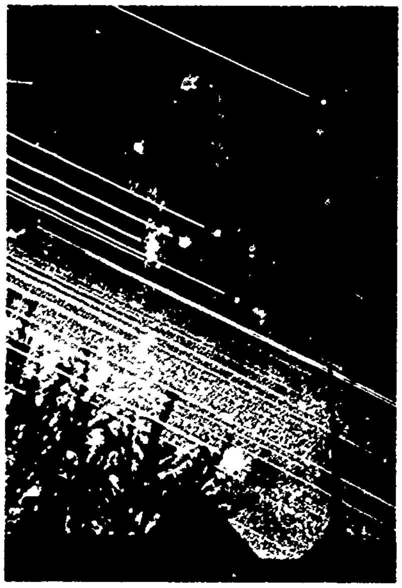


Fig. 5.3 Orion trails, a combination star and trai photograph. The camera shutter was opened for several hours while the stars trailed, then closed for a few minutes, then reopened while the camera was driven to follow the stars for a few minutes.

You may have noticed that a few stars are bright and many are faint. Some bright stars show colors, but most appear whitish. People have grouped many of the brighter stars into patterns, called constellations, such as the familiar Big Dipper and Orion. The brighter stars may seem to be larger, but if you look at them through binoculars, they still appear as points of light.

Have you noticed a particular star pattern overhead and then several hours later seen that it was near the western horizon? What happened? During the interval the stars in the western side of the sky moved down toward the horizon, while those in the eastern part of the sky moved up from the horizon. A photograph exposed for some time would show the trails of the stars, like those shown in Fig. 5.3. During the night, as seen from a point on the northern hemisphere of our earth, the stars appear to move counter-clockwise around a point in the sky called the North Celestial Pole, which is near the fairly bright star Polaris, as Fig. 5.4 suggests. Thus the stars like the sun show a daily motion across the sky.

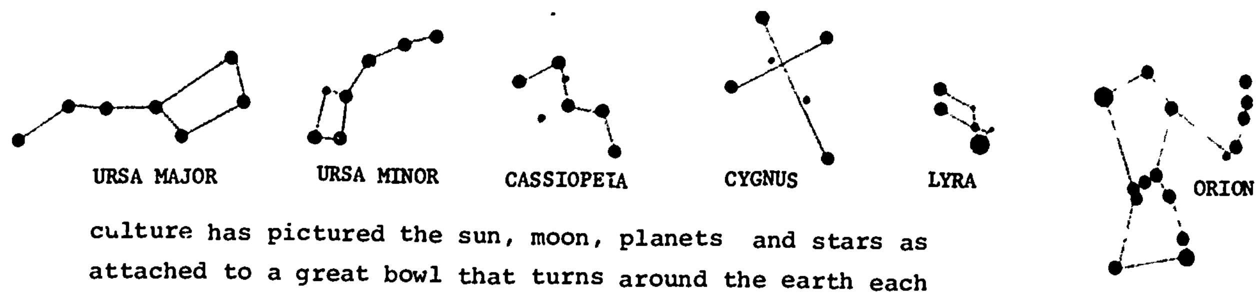
Fig. 5.4 Time exposure showing star trails around the North Celestial Pole. The diagonal line was caused by an artificial earth satellite. You can use a protractor to determine the duration of the exposure; the stars move about 15° per hour.

Which of the star trails in Fig. 5.4 might be that of Polaris? Defend your choice.

Ans. The bright central star to the left of the streak near the center of the photo is Polaris.

Some of the star patterns, such as Orion (the Hunter) and Cygnus (the Swan, also called the Northern Cross), were named thousands of years ago. Since we still see the same star patterns described by the ancients, we can conclude that these star patterns change very little, if at all, over the centuries. Thus in the heavens we observe both stability over the centuries as well as smooth, orderly daily motion. To explain this daily rising and setting almost every early

A: How long is a Sidereal day?
A: Globe! Sundial



culture has pictured the sun, moon, planets and stars as attached to a great bowl that turns around the earth each day.

But there are other motions in the sky. If you have observed the star patterns in the west just after twilight on two evenings several weeks apart, you have probably noticed during the second observation that the stars appeared nearer the horizon than when you made your first observation. As measured by sun-time, the stars set about four minutes earlier each day.

From these observations we can conclude that the sun is slowly moving eastward relative to the stars, even though we cannot see the stars when the sun is above the horizon. One complete cycle of the sun against the background of stars takes a year. The sun's yearly path among the stars is called the ecliptic. It is a great circle in the sky and is tilted at about 23½° to the equator. The point at which the sun, moving along the ecliptic, crosses the equator from south to north on March 21 is called the Vernal Equinox. Thus we have three motions of the sun to explain: the daily rising and setting, the annual eastward cycle among the stars, and the north-south seasonal motion.

If you told time by the stars, would the sun set earlier or later each day?

T14 will help visualize the relationships discussed in the section. If your students have made the suggested observations of the sky while studying Unit 1, they might summarize the motions described in Sec. 5.1 – 5.3. These are the major motions to be explained by any theory. Your students may have difficulty in understanding that either the sun or the stars can be taken as the reference system against which the other moves.

See "The Garden of Epicurus" in Project Physics Reader 2.

SG 5.2

Q1 What evidence do we have that the ancient people observed the sky?

Q2 For what practical purposes were calendars needed?

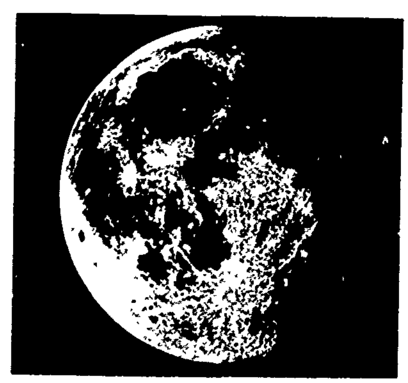
Q3 What are the observed motions of the sun during one year?

Q4 In how many years will the Gregorian calendar be off by one day?

5.2 Motions of the moon. The moon also moves eastward against the background of the stars and rises nearly an hour later each night. When the moon rises at sunset—when it is opposite the sun in the sky—the moon is bright and shows a full disc (full moon). About fourteen days later, when the moon is passing near the sun in the sky (new moon), we cannot see the moon at all. After new moon we first see the moon as a thin crescent low in the western sky at sunset. As the moon rapidly moves further eastward from the sun, the moon's crescent fattens to a half circle, called "quarter moon," and then within another week on to full moon again. After full moon, the pattern is reversed, with the moon slimming down to a crescent seen just before sunrise and then several days being invisible in the glare of sunlight.

As early as 380 B.C. the Greek philosopher Plato recognized that the phases of the moon could be explained by thinking of

These end of section questions are intended to help you check your understanding before going to the next section. Answers start on page 123.



Moon: 17 days old

A: Moon crater names

Summary 5.2
1. The moon moves eastward like the sun, but at a faster rate; upon careful inspection its motion shows irregularities.
2. Since antiquity the moon has been considered to be a sphere reflecting sunlight.

See Scientific American articles on moon's librations March 1954; and moon-star occultations. Jan 1955.

D28: Phases of the moon

Moon: 3 days old

E14: The height of Piton

the moon as a globe reflecting sunlight and moving around the earth, with about 29 days between new moons. Because the moon appears to move so rapidly relative to the stars, people early assumed the moon to be close to the earth.

The moon's path around the sky is close to the yearly path of the sun; that is, the moon is always near the ecliptic. The moon's path is tipped a bit with respect to the sun's path; if it were otherwise, we would have an eclipse of the moon at every full moon and an eclipse of the sun at every new moon. The moon's motion is far from simple and has posed persistent problems for astronomers as, for example, predicting accurately the times of eclipses.

Q5 Describe the motion of the moon during one month.

Q6 Why don't eclipses occur each month?

A: Making angular measurements

A: Stereo photo of the moon

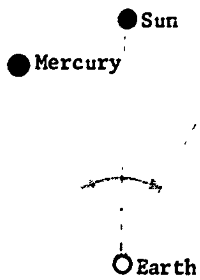
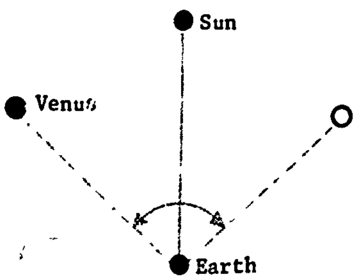


Fig. 5.5 The maximum angles from the sun at which we observe Mercury and Venus. Both planets can, at times, be observed at sunset or at sunrise. Mercury is never observed to be more than 28° from the sun, and Venus is never more than 48° from the sun.



5.3 The wandering stars. Without a telescope we can see, in addition to the sun and moon, five rather bright objects which move among the stars. These are the wanderers, or planets: Mercury, Venus, Mars, Jupiter and Saturn. (With the aid of telescopes three more planets have been discovered: Uranus, Neptune and Pluto; but none of these was known until nearly a century after the time of Isaac Newton.) Like the sun and moon, all the planets are observed to rise daily in the east and set in the west. Also like the sun and moon, the planets generally move eastward among the stars. But at certain times each planet stops moving eastward among the stars and for some months moves westward. This westward, or wrong-way motion, is called retrograde motion. Easy to describe, not so easy to explain.

In the sky Mercury and Venus are always near the sun. As Fig. 5.5 indicates, the greatest angular distance from the sun is 28° for Mercury and 48° for Venus. Mercury and Venus show retrograde motion after they have been farthest east of the sun and visible in the evening sky. Then they quickly move westward toward the sun, pass it, and reappear in the morning sky. During this motion they are moving westward relative to the stars, as is shown by the plot for Mercury in Fig. 5.7.

In contrast, Mars, Jupiter and Saturn may be found in any position in the sky relative to the sun. As these planets (and the three discovered with the aid of telescopes) move eastward they pass through the part of the sky opposite to the sun. When they pass through the point 180° from the sun, i.e., when they are opposite to the sun, as Fig. 5.6 indicates, they are said to be in opposition. When each of

Summary 5.3

1. Two planets, Mercury and Venus, are always within 48° of the sun. 2. Three other planets visible without a telescope, Mars, Jupiter and Saturn, move eastward and pass the point opposite the sun. Near the time of this opposition they cease their eastward motion and slowly move westward in retrograde motion for several months, then resume their eastward motion.

3. These planets are brightest when opposite the sun.
4. The planets, as well as the moon and sun, are always found near the ecliptic.

these planets nears the time of its opposition, it ceases its eastward motion and for several months moves westward (see Fig. 5.7). As Table 5.1 and Fig. 5.7 show, the retrograde motion of Saturn lasts longer, and has a smaller angular displacement than do the retrograde motions of Jupiter and Mars.

Fig. 5.6 Opposition of a planet occurs when, as seen from the earth, the planet is opposite to the sun, and crosses the meridian at midnight.

Table 5.1 Recent Retrograde Motions of the Planets

Planet	Duration of Retrograde	Days*	Westward Displacement*
Mercury	April 26 to May 30, 1963	34	15°
Venus	May 29 to July 11, 1964	43	19°
Mars	Jan. 29 to April 21, 1965	83	22°
Jupiter	Aug. 10 to Dec. 6, 1963	118	10°
Saturn	June 4 to Oct. 21, 1963	139	7°
Uranus	Dec. 25, '65 to May 24, 1966	152	4°
Neptune	Feb. 22 to Aug. 1, 1966	160	3°
Pluto	Dec. 28, '65 to June 2, 1966	156	2°

*These intervals and displacements vary somewhat from cycle to cycle.

The planets change considerably in brightness. When Venus is first seen in the evening sky as the "evening star," the planet is only fairly bright. But during the following four to five months as Venus moves farther eastward from the sun, Venus gradually becomes so bright that it can often be seen in the daytime if the air is clear. Later, when Venus scoots westward toward the sun, it fades rapidly, passes the sun, and soon reappears in the morning sky before sunrise as the "morning star." Then it goes through the same pattern of brightness changes, but in the opposite order: soon bright, then gradually fading. The variations of Mercury follow much the same pattern. But because Mercury is always seen near the sun during twilight, Mercury's changes are difficult to observe.

Mars, Jupiter and Saturn are brightest about the time that they are highest at midnight and opposite to the sun. Yet over many years their maximum brightness differs. The change is most noticeable for Mars; the planet is brightest when it is opposite the sun during August or September.

Not only do the sun, moon and planets generally move eastward among the stars, but the moon and planets (except Pluto) are always found within a band only 8° wide on either side of the sun's path.

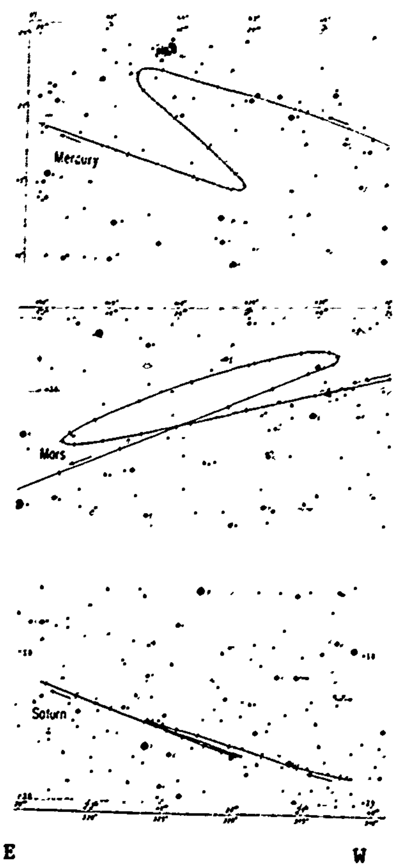


Fig. 5.7 The retrograde motions of Mercury (marked at 5-day intervals), Mars (at 10-day intervals), and Saturn (at 20-day intervals) in 1963, plotted on a star chart. The dotted line is the annual path of the sun, called the ecliptic.

A: Scale model of the solar system

Q7 In what part of the sky must you look to see the planets Mercury and Venus?

Q8 In what part of the sky would you look to see a planet which is in opposition?

Q9 When do Mercury and Venus show retrograde motion?

Q10 When do Mars, Jupiter and Saturn show retrograde motion?

Q11 In what ways do the retrograde motions of the planets differ? Is each the same at every cycle?

Summary 5.4
1. About 380 BC., Plato asked for a theory to account for the motions of the planets.

2. Reconstruction of Greek knowledge has been difficult because the few manuscripts available are copies and often translations.

3. Plato, and those who attempted to devise a theory of planetary motions, assumed that some combination of circular motions at uniform rates should account for the observed planetary motions.

4. "Self-evident" or obvious models or lines of interpretation may include or hide major assumptions.

5. Stress the two numbered points on the next page.

The library, established during the Hellenistic period, was probably more of a research center than a museum as we now use the term.

5.4 Plato's problem. About 400 B.C. Greek philosophers asked a new question: how can we explain the cyclic changes observed in the sky? Plato asked for a theory or general explanation to account for what was seen, or as he phrased it: "to save the appearances." The Greeks appear to have been the first people to desire theoretical explanations to account for natural phenomena. Their start was an important step toward science as we know it today.

How did the Greeks begin their explanation of the motions observed in the heavens? What were their assumptions?

At best, any answers to these questions must be tentative. While many scholars over the centuries have devoted themselves to the study of Greek thought, the documents available as the basis for our knowledge of the Greeks are mostly copies of copies and translations of translations in which errors and omissions occur. In some cases all we have are reports from later writers of what certain philosophers did or said, and these may be distorted or incomplete. The historians' task is difficult. Most of the original Greek writings were on papyrus or cloth scrolls, which have decayed through the ages. Many wars, much plundering and burning, have also destroyed many important documents. Especially tragic was the burning of the famous library of Alexandria in Egypt, which contained several hundred thousand documents. (Actually, it was burned three times: in part by Caesar's troops in 47 B.C.; then in 390 A.D. by a Christian fanatic; and the third time in 640 A.D. by Moslems when they overran the country.) Thus, while the general picture of Greek culture seems to be rather well established, many interesting particulars and details are not known.

The approach taken by the Greeks and their intellectual followers for many centuries was stated by Plato in the fourth century B.C. He stated the problem to his students in this way: the stars—eternal, divine, unchanging beings—move uniformly around the earth, as we observe, in that most perfect of all paths, the endless circle. But a few celestial objects, namely the sun, moon and planets, wander across the sky and trace out complex paths, including even retrograde motions. Yet, surely, being also heavenly bodies, they too must really move in a way that becomes their exalted status. This must be in some combination of circles. How then can we account for the observations of planetary motions and "save the appearances"? In particular, how can we explain the retrograde motions of the planets? Translated into more modern terms, the problem is: determine the combination of simultaneous uniform circular motions that must be assumed

How are the dates established for old manuscripts?
The detective work of restoration and dating is fascinating.

for each of the objects to account for the observed irregular motions. The phrase "uniform circular motion" means that the body moves around a center at a constant distance, and that the rate of angular motion around the center (such as one degree per day) is uniform or constant.

Note that the changes in the apparent brightness of the planets, what we would call physical evidence, was silently ignored. There is, however, some evidence that both Plato and Aristotle, as well as others, were not too happy with a purely geometric solution which failed to account for these observed changes in brightness.

Notice that the problem is concerned only with the positions of the sun, moon, and planets. The planets appear to be only points of light moving against the background of stars. From two observations at different times we obtain a rate of motion: a value of so many degrees per day. The problem then is to find a "mechanism," some combination of motions, that will reproduce the observed angular motions and lead to predictions in agreement with observations. The ancient astronomers had no observational evidence about the distances of the planets; all they had were directions, dates and rates of angular motion. Although changes in brightness of the planets were known to be related to their positions relative to the sun, these changes in brightness were not included in Plato's problem.

Probably because Plato did not consider this observation as germane to his problem, nothing was added to the usefulness of the theory, and so the phenomena were ignored.

Plato's statement of this historic problem of planetary motion illustrates the two main contributions of Greek philosophers to our understanding of the nature of physical theories:

1. According to the Greek view, a theory should be based on self-evident concepts. Plato regarded as self-evident the concept that heavenly bodies must have perfectly circular motions. Only in recent centuries have we come to understand that such commonsense beliefs may be misleading. More than that—we have learned that every assumption must be critically examined and should be accepted only tentatively. As we shall often see in this course, the identification of hidden assumptions in science has been extremely difficult. Yet in many cases, when the assumptions have been identified and questioned, entirely new theoretical approaches have followed.

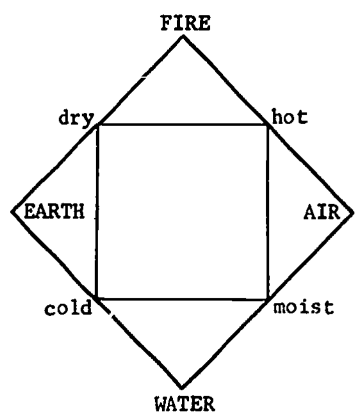
2. Physical theory is built on observable and measurable phenomena, such as the motions of the planets. Furthermore, our purpose in making a theory is to discover the uniformity of behavior, the hidden likenesses underlying apparent irregularities. For organizing our observations the language of number and geometry is useful. This use of mathematics, which is widely accepted today, was derived in part from the Pythagoreans, a group of mathematicians who lived in southern Italy about 500 B.C. and believed that "all things are numbers." Actually, Plato used the fundamental role of

Pythagoras (c 530 BC) and his followers taught the round-earth idea and introduced the transparent concentric-spheres model of the heavens.

numbers only in his astronomy, while Aristotle avoided measurements. This is unfortunate because, as the Prologue reported, the arguments of Aristotle were adopted by Thomas Aquinas, whose philosophy did not include the idea of measurement of change as a tool of knowledge.

It is impossible to describe THE theory of the Greeks. There were many variations.

Plato's assumption that heavenly bodies move with "uniform motion in perfect circles" was accepted for many centuries by those working on the problem of planetary motion. Not until around 1600 A.D. was this assumption abandoned.



Plato and many other Greek philosophers assumed that there were a few basic elements that mixed together to cause the apparent variety observed in the world. Although not everyone agreed as to what these elements were, gradually four were accepted as the explanation of phenomena taking place on earth. These elements were: Fire, Air, Water and Earth. Because substances found on earth contained various mixtures of these elements, these compound substances would have a wide range of properties and changes.

Again stress the "two world" aspect of Greek thought - earth and sky.

In the heavens, which were separate from the earth and were the abode of the gods, perfection must exist. Therefore motions in the heavens must be eternal and perfect, and the only perfect unending geometrical form was the circle. Also, the unchanging heavenly objects could not be composed of elements normally found at or near the earth, but were composed of a changeless fifth element of their own—the quintessence.

Summary 5.5
1. The earth was assumed to be fixed at the center of the universe.

Plato's astronomical problem remained the most significant problem for theoretical astronomers for nearly two thousand years. To appreciate the later efforts and consequences of the different interpretation developed by Kepler, Galileo and Newton, let us examine what solutions to Plato's problem were developed by the Greeks.

Q12 What assumptions did Plato make in his problem?

Q13 Why is our knowledge of Greek science incomplete?

Q14 What basic assumption did the Greeks make about the nature of a theory?

5.5 <u>The first earth-centered solution</u>. The Greeks observed that the earth was obviously large, solid and permanent, while the heavens seemed to be populated by small, remote, ethereal objects that continuously went through their various motions. What was more natural than to conclude that our big, heavy earth was the steady, unmoving center of the universe? Such an earth-centered model is called <u>geocentric</u>. With it the daily motion of the stars could easily be explained: they were attached to, or were holes in, a large surrounding spherical dome, and were all at the same distance from us.

2. By assigning several circular motions or rotating spheres of various sizes to each of the five planets and to the sun and the moon, a model could be made to fit roughly the observed motions. The sequence was: Stars, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Earth.

T13 : Stellar motion   A: Celestial sphere model
T14 : Celestial sphere

Daily, this celestial sphere would turn around an axis through the earth. As a result all the stars on it would move in circular paths around the pole of rotation. In this way the daily motions could be explained.

The observed motion of the sun through the year was explained by use of a more complex model. To explain the sun's motion among the stars a separate invisible shell was needed that carried the sun around the earth. To explain the observed annual north-south motion of the sun the axis of this sphere for the sun should be tipped from the axis of the dome of the stars. (See Fig. 5.8.)

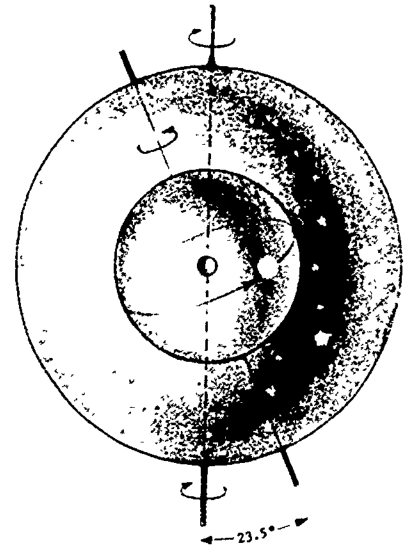


Fig. 5.8 The annual north-south (seasonal) motion of the sun was explained by having the sun on a sphere whose axis was tilted 23° from the axis of the sphere of the stars.

The motions of the planets—Mercury, Venus, Mars, Jupiter and Saturn—were more difficult to explain. Because Saturn moves most slowly among the stars, its sphere was assumed to be closest to the stars. Inside the sphere for Saturn would be spheres for faster-moving Jupiter and Mars. Since they all require more than a year for one trip around the sky, these three planets were believed to be beyond the sphere of the sun. Venus, Mercury and the moon were placed between the sun and the earth. The fast-moving moon was assumed to reflect sunlight and to be closest to the earth.

A: Armillary sphere

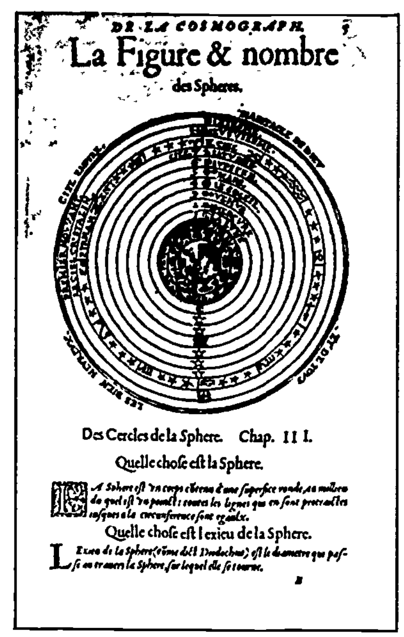


Fig. 5.9 A geocentric cosmological scheme. The earth is fixed at the center of concentric rotating spheres. The sphere of the moon (lune) separates the terrestrial region (composed of concentric shells of the four elements Earth, Water, Air and Fire) from the celestial region. In the latter are the concentric spheres carrying Mercury, Venus, Sun, Mars, Jupiter, Saturn and the stars. To simplify the diagram, only one sphere is shown for each planet. (From the DeGolyer copy of Petrus Apian's Cosmographie, 1551.)

Such an imaginary system of transparent shells or spheres can provide a rough "machine" to account for the general motions of heavenly objects. By choosing the sizes, rates and directions of motion of the supposed linkages between the various spheres a rough match could be made between the model and the observations (as in Fig. 5.9). If additional observations reveal other cyclic variations, more spheres and linkages can be added.

Eudoxus, Plato's friend, concluded that 27 spheres or motions would account for the general pattern of motions. Later Aristotle added 29 more motions to make a total of 56. An interesting description of this system is given by the poet Dante in the Divine Comedy, written in 1300 A.D., shortly after Aristotle's writings became known in Europe.

Aristotle was not happy with this system, for it did not get the heavenly bodies to their observed positions at quite the right times. In addition, it did not account at all for the observed variations in brightness of the planets. But we must not ridicule Greek science for being different from our science. The Greeks were just beginning the development of scientific theories and inevitably made assumptions that we now consider unsuitable. Their science was not "bad science," but in many ways it was a quite different kind of science from ours. Furthermore, we must

realize that to scientists 2000 years from now our efforts may seem strange and inept.

Even today scientific theory does not and cannot account for every detail of each specific case. As you have already seen, important general concepts, like velocity and acceleration, must be invented for use in organizing the observations. Scientific concepts are idealizations which treat selected aspects of observations rather than the totality of the raw data.

SG 5.3

This is an important point—do not overlook.

As you might expect, the history of science contains many examples in which the aspects neglected by one researcher turned out later to be quite important. But how would better systems for making predictions be developed unless there were first trials? Through tests and revisions theories may be improved or they may be completely replaced.

Q15 What is a geocentric system?

Q16 Describe the first solution to Plato's problem.

Summary 5.6

1. In the third century B.C. a quite different sort of theory was proposed, having the sun stationary with the earth and other planets revolving around it. All the daily motions observed in the sky were explained by a daily rotation of the earth.

2. This heliocentric system was not widely accepted for a variety of reasons. Some were philosophical, others were based on conflicts with expected results. The absence of the expected annual parallactic motion of the stars was the most significant conflict.

5.6 A sun-centered solution. For nearly two thousand years after Plato and Aristotle the geocentric model was generally accepted. However, another radically different model, based on different assumptions, had been proposed. In the third century B.C., Aristarchus, perhaps influenced by the writings of Heracleides, who lived a century earlier, suggested that a much simpler description of heavenly motion would result if the sun were considered to be at the center, with the earth, planets and stars all revolving around it. A sun-centered system is called heliocentric.

Unfortunately, because the major writings of Aristarchus have been lost, our knowledge of his work is based mainly on comments made by other writers. Archimedes wrote that Aristarchus found, from a long geometrical analysis, that the sun must be at least eighteen times farther away than the moon. Since the distance to the moon was known roughly, this put the sun several million miles away. Furthermore Aristarchus concluded that this distant sun must be much larger than the earth. He believed that the larger body, which was also the source of sunlight, should be at the center of the universe.

Aristarchus proposed that all the daily motions observed in the sky could be easily explained by assuming that the earth rotates daily on an axis. Furthermore, the annual changes in the sky, including the retrograde motions of the planets, could be explained by assuming that the earth and the five visible planets move around the sun. In this model

Additional information on Aristarchus' model
See Article Section.

the motion previously assigned to the sun around the earth was assigned to the earth moving around the sun. Also, notice that the earth became just one among several planets.

How such a system can account for the retrograde motions of Mars, Jupiter and Saturn can be seen from Figs 5.10 (a) and (b), in which an outer planet and the earth are assumed to be moving around the sun in circular orbits. The outer planet is moving more slowly than the earth. As a result, when we see the planet nearly opposite to the sun, the earth moves rapidly past the planet, and to us the planet appears to be moving backward, that is westward, or in retrograde motion, across the sky.

The heliocentric (sun-centered) hypothesis has one further advantage. It explains the bothersome observation that the planets are brighter and presumably nearer the earth during their retrograde motion.

Even so, the proposal by Aristarchus was attacked on three bases:

First, it did violence to the philosophical doctrines that the earth, by its very immobility and position, is different from the celestial bodies, and that the natural place of the earth is the center of the universe. In fact, his contemporaries considered Aristarchus impious for suggesting that the earth moved. Also, the new picture of the solar system contradicted common sense and everyday observations: certainly, the earth seemed to be at rest.

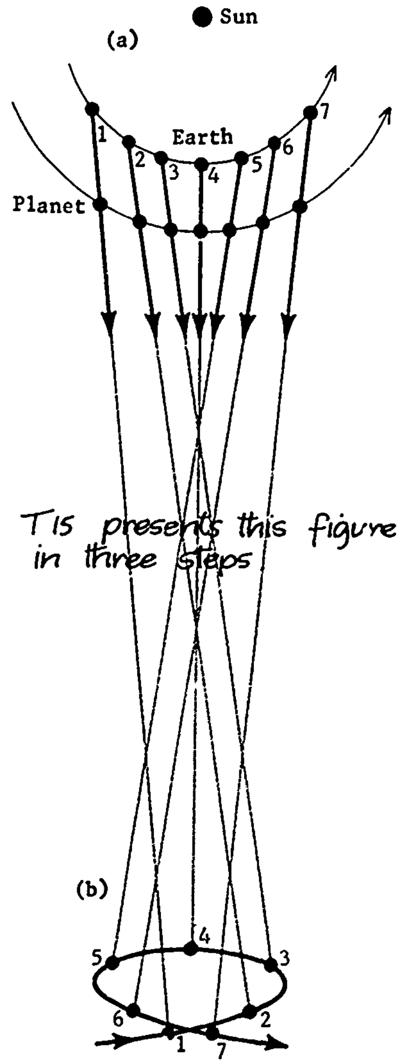
Second, the attackers offered observational evidence to refute Aristarchus. If the earth were moving in an orbit around the sun, it would also be moving back and forth below the fixed stars, such as the North Star. Then the angle from the vertical at which we have to look for any star would be different when seen from the various points in the earth's annual path (see Fig. 5.11). This shift, called the parallactic shift of the fixed stars, should occur on the basis of Aristarchus' heliocentric hypothesis. But it was not observed by the Greek astronomers.

This awkward fact could be explained in two ways. The stellar parallax could be too small to be observed with the naked eye, though this would require the stars to be either enormously distant, perhaps some hundreds of millions of miles away. Or the earth could be fixed, and the theory of Aristarchus was wrong.

Today with telescopes we can observe the parallactic shift of stars so we know that Aristarchus was correct. The

Fig. 5.10 Retrograde motion of an outer planet. The numbers in (b) correspond to the numbered positions in (a).

(a) Actual configurations of sun, earth and planet for retrograde motion.



T15 presents this figure in three steps

(b) Apparent path of the planet against the background stars as seen from the earth.

See Student Handbook for activity to help students understand parallax.

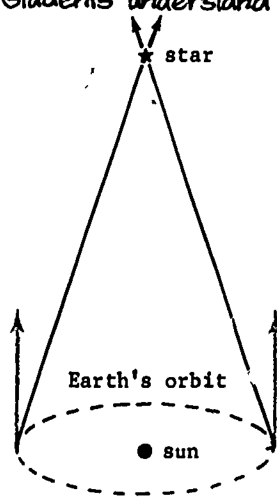


Fig. 5.11 How the changing position of the earth in its orbit should cause a parallactic shift in a star's position.

Parallax can be illustrated by having a student hold a finger vertically at arms length, and view, first with one eye, then the other, some object across the room — the finger will appear in a different place relative to the background.

stellar parallax is too small to be seen with the naked eye—and indeed so small that even with telescopes it was not measured until 1838. The largest parallactic shift is an angle of only 1.5 seconds of arc, equivalent to the diameter of a penny seen at a distance of about two miles! The parallax exists, but we can sympathize with the Greeks who rejected the heliocentric theory because the parallactic shift required by the theory could not be observed at that time.

Students should be cautioned to remember that the heliocentric idea was contrary to experience and to widely held assumptions which the geocentric model satisfied. No real empirical test could be used to decide between the two. To many students this possibility within science will be a shock. They have been led to believe that there is always at least one crucial experiment which will reveal the best conclusion.

Third, Aristarchus does not seem to have used his system for making predictions of planetary positions. His work seems to have been purely qualitative, a general picture of how things might be.

Here were two different ways to describe the same observations. But the new proposal required a drastic change in man's image of the universe—not to speak of the fact that the stellar parallax which it predicted was not observed. Actually Aristarchus' heliocentric hypothesis had so little influence on Greek thought that we might have neglected it here as being unimportant. But fortunately his arguments were recorded and eighteen centuries later stimulated the thoughts of Copernicus. Ideas, it seems, are not bound by space or time, and cannot be accepted or dismissed with final certainty.

Q17 What two radically new assumptions were made by Aristarchus?

Q18 How can the model proposed by Aristarchus explain retrograde motion?

Q19 What change predicted by Aristarchus' theory was not observed by the Greeks?

Q20 Why was Aristarchus considered impious?

Summary 5.7
Using several geometric devices, e.g., the eccentric, epicycle and equant, Ptolemy developed an earth-centered model in which each planet had a separate complex system of motions.

5.7 The geocentric system of Ptolemy. Disregarding the heliocentric model suggested by Aristarchus, the Greeks continued to develop their planetary theory as a geocentric system. As we have seen, the first solution in terms of crystalline spheres lacked accuracy. During the 500 years after Plato and Aristotle, astronomers began to want a more accurate theory for the heavenly timetables. Of particular importance were the positions of the sun, moon and planets along the ecliptic. A better theory must account for both the large general motions and the numerous smaller cyclic variations. To fit the observations, a complex theory was needed for each planet.

Several Greek astronomers made important contributions which resulted about 150 A.D. in the geocentric theory of Claudius Ptolemy of Alexandria. Ptolemy's book on the motions of the heavenly objects is a masterpiece of analysis, which used many new geometrical solutions.

Ptolemy wanted a system that would predict accurately the positions of each planet. But the type of system and motions he accepted was based on the assumptions of Aristotle. In the preface to The Almagest Ptolemy states:

> For indeed Aristotle quite properly divides also the theoretical [in contrast to the practical] into three immediate genera: the physical, the mathematical, and the theological....The kind of science which seeks after Him is the theological; for such an act can only be thought as high above somewhere near the loftiest things of the universe and is absolutely apart from sensible things. But the kind of science which traces through the material and the hot, the sweet, the soft, and such things, would be called physical, and such an essence... is to be found in corruptible things and below the the lunar sphere. And the kind of science which shows up quality with respect to forms and local motions, seeking figure, number, and magnitude, and also place, time, and similar things, would be defined as mathematical.

The Greeks made great progress in mathematics (Geometry) during the 500 years between Plato and Ptolemy. Ptolemy applied this "new math" in his efforts to predict the positions of the planets more accurately.

Then he defines the problem and states his assumptions:

> ...we wish to find the evident and certain appearances from the observations of the ancients and our own, and applying the consequences of these conceptions by means of geometrical demonstrations.
>
> And so, in general, we have to state, that the heavens are spherical and move spherically; that the earth, in figure, is sensibly spherical...; in position, lies right in the middle of the heavens, like a geometrical center; in magnitude and distance, [the earth] has the ratio of a point with respect to the sphere of the fixed stars, having itself no local motion at all.

Ptolemy then argues that each of these assumptions is necessary and fits with all our observations. The strength of his belief is illustrated by his statement: "...it is once for all clear from the very appearances that the earth is in the middle of the world and all weights move towards it." Notice that he has mixed the astronomical observations with the physics of falling bodies. This mixture of astronomy and physics became highly important when he referred to the proposal of Aristarchus that the earth might rotate and revolve:

> Now some people, although they have nothing to oppose to these arguments, agree on something, as they think, more plausible. And it seems to them there is nothing against their supposing, for instance, the heavens immobile and the earth as turning on the same axis [as the stars] from west to east very nearly one revolution a day....
>
> But it has escaped their notice that, indeed, as far as the appearances of the stars are concerned, nothing would perhaps keep things from

T16 helps explain use of the eccentric.

being in accordance with this simpler conjecture, but that in the light of what happens around us in the air such a notion would seem altogether absurd.

Here Ptolemy recognizes that the "simple conjecture" of a moving and rotating earth would perhaps satisfy the astronomical observations. But he rejects this conjecture by spelling out what would "happen around us in the air." The earth would spin at a great speed under the air, with the result that all clouds would fly past toward the west, and all birds or other things in the air would be carried away to the west. If, however, the air turned with the earth, "none the less the bodies contained in it would always seem to be outstripped by the movement of both [the earth and the air]."

Today we would use trigonometric terms - sines and cosines of angles and angle differences - where the Greeks used geometry.

On these assumptions and arguments Ptolemy developed very clever and rather accurate procedures by which the positions of each planet could be derived on a geocentric model. In the solutions he used circles and three other geometrical devices. Each device provided for variations in the rate of angular motion as seen from the earth. To appreciate Ptolemy's problem, let us examine one of the many small variations he was attempting to explain.

L10 - Retrograde motion, Geocentric Model, demonstrates how an epicycle can produce retrograde motion.

One irregularity that must be explained will be immediately apparent if you consult a calendar. Let us divide the sun's total path around the sky into four equal segments, each an arc of 90°, and start from where the sun's path crosses the celestial equator on March 21. Although there may be a variation of one day between years, due to the introduction of a "leap day," the sun is usually farthest north on June 21, back at the equator on September 23, then farthest south on December 22.

As Table 5.2 shows, the actual motion of the sun during the year is not at a uniform rate.

Table 5.2

Irregular Motion of the Sun in Moving through 90° Arcs

	Day count of year	Difference in days
March 21	80th day	
		92
June 21	172nd day	
		94
September 23	266th day	
		90
December 22	356th day	
		89
March 21 (80 + 365)	445th day	

Center a protractor on point C of Fig. 5.12 and measure the degrees in the arcs 1-2, 2-3, 3-4 and 4-1. Consider each 1° around C as one day. Make a graph of the days needed for the planet to move through the four arcs as seen from the earth, E.

Ptolemy was willing to sacrifice Plato's assumption of uniform angular motion at a constant distance around the earth in order to gain greater precision in his predictions.

A: Epicycle machine.

Do not stress the details of the various geometrical devices used by Ptolemy. It is more important to show his ability to modify the model to fit the increasingly more precise observations.

1. The eccentric. Previously, astronomers had held, with Plato, that the motion of a planet must be at a uniform angular rate and at a constant distance from the center of the earth. This is what we defined in Sec. 5.4 as uniform circular motion. Although Ptolemy believed that the earth was at the center of the universe, it need not be at the center around which the radius turned at a uniform rate. He used a new arrangement called an eccentric (Fig. 5.12) which has the radius of constant length moving uniformly around the center C, but with the earth E located off-center. As seen from the off-center earth, the planets, sun, etc., would require unequal numbers of days to move through the quadrants, 1-2, 2-3, etc.

An eccentric motion, as shown in Fig. 5.12, will account for the type of irregularity reported in Table 5.2. However, the scale of Fig. 5.12 is misleading; the earth need be off-set from the center by only a small amount to satisfy the data of Table 5.2. Notice that Ptolemy was giving up the old notion that the earth must be at the center of the motion.

2. The epicycle. While the eccentric can account for small variations in the rate of motion, it cannot describe any such radical change as retrograde motion. To account for retrograde motion, Ptolemy used another type of motion, the epicycle (see Fig. 5.13). The planet P is considered to be moving at a uniform rate on the circumference of a small circle, called the epicycle. The center of the epicycle D moves at a uniform rate on a large circle, called the deferent, around its center C.

With a relatively large radius or short period for the epicycle, the planet would be seen to move through loops. If from the center C we look out nearly in the plane of the motion, these loops would look like retrograde motions. Fig. 5.14 shows the motions produced by a simple mechanical model, an "epicycle machine."

An epicycle can be used to describe many kinds of motion. We may select the ratio of the radius of the epicycle to that of the deferent. Also we may choose the directions and rates of angular motion of the epicycle. To obtain apparent retrograde motion as seen from the center of the deferent, the epicycle must turn rapidly or have a radius which is a sizable fraction of the radius of the deferent.

To describe the three outer planets Ptolemy had to make a strange assumption. As Fig. 5.15 shows, he had to have the radius of each epicycle always parallel to the line from

A: Make photographs of epicycles like these. See Student Handbook.

Epicycles: See Resource p.79.

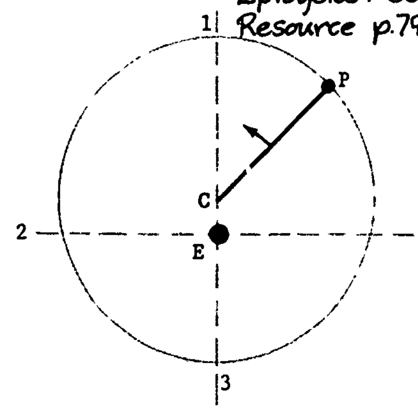


Fig. 5.12 An eccentric. The angular motion is at a uniform rate around the center, C. But the earth, E, is off-center.

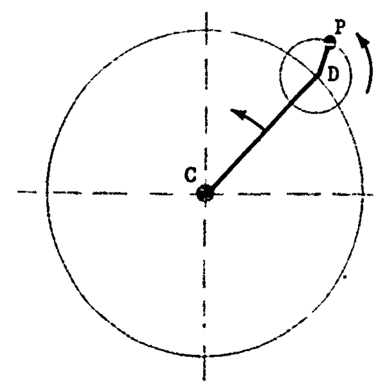
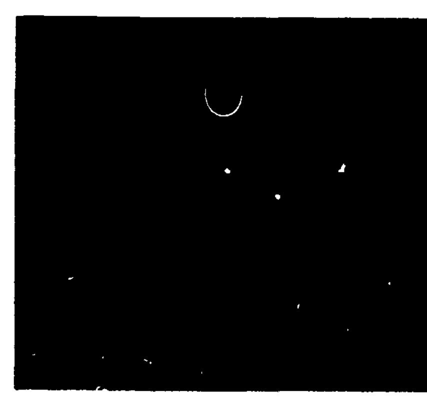
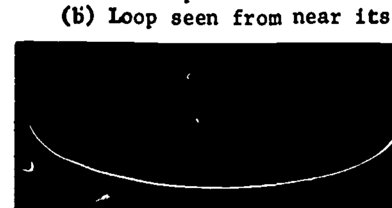


Fig. 5.13 An epicycle. The planet P revolves on its epicycle about D. D revolves on the deferent (large circle) centered at the Earth C.



(a)

Fig. 5.14 Retrograde motion created by a simple epicycle machine.
(a) Stroboscopic photograph of epicyclic motion. The flashes were made at equal time intervals. Note that the motion is slowest in the loop.
(b) Loop seen from near its plane.



(b)

The very large epicycle for Venus occupies about three fourths of the space between the earth and sun.

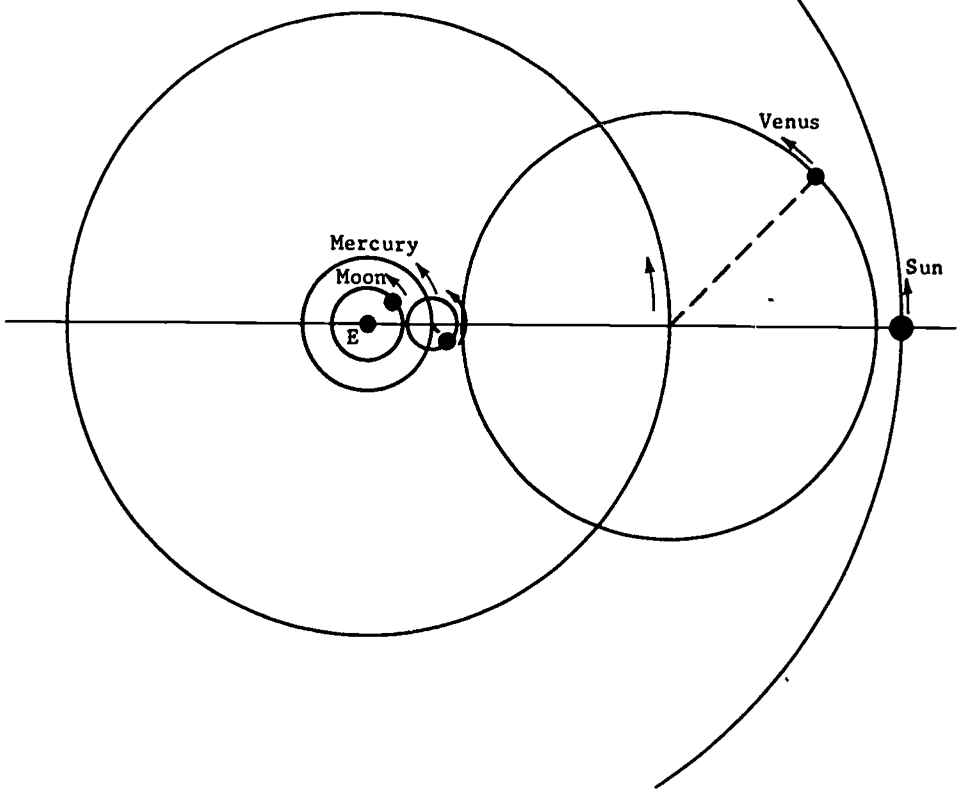
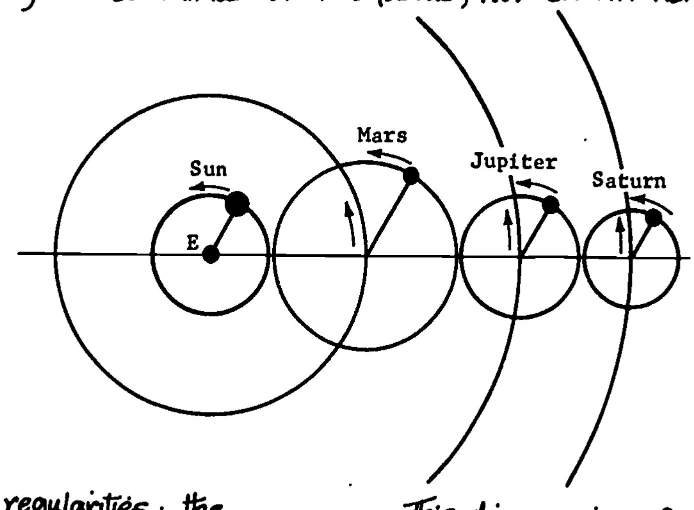


Fig. 5.15 Simplified representation of the Ptolemaic system. The scale of the upper drawing, which shows the earth and sun, is ten times that of the lower drawing, which shows the planets that are further than the sun. The planets are shown along a line to emphasize the relative sizes of the epicycles. The epicycles of the moon are not included.

Notice that each planet has only one epicycle. All other epicycles are represented by the eccentrics and equants, not shown here.



Note two suggestive regularities: the epicycles have one-year periods and are always parallel to the earth-sun radius.

This diagram is referred to in Chapter 6 with a discussion of how Copernicus replaced all the epicycles with one anual motion of the earth.

the sun to the earth. This required that each epicycle have a period of exactly one year. As we shall see in Ch. 6, Copernicus wondered about this limitation of the epicycles.

T16 helps explain use of the equ

Not only may the use of an epicycle describe retrograde motion, it also could explain the greater brightness of the planets when they were near opposition, as you can see from Fig. 5.14. However, to accept this explanation of the changes in brightness would oblige us to assume that the planets actually moved through space on epicycles and deferents. This assumption of "real motions" is quite different from that of considering the epicycles and deferents as only useful computing devices, like algebraic equations.

3. The equant. But even with combinations of eccentrics and epicycles Ptolemy was not able to fit the motions of the five planets. There were more variations in the rates at which the planets moved than could be fitted by eccentrics and epicycles. For example, as we see in Fig. 5.16, the retrograde motion of Mars is not always of the same angular size or duration. To allow for these motions Ptolemy introduced a third geometrical device, called the equant (Fig. 5.17), which is a modified eccentric. The uniform angular motion is around an off-center point C, while the earth E (and the observer) is equally off-center, but in the opposite direction.

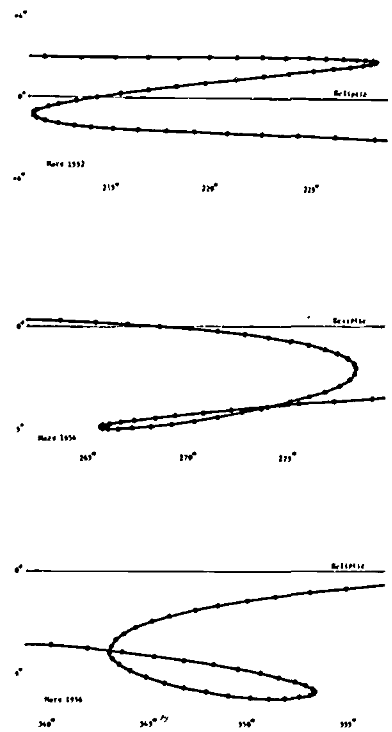


Fig. 5.16 Mars is plotted at four-day intervals on three consecutive oppositions. Note the different sizes and shapes of the retrograde curves.

Although Ptolemy displaced the earth from the center of the motion, he always used a uniform rate of angular motion around some center. To that extent he stayed close to the assumptions of Plato. By a combination of eccentrics, epicycles, and equants he described the motion of each planet separately. His geometrical analyses were equivalent to a complicated equation for each individual planet. Ptolemy did not picture these motions as an interlocking machine where each planet moved the next. However, Ptolemy adopted the old order of distances: stars, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Earth. Because there was no information about the distances of the planets, the orbits are usually shown nested inside each other so that their epicycles do not quite overlap (see Fig. 5.15).

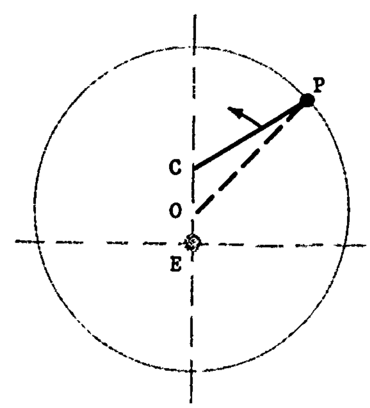


Fig. 5.17 An equant. The planet P moves at a uniform rate around the off-center point C. The earth E is equally off-center in the opposite direction.

Notice how radically this set of geometrical motions differed from the propositions of Plato stated 500 years earlier. Although Ptolemy used uniform angular motions and circles, the centers and radii of these motions could now be adjusted and combined to provide the best fit with observations. No longer was the center of the motion at the earth,

but the center could be offset by whatever amount was needed. For each planet separately Ptolemy had a combination of motions that predicted its observed positions over long periods of time to within about two degrees.

There is little evidence that anyone believed that the planets actually moved through space in paths described by Ptolemy.

However, there were some difficulties. For example, his proposed motions for the moon involved such large epicycles that during a month the observed angular diameter of the moon should change by a factor of two.

The Ptolemaic description was a series of mathematical devices to match and predict the motion of each planet separately. A recently discovered manuscript of Ptolemy's describes his picture of how the planet orbits were related in a way similar to that shown in Fig. 5.15. Nevertheless, in the following centuries most people—including Dante—believed that the planets really moved on some sort of crystalline spheres as Eudoxus had suggested, but that the motions were described mathematically by Ptolemy's combinations of geometric devices.

In Ptolemy's theory of the planetary motions there were, as in all theories, a number of assumptions:

1. that the heaven is spherical in form and rotates around the earth once a day;
2. that the earth is spherical;
3. that the earth is at the center of the heavenly sphere;
4. that the size of the earth is negligible compared to the distance to the stars;
5. that the earth has no motions;
6. that uniform angular motion along circles is the only proper behavior for celestial objects.

Although now discarded, the Ptolemaic system, proposed in 150 A.D., was used for about 1500 years. What were the major reasons for this long acceptance? Ptolemy's theory:

1. predicted fairly accurately the positions of the sun, moon and planets;
2. did not predict that the fixed stars should show a parallactic shift;•
3. agreed in most details with the philosophical doctrines developed by the earlier Greeks, including the idea of "natural motion" and "natural place";
4. had commonsense appeal to all who saw the sun, moon, planets and stars moving around them;
5. agreed with the comforting assumption that we live on an immovable earth at the center of the universe.

• In fact, it was the positive prediction that there would be no parallax.

Also:

6. could survive better because at that time there were very few theoretical astronomers.
7. fitted in with Thomas Aquinas' widely accepted synthesis of Christian belief and Aristotelian physics. SG 5.4

Yet Ptolemy's theory was eventually displaced by a heliocentric one. Why did this occur? What advantages did the new theory have over the old? From our present point of view, on what basis do we say that a scientific theory is successful or unsuccessful? We shall have to face such persistent questions in what follows.

Q21 What assumptions did Ptolemy make for his theory?

Q22 What arguments did Ptolemy use against the idea that the earth rotated?

Q23 What limitation did Ptolemy have to assign to his epicycles?

Q24 Was the Ptolemaic system proposed as a description of real planetary orbits, or only as a means for computing positions?

Q25 In what way did Ptolemy disregard the assumptions of Plato?

Q26 Why was Ptolemy's system accepted for more than a thousand years?

Ptolemy wrote the Tetrabilos, which is a handbook for making and interpreting astrological predictions. The increased desire for more accurate predictions of planetary positions, both past and future, arose in part from a greater acceptance of astrology as a way of explaining the behavior and personalities of people.

Study Guide

5.1 a) List the observations of the motions of heavenly bodies that you might make which would have been possible in ancient Greek times. *Diurnal, Annual*

b) For each observation, list some reasons why the Greeks thought these motions were important. *Discussion*

5.2 Describe the apparent motions of the stars and their times of rising and setting if the earth's shape were:

a) saucer-shaped,
b) flat,
c) a pyramid or cube,
d) a cylinder having its axis north-south. *Discussion*

5.3 Throughout Chapter 5, many references are made to the importance of recording observations accurately.

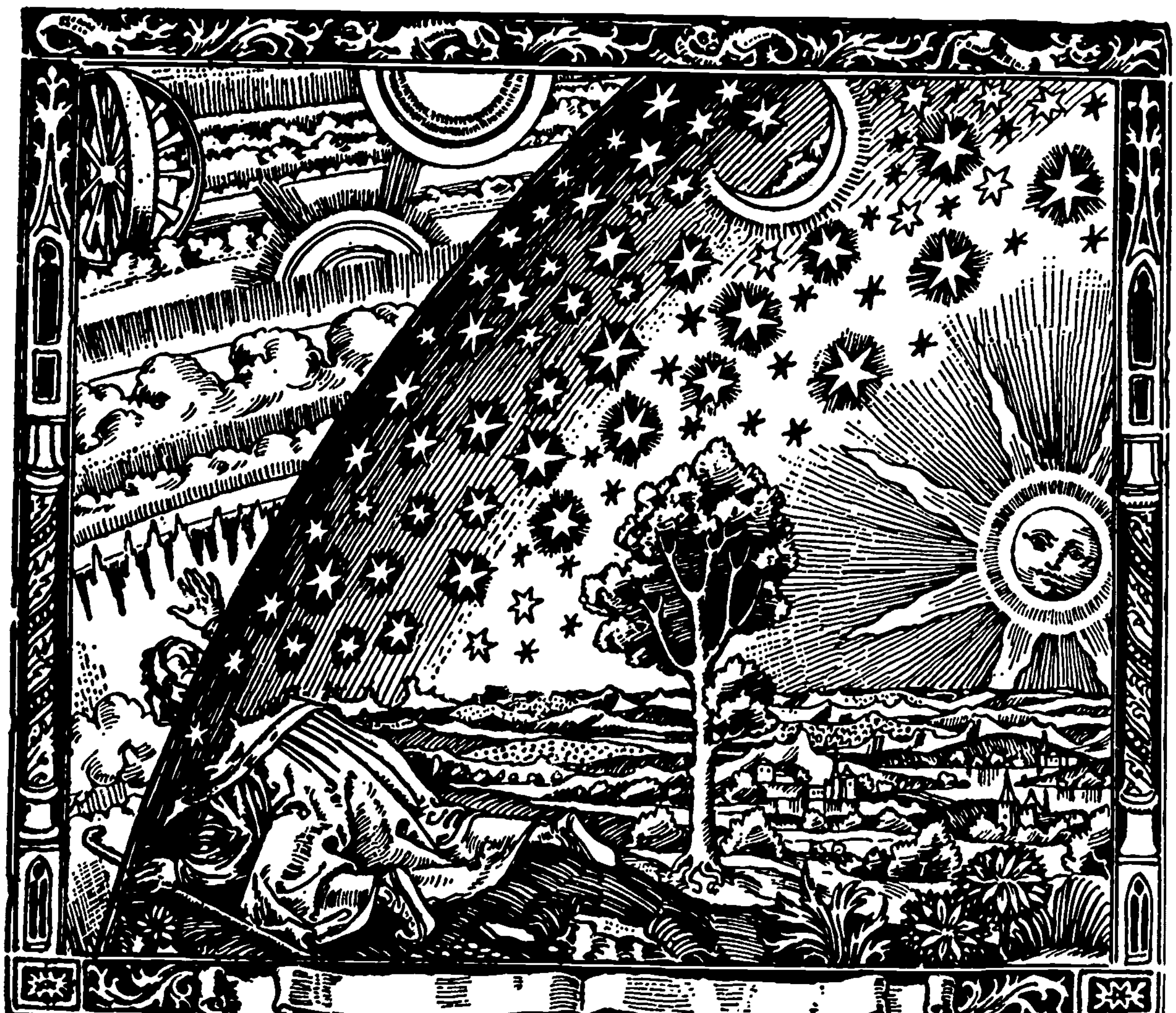
a) Why is this so important in astronomy?
b) Why are such records more important in astronomy than in other areas of physics you have already studied? *Discussion*

5.4 As far as the Greeks were concerned, and indeed as far as we are concerned, a reasonable argument can be made for either the geocentric or the heliocentric theory of the universe.

a) In what ways were both ideas successful?
b) In terms of Greek science, what are some advantages and disadvantages of each system?
c) What were the major contributions of Ptolemy? *Discussion*

Chapter 6 Does the Earth Move? — The Works of Copernicus and Tycho

This chapter summarizes the work of Copernicus and relates it to the Almagest of Ptolemy. Tycho is presented as the first major astronomical observer of Europe.



Man breaking through the vault of the heavens to new spheres. (Woodcut, circa 1530.)

6.1 The Copernican system. Nicolaus Copernicus (1473-1543) was still a young student in Poland when Columbus discovered America. Copernicus was an outstanding astronomer and mathematician, and also was a talented and respected churchman, jurist, administrator, diplomat, physician, classicist and economist. During his studies in Italy he learned Greek and read the writings of earlier philosophers and astronomers. As a canon of the Cathedral of Frauenberg he was busy with civic and church affairs, but increasingly he worked on his astronomical studies. On the day of his death in 1543, he saw the first copy of his great book which opened a whole new vision of the universe.
6.1).

Fig. 6.1 Nicolaus Copernicus (1473-1543). In Polish his name was Kopernik.

Copernicus titled his book De Revolutionibus Orbium Coelestium, or On the Revolutions of the Heavenly Spheres, which suggests an Aristotelian notion of concentric spheres. Copernicus was indeed concerned with the old problem of Plato: the construction of a planetary system by combinations of the fewest possible uniform circular motions. He began his study to rid the Ptolemaic system of the equants which were contrary to Plato's assumptions. In his words, taken from a short summary written about 1530,

> ...the planetary theories of Ptolemy and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind.
>
> Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uniformly about its proper center, as the rule of absolute motion requires.

In De Revolutionibus he wrote:

> We must however confess that these movements [of the sun, moon, and planets] are circular or are composed of many circular movements, in that they maintain these irregularities in accordance with a constant law and with fixed periodic returns, and that could not take place, if they were not circular. For it is only the circle which can bring back what is past and over with....
>
> I found first in Cicero that Nicetas thought that the Earth moved. And afterwards I found in Plutarch that there were some others of the same opinion.... Therefore I also...began to meditate upon the mobility of the Earth. And although the opinion seemed absurd, nevertheless, because I knew that others before me had

Summary 6.1

1. Copernicus was motivated in part by a desire to avoid the use of the equant in the Ptolemaic system.

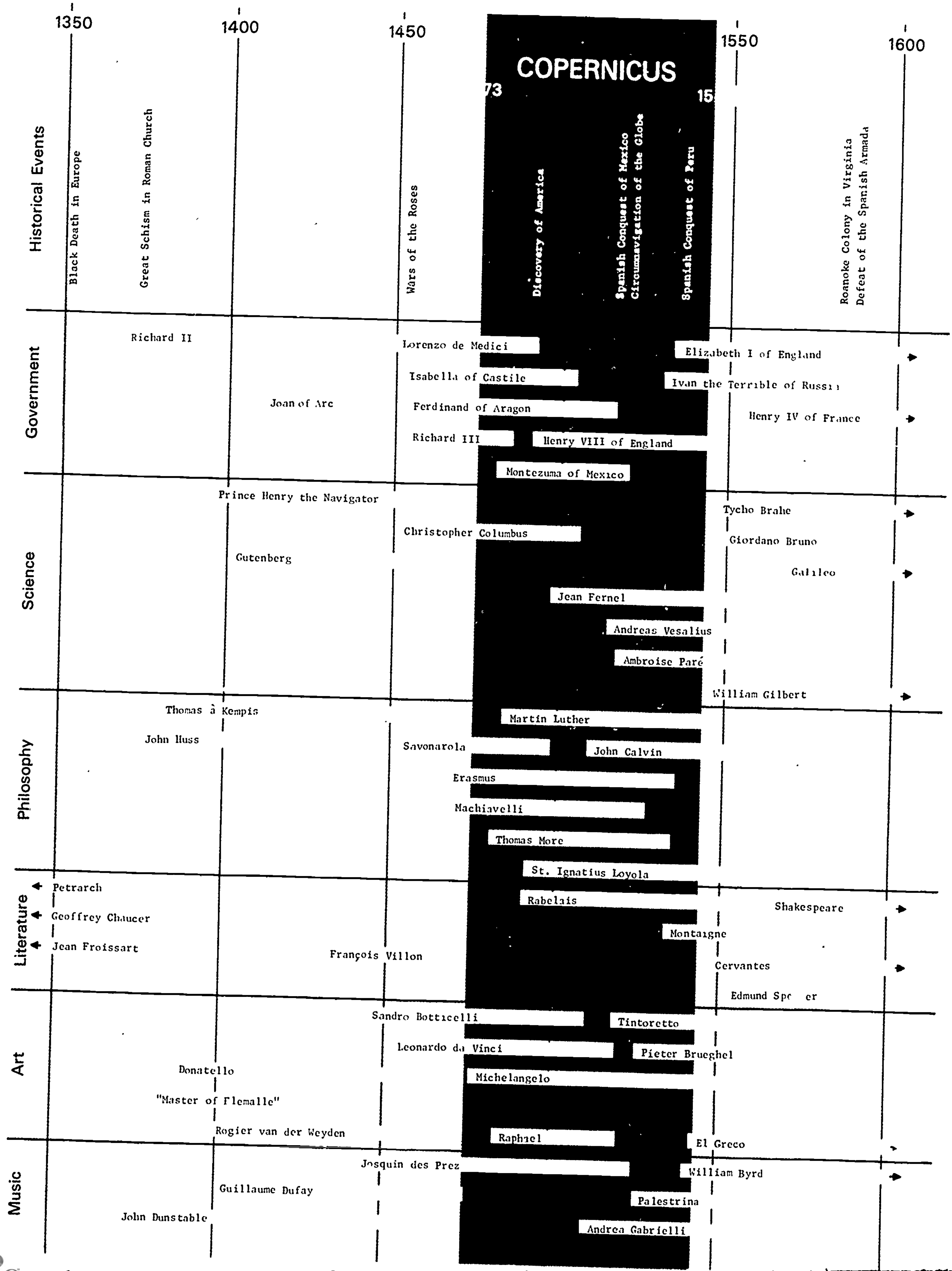
2. Copernicus knew that Aristarchus had proposed a heliocentric system as was described in the Almagest.

3. Copernicus was delighted by the symmetry provided by the heliocentric model.

What was "the rule of absolute motion"?

Copernicus was one of the few men of his time who learned Greek. As a result he was able to read some of the newly discovered Greek manuscripts. From them he learned the arguments for a sun-centered system, as well as the arguments given by Ptolemy against such a system.

You might ask your students which languages they think would be valuable to a modern scientist: Chinese? German? Russian?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1400
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Historical Events
Black Death in Europe
Great Schism in Roman Church
Wars of the Roses
Discovery of America
Spanish Conquest of Mexico
Circumnavigation of the Globe
Spanish Conquest of Peru
Roanoke Colony in Virginia
Defeat of the Spanish Armada
Government
Richard II
Lorenzo de Medici
Elizabeth I of England
Isabella of Castile
Ivan the Terrible of Russia
Joan of Arc
Ferdinand of Aragon
Henry IV of France
Richard III
Henry VIII of England
Montezuma of Mexico
Science
Prince Henry the Navigator
Tycho Brahe
Christopher Columbus
Giordano Bruno
Gutenberg
Galileo
Jean Fernel
Andreas Vesalius
Ambroise Paré
William Gilbert
Philosophy
Thomas à Kempis
Martin Luther
John Huss
Savonarola
John Calvin
Erasmus
Machiavelli
Thomas More
St. Ignatius Loyola
Literature
Petrarch
Rabelais
Geoffrey Chaucer
Shakespeare
Montaigne
Jean Froissart
François Villon
Cervantes
Edmund Spenser
Art
Sandro Botticelli
Tintoretto
Leonardo da Vinci
Pieter Brueghel
Donatello
Michelangelo
"Master of Flemalle"
Rogier van der Weyden
Raphael
El Greco
Music
Josquin des Prez
William Byrd
Guillaume Dufay
Palestrina
John Dunstable
Andrea Gabrielli

been granted the liberty of constructing whatever circles they pleased in order to demonstrate astral phenomena, I thought that I too would be readily permitted to test whether or not, by the laying down that the Earth had some movement, demonstrations less shaky than those of my predecessors could be found for the revolutions of the celestial spheres....I finally discovered by the help of long and numerous observations that if the movements of the other wandering stars are correlated with the circular movement of the Earth, and if the movements are computed in accordance with the revolution of each planet, not only do all their phenomena follow from that but also this correlation binds together so closely the order and magnitudes of all the planets and of their spheres or orbital circles and the heavens themselves that nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole.

After nearly forty years of study Copernicus proposed a system of more than thirty eccentrics and epicycles which would "suffice to explain the entire structure of the universe and the entire ballet of the planets." Like the Almagest, De Revolutionibus uses long geometrical analyses and is difficult to read. Examination of the two books strongly suggests that Copernicus thought he was producing an improved version of the Almagest. He used many of Ptolemy's observations plus a few more recent ones. Yet his system, or theory, differed from that of Ptolemy in several fundamental ways. Like all scientists, Copernicus made many assumptions as the basis for his system:

"1. There is no one center of all the celestial circles or spheres.

2. The center of the earth is not the center of the universe, but only of [gravitation] and of the lunar sphere.

3. All the spheres revolve about the sun...and therefore the sun is the center of the universe.

4. The ratio of the earth's distance from the sun to the [sphere of the stars] is so much smaller than the ratio of the earth's radius to its distance from the sun that the distance from the earth to the sun is imperceptible in comparison with the [distance to the stars].

5. Whatever motion appears in the [sky] arises not from any motion of the [sky], but from the earth's motion. The earth together with its [water and air] performs a complete rotation on its fixed poles in a daily motion, while the [sky remains] unchanged.

6. What appear to us as motions of the sun arise not from its motion but from the motion of the earth and...we revolve about the sun like any other planet. The earth has, then, more than one motion.

See the Preface to Copernicus' Revolutionibus in Project Physics Reader 2.

In the first quotation Copernicus makes his case against the system of Ptolemy. Develop the point that Copernicus was attempting to purify the Ptolemaic system by eliminating the use of the equant. By having the planets move around the sun he was able to eliminate the major epicycle from the Ptolemaic system. Yet to account for other smaller variations in the motions of the planets, he still had to include eccentrics and small epicycles not shown in the figures.

LII: Retrograde motion, heliocentric model.

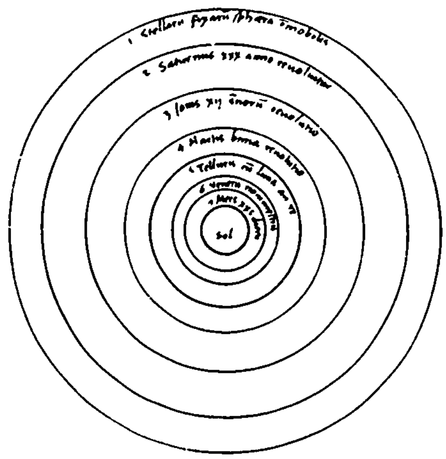


Fig. 6.2 Copernicus' diagram of his heliocentric system. (From his manuscript of De Revolutionibus, 1543.) This simplified representation omits the many epicycles actually used in the system.

Figure 6.2 having the sun (sol) at the center, is only a rough sketch of the major idea of the Copernican system. Figure 6.5 suggests the variety of motions actually used.

7. The apparent retrograde...motion of the planets arises not from their motion but from the earth's. The motions of the earth alone, therefore, are sufficient to explain so many apparent [motions] in the [sky]."

Comparison of this list with the assumptions of Ptolemy, given in Chapter 5, will show some identities as well as some important differences.

Notice that Copernicus proposed that the earth rotates daily. As Aristarchus and others had realized, this rotation would account for all the daily risings and settings observed in the sky. Also Copernicus proposed, as Aristarchus had done, that the sun was stationary and the center of the universe. The earth, like the other planets, moved about some point near the sun. Thus the Copernican system is a heliostatic (fixed sun) system in which the sun is located near, but not at, the various centers around which the planets moved.

Figure 6.2 shows the main concentric spheres carrying the planets around the sun. His text explains the outlines of the system:

> The ideas here stated are difficult, even almost impossible, to accept; they are quite contrary to popular notions. Yet with the help of God, we will make everything as clear as day in what follows, at least for those who are not ignorant of mathematics....
>
> The first and highest of all the spheres is the sphere of the fixed stars. It encloses all the other spheres and is itself self-contained; it is immobile; it is certainly the portion of the universe with reference to which the movement and positions of all the other heavenly bodies must be considered. If some people are yet of the opinion that this sphere moves, we are of a contrary mind; and after deducing the motion of the earth, we shall show why we so conclude. Saturn, first of the planets, which accomplishes its revolution in thirty years, is nearest to the first sphere. Jupiter, making its revolution in twelve years, is next. Then comes Mars, revolving once in two years. The fourth place in the series is occupied by the sphere which contains the earth and the sphere of the moon, and which performs an annual revolution. The fifth place is that of Venus, revolving in nine months. Finally, the sixth place is occupied by Mercury, revolving in eighty days.
>
> In the midst of all, the sun reposes, unmoving. Who, indeed, in this most beautiful temple would place the light-giver in any other part than that whence it can illumine all other parts...?

E15: The shape of the earth's orbit

> In this ordering there appears a wonderful symmetry in the world and a precise relation between the motions and sizes of the spheres which no other arrangement offers.

The period of the outer planets around the sun can be estimated by dividing its number of observed cycles in the sky by the number of years of observation.

Q1 What reason did Copernicus give for rejecting the use of equants?

Q2 What could Copernicus have meant when he said, "There is no one center of all the celestial circles or spheres," and yet "All the spheres revolve around the sun as their mid-point, and therefore the sun is the center of the universe"?

Q3 In the following table mark with a P the assumptions made by Ptolemy, and with a C those made by Copernicus.

a) The earth is spherical.

b) The earth is only a point compared to the distances to the stars.

c) The heavens rotate daily around the earth.

d) The earth has one or more motions.

e) Heavenly motions are circular.

f) The observed retrograde motion of the planets results from the earth's motion around the sun.

6.2 New conclusions. As often happens in science, a new way of looking at the observations—a new theory—leads to new types of conclusions. Copernicus used his moving-earth model to get two results not possible with the Ptolemaic theory. He found the periods of motion of each planet around the sun. Also he found the distance of each planet from the sun in terms of the distance of the earth from the sun. The distance between the earth and sun is known as the astronomical unit (A.U.).

If a body is observed to make 6 full cycles of the sky in 24 years, what approximately is its orbital period?

Summary 6.2

1. A new way of looking at observations, that is, a new theory, often leads to new types of conclusions.

2. From his model Copernicus found the periods of the planets' motions around the sun, and also their relative distances from the sun.

To get the periods of the planets around the sun Copernicus used observations that had been recorded over many years. For the outer planets, Mars, Jupiter and Saturn, he found the average number of years needed for the planet to make one trip around the sky, as Table 6.1 shows. When averaged over many years the period is rather close to the planet's actual orbital period.

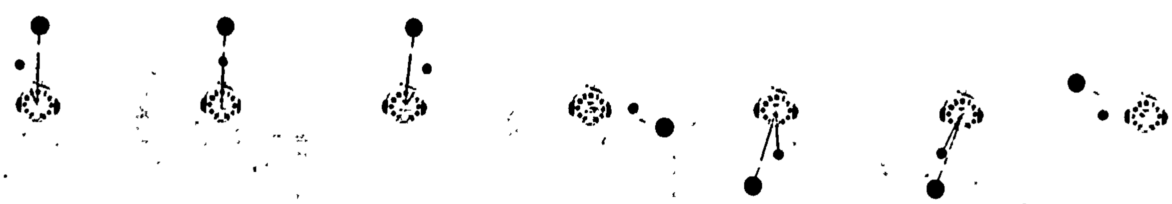
Table 6.1 Copernicus' Derivation of the Period of Mars, Jupiter and Saturn around the Sun.

Planet	Years of Obs.	Cycles among the Stars	Ratio	Period Copernicus'	Period Modern
Mars	79	42	79/42	687d	687.0d
Jupiter	71	6	71/6	11.8y	11.86y
Saturn	59	2	59/2	29.5y	29.46y

For the quick-moving inner planets, Mercury and Venus, the procedure of Copernicus had a form which we call the "chase problem." As an example of such a chase, consider the hour and minute hands of a clock or watch, as shown in

Fig. 6.3 Clock analogy of the "chase" problem. The small disk, representing the earth, is on an extension of the minute hand. The larger disk, representing a planet, is on an extension of the hour hand. The sun is at the center. The sequence shows the earth overtaking and passing the planet.

(a) 11:55 (b) 12:00 (c) 12:05 (d) 3:15 (e) 6:30 (f) 6:35 (g) 9:45



Additional information. See Article Section.

Fig. 6.3. If you were riding on the long hour hand shown in Fig. 6.3, how many times in 12 hours would the minute hand pass between you and the center? If you are not certain, slowly turn the hands of a clock or watch and keep count. From this information, can you derive a relation by which you would conclude that the period of the minute hand around the center was one hour?

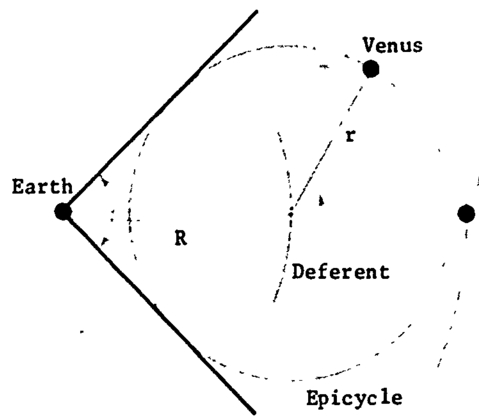


Fig. 6.4(a) The orbit of Venus according to Ptolemy. The maximum angle between the sun and Venus is about 45° east and west of the line connecting the fixed earth and the revolving sun. $r = 0.7R$

Now for a planetary application. We assume that Mercury and Venus are closer than the earth is to the sun, and that they have orbital periods less than one year. Because the earth is moving in the same direction as the planets, they have to chase the earth to return to the same apparent position in the sky—such as being farthest eastward from the sun. We can solve such a chase problem by counting for an interval of T years the number of times N a planet attains some particular position relative to the sun. The actual number of trips the planet has made around the sun in this interval of T years is the sum of N and T. The planet's period, years per revolution, is then $T/(T + N)$. From observations available to him Copernicus formed the ratios $T/(T + N)$ and found the periods shown in Table 6.2. His results were remarkably close to our present values.

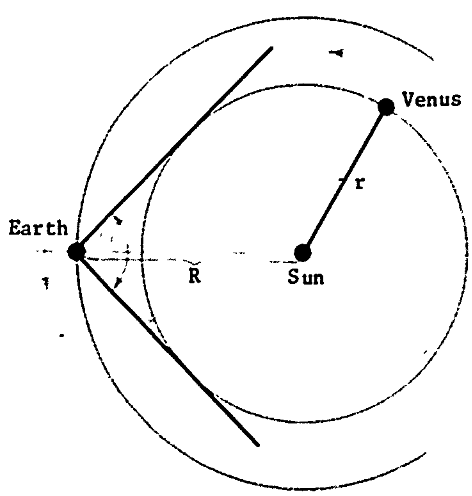


Fig. 6.4(b) The orbit of Venus, according to Copernicus. With the earth in orbit around the sun the same maximum angle between Venus and the earth-sun line is observed. Here $r = 0.7R$ in astronomical units.

Table 6.2 Copernicus' Derivation of Periods of Mercury and Venus around the Sun

Planet	Years of Orbits T	Number of Times Farthest East of the Sun N	(T+N)	$\frac{T}{(T+N)}$	Period Copernicus'	Period Modern
Mercury	46	145	191	46/191y	88d	88.0d
Venus	8	5	13	8/13y	224d	224.7d

For the first time in history, Copernicus was able to derive distances to the planets in terms of the distance of the earth from the sun (the astronomical unit). Remember that the Ptolemaic system had no distance scale; it provided only a way of deriving the directions to the planets. If, as Copernicus proposed, the earth moved around the sun, the distances of the inner planets to the sun could be found in terms of the earth's distance, as Fig. 6.4 indicates, from their maximum angles from the sun. The values found by Copernicus (with the modern values in parentheses) are: Venus 0.72 (0.72 A.U.), Mercury 0.38 (0.39 A.U.). The earth's distance from the sun has been taken as 1.00, or one A.U.

Might be a good place to point out the arbitrariness of units. The AU is a perfectly good unit for local astronomy - even better than the m or km. Until recently, the conversion factor AU/km was not well known.

In the Copernican system, the large epicycles of Ptolemy, shown in Fig. 5.15, p. 22, were replaced by the orbit of

the earth. The radius r of Ptolemy's epicycle was given by Copernicus in terms of the radius R of the deferent, which was taken as 10,000 (see Table 6.3). From these numbers we can find for each planet the ratio of the radius of the deferent to that of the epicycle; these are listed in the third column of Table 6.3.

As Figs. 6.5(a) and (b) show, we can in imagination expand the scale of the planet's orbit (according to Ptolemy), both the deferent and epicycle, until the radius r of the epicycle is the same size as the orbit of the sun about the earth (or the earth about the sun). Figure 6.5c shows that we can then displace all three bodies: planet, sun and earth, along parallel lines and through equal distances. By this displace-

A: Retrograde motion

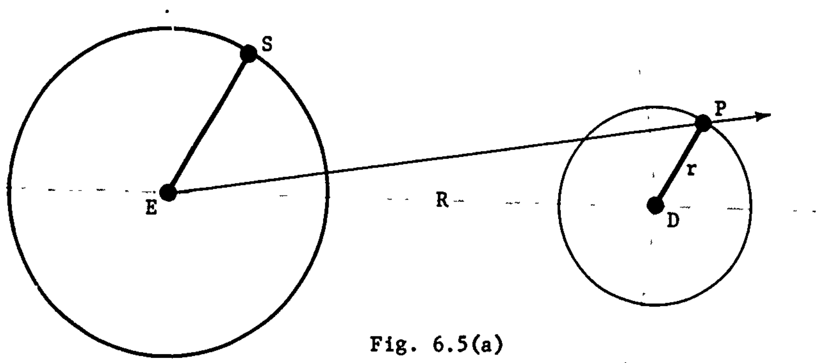


Fig. 6.5(a)

Fig. 6.5(a) The orbit of the sun S around the earth E and the deferent and epicycle of an outer planet P, as shown in Fig. 5.15.

(b) The deferent and epicycle are enlarged while maintaining the same maximum angle of displacement for the epicycle until the epicycle has the same radius, r', as ES, the earth-to-sun distance.

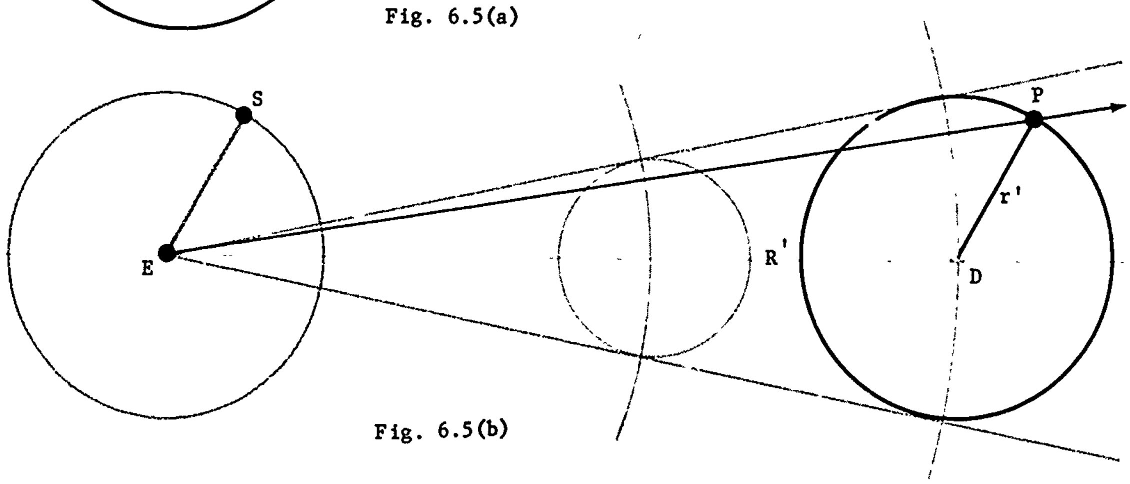


Fig. 6.5(b)

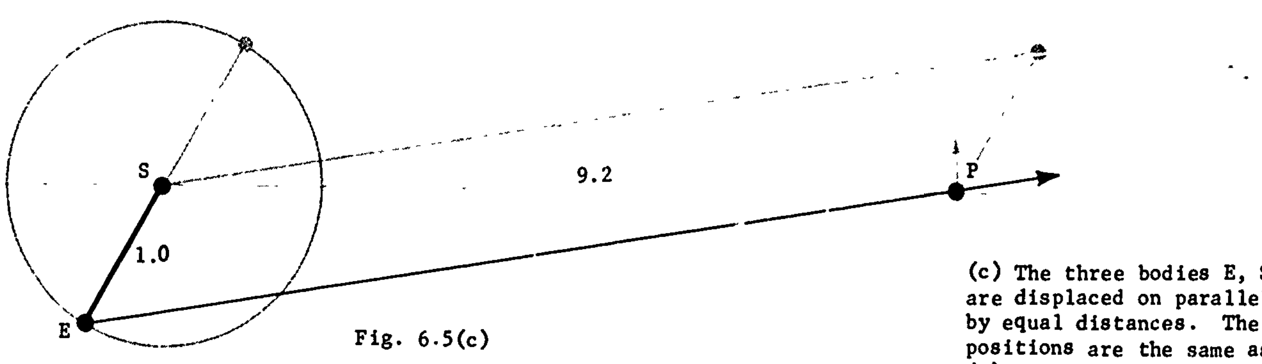


Fig. 6.5(c)

(c) The three bodies E, S and P are displaced on parallel lines by equal distances. The relative positions are the same as in part (a), but now the sun is the center of the system and the earth's orbit replaces the epicycle of the planet P.

A: Planetary positions relative to the sun

ment we move the earth from the center of all the motions and put the sun at the center. Also, we have eliminated the planet's epicycle and replaced it by the orbit of the earth. In addition, we have changed the frame of reference and put the sun instead of the earth at the origin of the coordinate system and the center of the planetary motions. Now the model resembles the Copernican system. Furthermore, the relative distance from the sun to each planet is set.

Table 6.3 The Sizes of Planetary Orbits

	Ptolemy's Ratios Deferent/Epicycle (R/r)	Copernicus' R, when r = 1	Modern Values
Saturn	10,000/1090	9.2	9.54 A.U.
Jupiter	10,000/1916	5.2	5.20
Mars	10,000/6580	1.52	1.52
Earth	(one astronomical unit)	1.00	1.00
Venus		0.72	0.72
Mercury		0.38	0.39

Q4 What new results did Copernicus obtain with a moving-earth model which were not possible with a geocentric model for the planetary system?

6.3 Arguments for the Copernican System. Since Copernicus knew that to many his work would seem absurd, "nay, almost contrary to ordinary human understanding," he tried to meet the old arguments against a moving earth in several ways.

Note here the types of arguments used by Copernicus. The only new observation predicted by the sun-centered system was the annual parallax of the stars - which was not observed. Even Ptolemy knew from the work of Aristarchus, Section 5.6, that the distance between the earth and sun was at least several million miles. If the stars showed no annual parallactic shift, the stars would have to be at an incomprehensibly large distance.

1. Copernicus argued that his assumptions agreed with dogma at least as well as Ptolemy's. Copernicus has many sections on the limitations of the Ptolemaic system (most of which had been known for centuries). Other sections pointed out how harmonious and orderly his own system seems and how pleasingly his system reflects the mind of the Divine Architect. To Copernicus, as to many scientists, the complex events they saw were but symbols of the working of God's mind. To seek symmetry and order in the observed changes was to Copernicus an act of reverence. To him the symmetry and order were renewed proof of the existence of the Deity. As a highly placed and honored church dignitary, he would have been horrified if he had been able to foresee that his theory would contribute to the sharp clash, in Galileo's time, between religious dogma and the interpretations that scientists gave to their experiments.

2. Copernicus' analysis was as thorough as that of Ptolemy. He calculated the relative radii and speeds of the circular motions in his system so that tables of planetary motion could be made. Actually the theories of

Ptolemy and Copernicus were about equally accurate in predicting planetary positions, which for both theories often differed from the observed positions by as much as 2°, or four diameters of the moon.

3. Copernicus cleverly tried to answer several objections that were certain to be raised against his heliocentric system—as they had been, long ago, against that of Aristarchus. To the argument that the earth, rotating so rapidly about its own axis, would surely burst like a flywheel driven too fast, he asked, "Why does the defender of the geocentric theory not fear the same fate for his rotating celestial sphere—so much faster because so much larger?" To the argument that birds in flight and clouds should be left behind by the rapidly rotating and revolving earth, he answered that the atmosphere is dragged along with the earth.

Summary 6.3
Among Copernicus' arguments for his system:
1) It was just as in keeping with reverence for God as was Ptolemy's.
2) It was as accurate as Ptolemy's.
3) Certain "common sense" objections were easily answered.
4) The absence of observable stellar parallax showed only the immense distance of the stars.
5) His system was beautifully simple.

To the old question of the absence of parallax for the fixed stars, he could only give the same answer as Aristarchus:

SG 6.1

A: Proof that the earth rotates

> ...the dimensions of the world [universe] are so vast that though the distance from the sun to the earth appears very large as compared with the size of the spheres of some planets, yet compared with the dimensions of the sphere of the fixed stars, it is as nothing.

However, would you expect that those who believed in a small earth-centered universe would be persuaded that the stars were far away because their parallax was <u>not</u> observed? The argument was logical, but not convincing.

You might mention Tycho's comment that he could not see why God would have created so much space only to leave it empty.

4. Copernicus claimed that the greatest advantage of his scheme was its simple description of the general motions of the planets. Figure 5.7, p. 11, shows how the retrograde motions will appear from a moving earth.

Yet for computations, because Copernicus would not use equants, he needed more small motions than did Ptolemy to account for the observations.

Q5 What arguments did Copernicus use in favor of his system?

Q6 What were the largest differences between observed planetary positions and those predicted by Ptolemy and Copernicus?

Q7 In what way was Copernicus' conclusion about the distance to the stars not convincing?

Q8 Did the Copernican system provide simple calculations of where the planets should be seen?

6.4 <u>Arguments against the Copernican system</u>. Copernicus' hopes for acceptance of his theory were not quickly fulfilled. More than a hundred years passed before the heliocentric system was generally accepted even by astronomers. In the meantime the theory and its few champions met power-

2. Copernicus clearly stated that the basic difference between a heliocentric and geocentric model was in the frame of reference.

Summary 6.4
1. The same arguments given by Ptolemy against Aristarchus were applied to the Copernican model. These arguments were based on religious dogma and conflict with Aristotelian physics of motion.

ful opposition. Most of the arguments were the same as those used by Ptolemy against the heliocentric ideas of Aristarchus.

1. Apart from its apparent simplicity, the Copernican system had no scientific advantages over the geocentric theory. There was no known observation that was explained by one system and not by the other. Copernicus introduced no new types of observations into his work. Furthermore, the accuracy of his final predictions was little better than that of Ptolemy's results. As Francis Bacon wrote in the early seventeenth century: "Now it is easy to see that both they who think the earth revolves and they who hold the old construction are about equally and indifferently supported by the phenomena."

Basically, the rival systems differed in their choice of reference systems used to describe the observed motions. Copernicus himself stated the problem clearly:

In the quotation from Copernicus, notice how clearly he makes the point that the relativity of motion depends upon the position and motion of the observer. This could be related to the discussion, in Unit I, of Galilean relativity, but make the point that Galileo wrote after Copernicus and that Galileo probably was influenced by the statement of Copernicus.

> Although there are so many authorities for saying that the Earth rests in the centre of the world that people think the contrary supposition...ridiculous; ...if, however, we consider the thing attentively, we will see that the question has not yet been decided and accordingly is by no means to be scorned. For every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. For no movement is perceptible relatively to things moved equally in the same directions—I mean relatively to the thing seen and the spectator. Now it is from the Earth that the celestial circuit is beheld and presented to our sight. Therefore, if some movement should belong to the Earth...it will appear, in the parts of the universe which are outside, as the same movement but in the opposite direction, as though the things outside were passing over. And the daily revolution...is such a movement.

F8 · Frames of reference

In that statement Copernicus invites the reader to shift the frame of reference from that of an observer on the earth to one at a remote position looking upon the whole system with the sun at the center. As you may know from personal experience, such a shift is not easy for us even today. Perhaps you can sympathize with those who preferred to hold to an earth-centered system for describing what they actually saw.

equivalent, that is, as far as physics is concerned.

Physicists now generally agree that all systems of reference are equivalent, although some may be more complex to use or think about. The modern attitude is that the choice of a frame of reference depends mainly on which will provide the simplest solution to the problem

being studied. We should not speak of reference systems as being right or wrong, but rather as being convenient or inconvenient. However, a reference system that may be acceptable to one person may involve philosophical assumptions that are unacceptable to another.

2. The lack of an observable parallax for the fixed stars was against Copernicus' model. His only reply was unacceptable because it meant that the stars were practically an infinite distance away from the earth. To us this is no shock, because we have been raised in a society that accepts the idea. Even so, such distances do strain our imagination. To the opponents of Copernicus such distances were absurd.

The Copernican system led to other conclusions that were also puzzling and threatening. Copernicus found actual distances between the sun and the planetary orbits. Perhaps the Copernican system was not just a mathematical procedure for predicting the positions of the planets. Perhaps Copernicus had revealed a real system of planetary orbits in space. This would be most confusing, for the orbits were far apart. Even the few small epicycles needed to account for variations in the motions did not fill up the spaces between the planets. Then what did fill up these spaces? Because Aristotle had stated that "nature abhors a vacuum," there had to be something in all that space. As you might expect, those who felt that space should be full of something invented various sorts of invisible fluids and ethers to fill up the emptiness. More recently analogous fluids have been used in theories of chemistry, and of heat, light and electricity.

3. Since no definite decision between the Ptolemaic and the Copernican theories could be made on the astronomical evidence, attention focused on the argument concerning the immobility and central position of the earth. For all his efforts, Copernicus was unable to persuade most of his readers that the heliocentric system was at least as close as the geocentric system to the mind and intent of God. All religious faiths in Europe, including the new Protestants, found enough Biblical quotations (e.g., Joshua 10: 12-13) to assert that the Divine Architect had worked from a Ptolemaic blueprint. Indeed, the religious reformer Martin Luther branded Copernicus as "the fool who would overturn the whole science of astronomy."

Eventually, in 1616, when storm clouds were raised by the case of Galileo, the Papacy put De Revolutionibus on

A:Trial of Copernicus

the Index of forbidden books as "false and altogether opposed to Holy Scriptures," and withdrew its approval of an earlier outline of Copernicus' work. Some Jewish communities forbade the teaching of the heliocentric theory. It was as if man insisted on the middle of the stage for his earth, the scene of both his daily life and prayer in a world he felt was created especially for him.

The assumption that the earth was not the center of the universe was offensive. But worse, the Copernican system suggested that the other planets were similar to the earth. Thus the concept of the heavenly ether was threatened. Who knew but what some fool might next suggest that the sun and possibly even the stars were made of earthly materials? If the other celestial bodies, either in our solar system or beyond, were similar to the earth, they might even be inhabited, no doubt by heathens beyond the power of salvation! Thus the whole Copernican scheme led to profound philosophical questions.

4. The Copernican theory conflicted with the basic propositions of Aristotelian physics. This conflict is well described by H. Butterfield:

> ...at least some of the economy of the Copernican system is rather an optical illusion of more recent centuries. We nowadays may say that it requires smaller effort to move the earth round upon its axis than to swing the whole universe in a twenty-four hour revolution about the earth; but in the Aristotelian physics it required something colossal to shift the heavy and sluggish earth, while all the skies were made of a subtle substance that was supposed to have no weight, and they were comparatively easy to turn, since turning was concordant with their nature. Above all, if you grant Copernicus a certain advantage in respect of geometrical simplicity, the sacrifice that had to be made for the sake of this was tremendous. You lost the whole cosmology associated with Aristotelianism—the whole intricately dovetailed system in which the nobility of the various elements and the hierarchical arrangement of these had been so beautifully interlocked. In fact, you had to throw overboard the very framework of existing science, and it was here that Copernicus clearly failed to discover a satisfactory alternative. He provided a neater geometry of the heavens, but it was one which made nonsense of the reasons and explanations that had previously been given to account for the movements in the sky.

Although the sun-centered Copernican scheme was equivalent to the Ptolemaic in explaining the astronomical observations, to abandon the geocentric hypothesis seemed "philosophically false and absurd," dangerous, and fan-

tastic. What other reaction could one have expected? Learned Europeans at that time recognized the Bible and the writings of Aristotle as their two supreme sources of authority. Both appeared to be challenged by the Copernican system. Although the freedom of thought that marked the Renaissance was just beginning, the old image of the world provided security and stability to many.

Similar conflicts between the philosophical assumptions underlying accepted beliefs and those arising from scientific studies have occurred many times. During the last century there were at least two such conflicts. Neither is completely resolved today. In biology the evolutionary theory based on Darwin's work has had major philosophical and religious overtones. In physics, as Units 4, 5 and 6 indicate, evolving theories of atoms, relativity, and quantum mechanics have challenged other long-held philosophical assumptions about the nature of the world and our knowledge of reality.

Q9 Why did many people, such as Francis Bacon, adopt a ho-hum attitude toward the arguments about the correctness of the Ptolemaic or Copernican systems?

Q10 What was the major difference between the Ptolemaic and the Copernican systems?

Q11 How did the astronomical argument become involved with religious beliefs?

Q12 In what way did the Copernican system conflict with the accepted physics of the time?

Q13 List some conflicts between scientific theories and philosophical assumptions of which you are aware.

6.5 Historical consequences. Eventually, the moving-earth model of Copernicus was accepted. However, the slowness of that acceptance is illustrated by a recent discovery in the diary of John Adams, the second President of the United States: he wrote that at Harvard College on June 19, 1753, he attended a lecture where the correctness of the Copernican system was disputed.

Summary 6.5
The new viewpoint of Copernicus opened the way for further modification and improvements.

Soon we shall follow the work which gradually led to the general acceptance of the heliocentric theory. Yet within a century the detailed Copernican system of uniform circular motions with eccentrics and epicycles was replaced. We shall see that the real scientific significance of Copernicus' work lies in the fact that a heliocentric formulation opened a new way for understanding planetary motion. This was through the simple laws of ordinary (terrestrial) mechanics which were developed during the 150 years that followed.

The Copernican model with moving earth and fixed sun opened a floodgate of new possibilities for analysis and description. According to this model the planets could be considered as real bodies moving along actual orbits.

F9: Planets in orbit

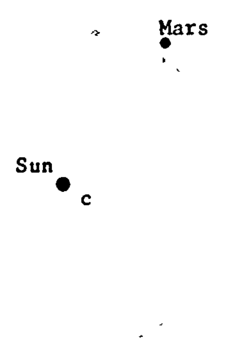


Fig. 6.6 In the Copernican system, the center of the deferent of Mars was offset from the sun to point C. In addition, a small epicycle was needed to account for minor variations in the planet's motion.

This section directs attention to the importance of new ways of looking at old problems. For a considerable time, even well after the work of Newton, as the diary notation of John Adams indicates, educated people knew about the Copernican system but did not necessarily believe that it represented the real motion of the planets.

What other scientific theories do you know which challenge the assumption that man he summit of creation?

Now Kepler and others could consider these planetary paths in quite new ways.

In science, a new set of assumptions often leads to new interpretations and unexpected results. Usually the sweep of possibilities cannot be foreseen by those who begin the revolution—or by their critics. For example, the people who laughed at the first automobiles, which moved no faster than a walking horse, failed to realize that those automobiles were but a crude beginning and could soon be improved, while the horse was in its "final edition."

The memory of Copernicus is honored for two additional reasons. First, he was one of those giants of the fifteenth and sixteenth centuries who challenged the contemporary world-picture. Second, his theory became a main force in the intellectual revolution which shook man out of his self-centered view of the universe.

As men gradually accepted the Copernican system, they necessarily found themselves accepting the view that the earth was only one among several planets circling the sun. Thus it became increasingly difficult to assume that all creation centered on mankind.

Acceptance of a revolutionary idea based on quite new assumptions, such as Copernicus' shift of the frame of reference, is always slow. Sometimes compromise theories are proposed as attempts to unite two conflicting alternatives, that is, "to split the difference." As we shall see in various Units, such compromises are rarely successful. Yet the conflict usually stimulates new observations that may be of long-term importance. These may lead to the development or restatement of one theory until it is essentially a new theory, as we shall see in Chapter 7.

Such a restatement of the heliocentric theory came during the years after Copernicus. While many men provided observations and ideas, we shall see that major contributions were made by Tycho Brahe, Kepler, Galileo and then Isaac Newton. New and better solutions to the theoretical problems required major improvements in the precision with which planetary positions were observed. Such improvements and the proposal of a compromise theory were the life work of the astronomer Tycho Brahe.

Q14 In terms of our historical perspective, what was probably the greatest contribution of Copernicus?

Q15 How did the Copernican system encourage the suspicion that there might be life on objects other than the earth? Is such a possibility seriously considered today?

6.6 Tycho Brahe. Tycho Brahe (Fig. 6.7) was born in 1546 of a noble, but not particularly rich, Danish family. By the time Tycho was thirteen or fourteen, he had become intensely interested in astronomy. Although he was studying law, he secretly spent his allowance money on astronomical tables and books such as the Almagest and De Revolutionibus. Soon he discovered that both Ptolemy and Copernicus had relied upon tables of planetary positions that were inaccurate. He concluded that before a satisfactory theory of planetary motion could be created new astronomical observations of the highest possible accuracy gathered during many years would be necessary.

Summary 6.6
Tycho Brahe created a major observatory with large instruments to collect astronomical observations of greater precision.

See "The Boy Who Redeemed His Father's Name" in Project Physics Reader 2.

Tycho's interest in studying the heavens was increased by an exciting observation in 1572. Although the ancients had taught that the stars were unchanging, Tycho observed a "new star" in the constellation Cassiopeia. It soon became as bright as Venus and could be seen even during the daytime. Then over several years it faded until it was no longer visible. To Tycho these changes were astonishing: change in the starry sky! Since the ancients had firmly believed that no changes were possible in the starry heavens, at least one assumption of the ancients was wrong. Perhaps other assumptions were wrong, too. What an exciting life he might have if he could study the heavens, searching for other changes of the stars and planets.

The brief sketch of the life of Tycho Brahe suggests the education of a man in the middle of the sixteenth century. Also it indicates the variety of events, some accidental, which apparently encouraged him to devote his life to astronomical observations.

After observing and writing about the "new star," Tycho travelled through northern Europe where he met many other astronomers and collected books. Apparently he was considering moving to Germany or Switzerland where he could easily meet other astronomers. To keep the young scientist in Denmark, King Frederick II made Tycho an offer that was too attractive to turn down. Tycho was given an entire small island, and also the income derived from various farms to allow him to build an observatory on the island and to staff and maintain it. The offer was accepted, and in a few years Uraniborg ("Castle of the Heavens") was built (Fig. 6.8). It was a large structure, having four large observatories, a library, a laboratory, shops, and living quarters for staff, students and observers. There was even a complete printing plant. Tycho estimated that the observatory cost Frederick II more than a ton of gold. In terms of the time of its building, this magnificent laboratory was at least as significant, complex and expensive as some of the great research establishments of our own time. Primarily

See "The Great Comet of 1965" in Project Physics Reader 2.

This paragraph comments on the observation of a "new star", or nova. This was a startling observation showing that changes could and did occur in the starry firmament. An observation of a similar nova is reported by Hipparchus, about 150 B.C., and apparently also motivated him to make the series of observations which 300 years later was the basis for Ptolemy's star catalogue.

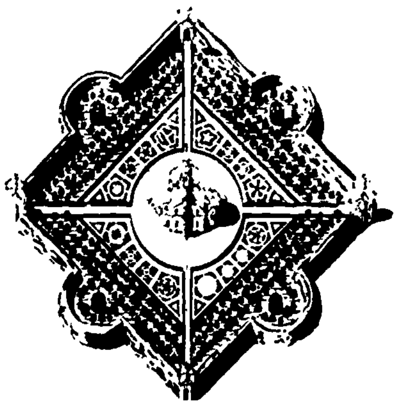
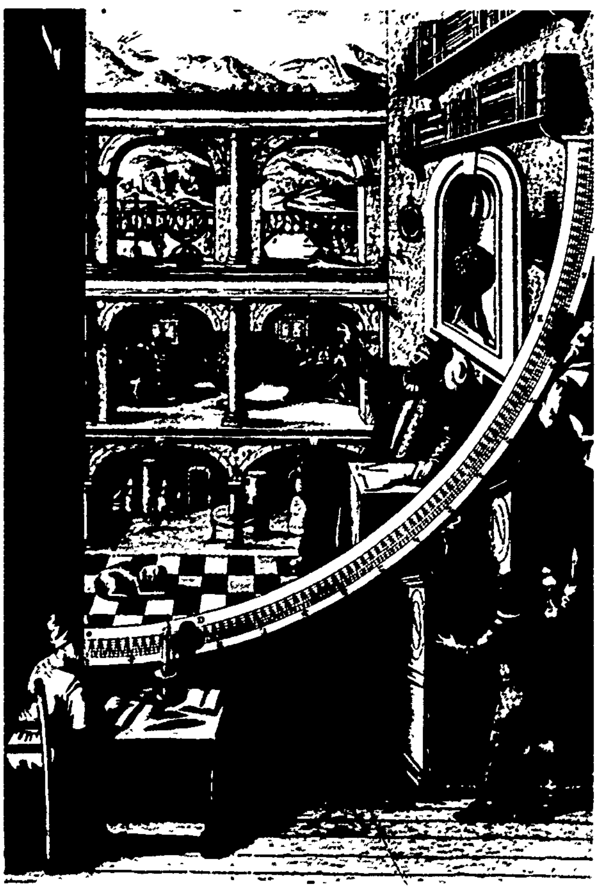


Fig. 6.8



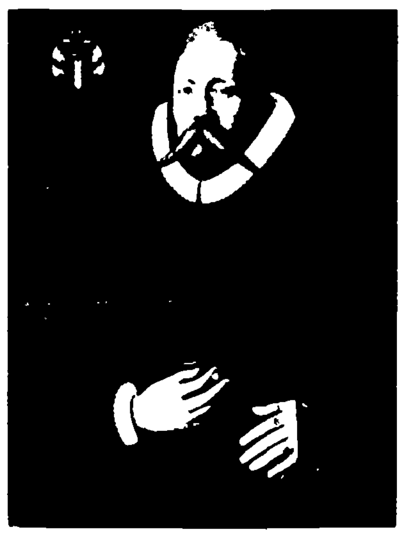


Fig. 6.7

At the top left is a plan of the observatory, gardens and wall built for Tycho Brahe at Uraniborg, Denmark.

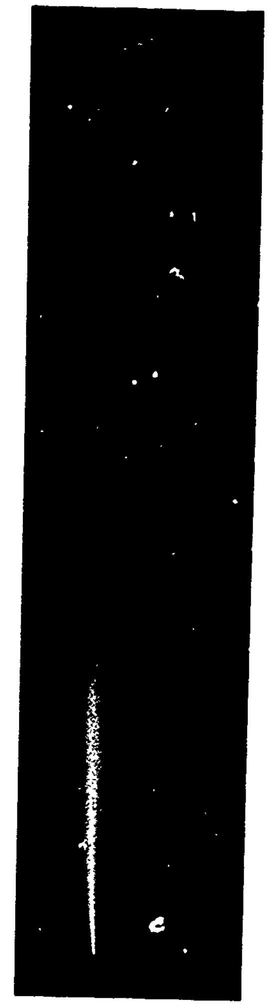
The cross section of the observatory, above center, shows where most of the important instruments were housed. Under the arch near the left is Tycho's largest celestial sphere.

At the left is the room containing Tycho's great quadrant. On the walls are pictures of Tycho and some of his instruments.

Above is a portrait of Tycho painted about 1597.

a research center, Uraniborg was a place where scientists, technicians and students from many lands could gather to study astronomy. Here was a unity of action, a group effort under the leadership of an imaginative scientist to advance the boundaries of knowledge in one science.

In 1577 Tycho observed a bright comet, a fuzzy object which moved across the sky erratically, unlike the orderly motions of the planets. To find the distance to the comet Tycho compared observations of its position seen from Denmark with its positions observed from elsewhere in Europe. At a given time, the comet had the same position among the stars even though the observing places were many hundreds of miles apart. Yet the moon's position in the sky was different when viewed from the ends of such long baselines. Therefore, Tycho concluded, the comet must be at least six times farther away than the moon. This was an important conclusion. Up to that time comets had been believed to be some sort of local event, like a cloud in the sky. Now comets had to be considered as distant astronomical objects which seemed to move right through the crystalline spheres. Tycho's book on this comet was widely read and helped to undermine belief in the old assumptions about the nature of the heavens.



This figure shows the bright comet of 1965 which passed the sun at only some 300,000 km. This sun-grazing comet moved in nearly the same orbit as similar comets in 1880, 1882 and 1887. All have periods of many thousands of years.

Fig. 6.9 The bright comet of 1965.

The smooth arc is the path of the sun – the ecliptic.

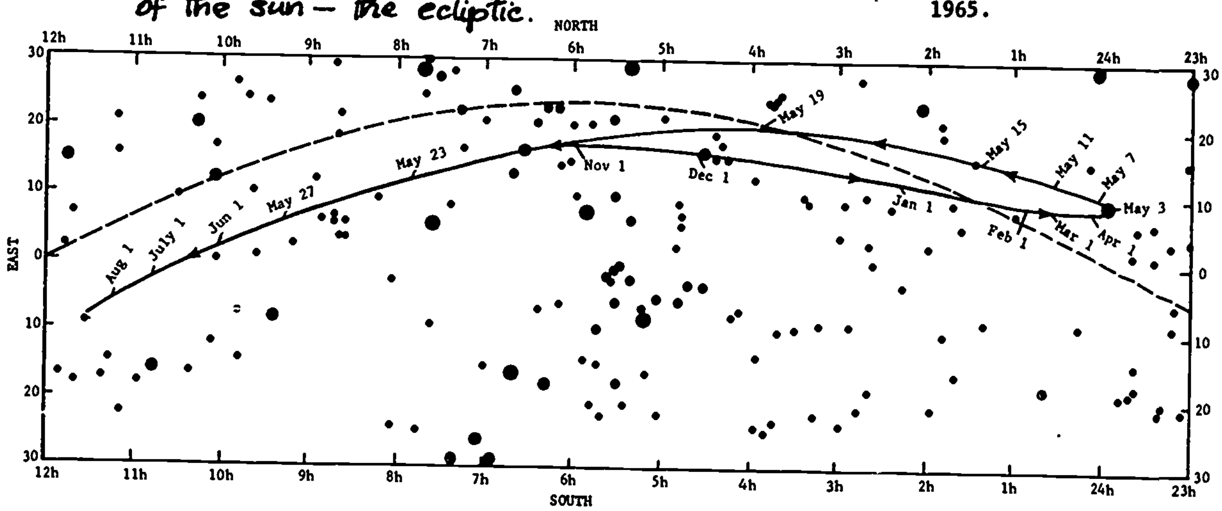


Fig. 6.10 Motion of Halley's Comet in 1909-10.

Fig. 6.10 has been inserted here to direct attention to the peculiar observed motions of comets. Also it will be referred to again in Chapter 8 where a three-dimensional

Q16 What stimulated Tycho to become interested in astronomy?

Q17 Why were Tycho's conclusions about the comet of 1577 important?

Q18 In what ways was Tycho's observatory like a modern research institute?

Q19 What evidence can you find that comets had been considered as omens of some disaster?

Q20 How can you explain the observed motion of Halley's comet during 1909-1910 as shown in Fig. 6.10?

model of the orbit of this comet is suggested for demonstration. Then the strange motions observed here can be explained.

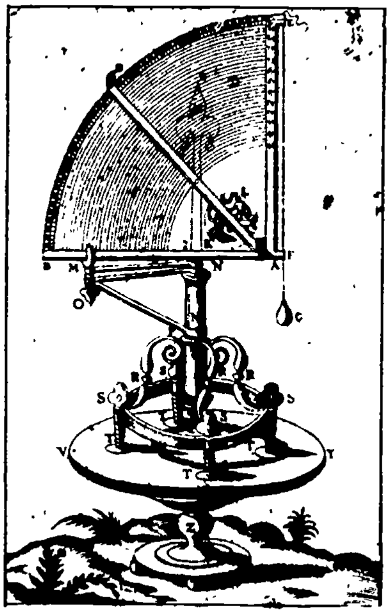


Fig. 6.11 One of Tycho's instruments, a quadrant; a device for measuring the angular altitude of heavenly objects. Unfortunately all of Tycho's instruments have been destroyed or lost.

See "A Night at the Observatory" in Project Physics Reader 2.

For a more modern example of this same problem of instrumentation, you may wish to read about the development and construction of the 200-inch Hale telescope on Mt. Palomar.

Summary 6.7
The skill and care which Tycho devoted to his instruments deserve emphasis. He was the first great astronomical observer of northern Europe. He developed the equivalent of the vernier. Also he investigated many systematic errors between his instruments. Furthermore, he redetermined and applied corrections for atmospheric refraction. The observation of Ptolemy and others were reliable only to about $\frac{1}{6}°$ but Tycho's were known to be precise to about $\frac{1}{30}°$ As often happens, increased precision in observations makes apparent and significant differences that were previously hidden.

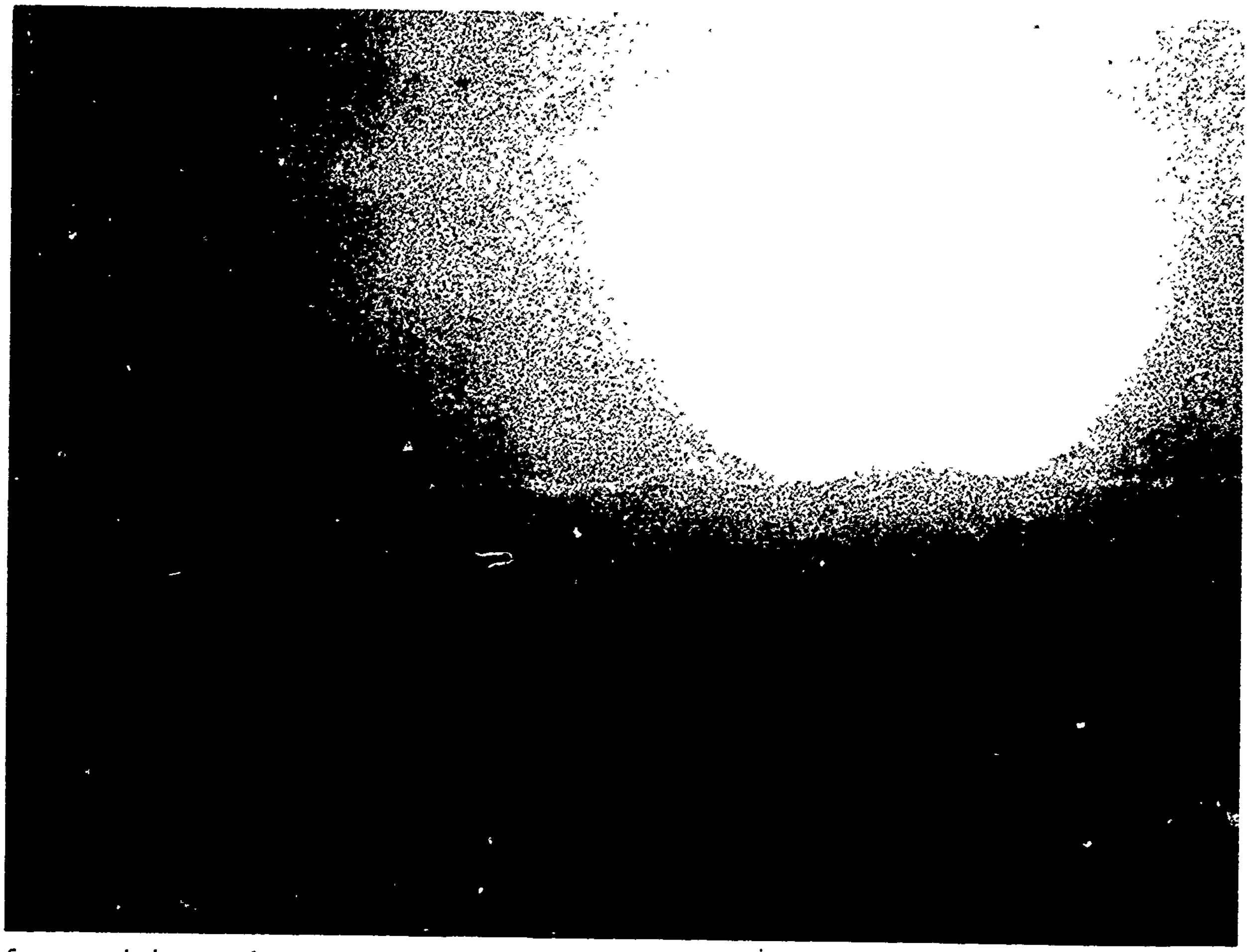
6.7 Tycho's observations. Tycho's fame results from his lifelong devotion to making unusually accurate observations of the positions of the stars, sun, moon, and planets. He did this before the telescope was invented. Over the centuries many talented observers had been recording the positions of the celestial objects, but the accuracy of Tycho's work was much greater than that of the best astronomers before him. How was Tycho Brahe able to do what no others had done before?

Singleness of purpose was certainly one of Tycho's assets. He knew that observations of the highest precision must be made during many years. For this he needed improved instruments that would give consistent readings. Fortunately he possessed both the mechanical ingenuity to devise such instruments and the funds to pay for their construction and use.

Tycho's first improvement on the astronomical instruments of the day was to make them larger. Most of the earlier instruments had been rather small, of a size that could be moved by one person. In comparison, Tycho's instruments were gigantic. For instance, one of his early devices for measuring the angular altitude of planets was a quadrant having a radius of about six feet (Fig. 6.11). This wooden instrument was so large that it took many men to set it into position. Tycho also put his instruments on heavy, firm foundations. Another huge instrument was attached to a wall that ran exactly north-south. By increasing the stability of the instruments, Tycho increased the reliability of the readings over long periods of time. Throughout his career Tycho also created better sighting devices, more precise scales and stronger support systems, and made dozens of other changes in design which increased the precision of the observations.

Not only did Tycho devise better instruments for making his observations, but he also determined and specified the actual limits of precision of each instrument. He realized that merely making larger and larger instruments does not always result in greater precision; ultimately, the very size of the instrument introduces errors since the parts will bend under their own weight. Tycho therefore tried to make his instruments as large and strong as he could without at the same time introducing errors due to bending. Furthermore, in the modern tradition, Tycho calibrated each instrument and determined its range of systematic error. (Nowadays most scientific instruments designed

E16: Using lenses, making a telescope, using a telescope
A: Telescope or binocular observations



for precision work are accompanied by a statement, usually in the form of a table, of systematic corrections to be applied to the readings.)

Fig. 6.12 The oblate setting sun. The light's path through the earth's atmosphere caused the sun to appear both oval and rough-edged.

Like Ptolemy and the Moslem observers, Tycho knew that the light from each heavenly body was bent downward increasingly as the object neared the horizon (Figs. 6.12 and 6.13), an effect known as atmospheric refraction. To increase the precision of his observations, Tycho carefully determined the amount of refraction so that each observation could be corrected for refraction effects. Such careful work was essential if improved records were to be made.

At Uraniborg, Tycho worked from 1576 to 1597. After the death of King Frederick II, the Danish government became less interested in helping to pay the cost of Tycho's observatory. Yet Tycho was unwilling to consider any reduction in the costs of his activities. Because he was promised support by Emperor Rudolph of Bohemia, Tycho moved his records and several instruments to Prague. There, fortunately, he took on as an assistant an able, imaginative young man named Johannes Kepler. When Tycho died in 1601, Kepler obtained all his records about Mars. As Chapter 7 reports, Kepler's analysis of Tycho's observations solved much of the ancient problem.

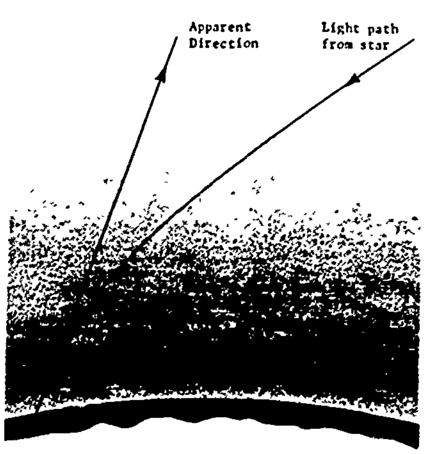


Fig. 6.13 Refraction, or bending, of light from a star by the earth's atmosphere. The amount of refraction shown in the figure is greatly exaggerated over what actually occurs.

A : Observing sunspots

See Atmospheric Refraction in Article Section.

Q21 What improvements did Tycho make in astronomical instruments?

Q22 In what way did Tycho correct his observations to provide records of higher accuracy?

6.8 Tycho's compromise system. Tycho's observations were intended to provide a basis for a new theory of planetary motion which he had outlined (Fig. 6.14) in an early publication. Tycho saw the simplicity of the Copernican system by which the planets moved around the sun, but he could not accept the idea that the earth had any motion. In Tycho's system, all the planets except the earth moved around the sun, which in turn moved around the stationary earth. Thus he devised a compromise theory which, as he said, included the best features of both the Ptolemaic and the Copernican systems, but he did not live to publish a quantitative theory. As we look at it today, his system is equivalent to either the Copernican or the Ptolemaic system. The difference between the three systems is the choice of what is regarded as stationary, that is, what frame of reference is chosen.

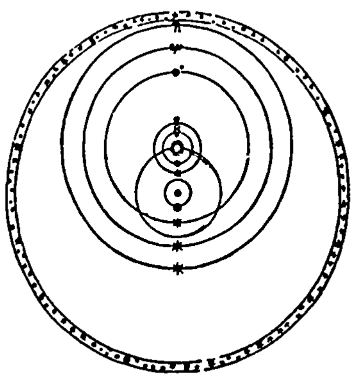


Fig. 6.14 Main spheres in Tycho Brahe's system of the universe. The earth was fixed and was at the center of the universe. The planets revolved around the sun, while the sun, in turn, revolved around the fixed earth.

Summary 6.8

1. Tycho could accept that the planets moved around the sun, but he could not conceive that the earth, too, might move. His position was justified because no observations suggested a heliocentric model; on the contrary, such a possibility defied ancient philosophical and religious traditions.

2. The observational evidence could be satisfied by either theory; there was no critical observation, other than the lack of stellar parallax.

The compromise Tychonic system was accepted by some people and rejected by others. Those who accepted Ptolemy objected to Tycho's proposal that the planets moved around the sun. Those who were interested in the Copernican model objected to having the earth held stationary. Thus the argument continued between those holding the seemingly self-evident position that the earth was stationary and those who accepted, at least tentatively, the strange, exciting proposals of Copernicus that the earth might rotate and revolve around the sun. These were philosophical or aesthetic preferences, for the scientific evidence did not yet provide an observational basis for a choice. To resolve the conflict and to produce a drastically revised sun-centered model was the work of Kepler who analyzed Tycho's high-quality observations of Mars.

All planetary theories up to this time had been developed only to provide some system by which the positions of the planets could be predicted fairly precisely. In the terms used in Unit I, these would be called kinematic descriptions. The causes of the motions—what we now call the dynamics of the motions—had not been questioned. The motions were, as Aristotle said, "natural." The heavens were still considered to be completely different from earthly materials and to change in quite different ways. That a common physics could describe both earthly and heavenly motions was a revolutionary idea yet to be proposed.

The Copernican system opened again the argument mentioned at the end of Chapter 5: were the Copernican orbits actual paths in space, or only convenient computational devices? We shall see that the eventual success of the Newtonian synthesis led to the confident assumption that scientists were describing the real world. However, later chapters of this text, dealing with recent discoveries and theories, will indicate that today scientists are much less certain that they know what is meant by the word reality.

The status of the problem in the early part of the seventeenth century was later well described by the English poet, John Milton, in Paradise Lost:

...He his fabric of the Heavens
Hath left to their disputes, perhaps to move
His laughter at their quaint opinions wide
Hereafter, when they come to model Heaven
And calculate the stars, how they will wield
The mighty frame, how build, unbuild, contrive
To save appearances, how gird the sphere
With centric and eccentric scribbled o'er,
Cycle and epicycle, orb in orb.

Q23 In what ways did Tycho's system for planetary motions resemble either the Ptolemaic or the Copernican systems?

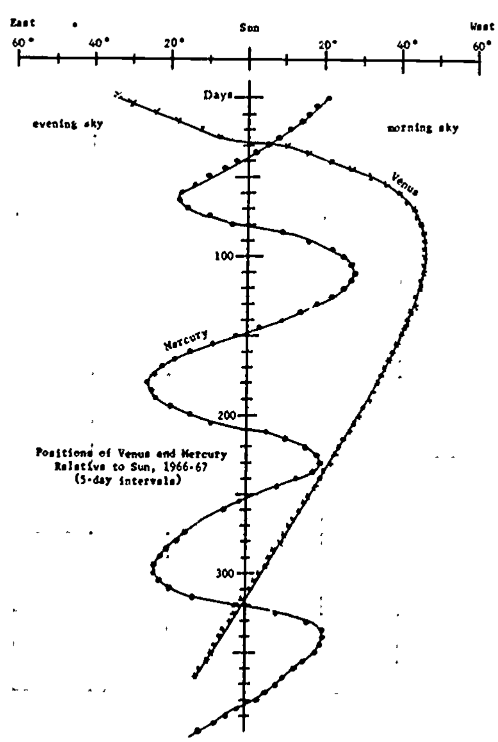
Q24 To what degree do you feel that the Copernican system, with its many motions on eccentrics and epicycles, reveals real paths in space rather than being only another means of computing planetary positions?

Study Guide

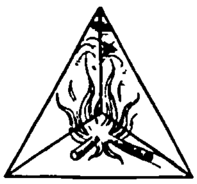
6.1 The diagram to the right shows the motions of Mercury and Venus east and west of the sun as seen from the earth during 1966-67. The time scale is indicated at 10-day intervals along the central line of the sun's position.

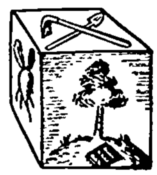
a) Can you explain why Mercury and Venus appear to move from farthest east to farthest west more quickly than from farthest west to farthest east?
b) From this diagram can you find a period for Mercury's apparent position in the sky relative to the sun?
c) With the aid of the "watch model" can you derive a period for Mercury's orbital motion around the sun?
d) What are the major sources of uncertainty in the results you derived?
e) Similarly can you estimate the orbital period of Venus?

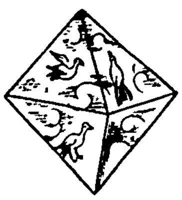
Discussion

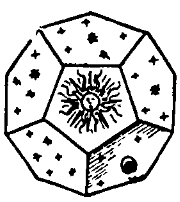


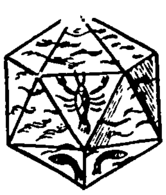
Chapter 7 A New Universe Appears — The Work of Kepler and Galileo

1. 

2. 

3. 

4. 

5. 

7.1 The abandonment of uniform circular motion. Kepler's life-long desire was to perfect the heliocentric theory. He viewed the harmony and simplicity of that theory with "incredible and ravishing delight." To Kepler, Fig. 7.1, such patterns of geometric order and numerical relation were clues to God's mind. Therefore, to unfold these patterns further through the heliocentric theory Kepler attempted in his first major paper to explain the spacing of the planetary orbits, as found by Copernicus (Table 6.3, p. 34).

From the time of the Greeks learned men knew that there were just five regular geometrical solids. The regular solid with six square faces is the cube. The dodecahedron has twelve five-sided faces. The other three regular solids have faces which are equilateral triangles. The tetrahedron has four triangular faces, the octahedron has eight triangular faces, and the icosahedron has twenty triangular faces.

Kepler wondered whether there was any relation between these five solids and the six known planets. He realized that these five regular solids could be nested, one inside the other like a set of mixing bowls. Between the five solids would be four spaces for planetary spheres. A fifth sphere could be inside the whole nest and a sixth sphere could be around the outside. Perhaps then some sequence of the five solids just touching the spheres would actually be spaced at the same relative distances from the center as were the planetary orbits. Kepler said:

> I took the dimensions of the planetary orbits according to the astronomy of Copernicus, who makes the sun immobile in the center, and the earth movable both around the sun and upon its own axis; and I showed that the differences of their orbits corresponded to the five regular Pythagorean figures....

By trial and error Kepler found a fit within about five percent of e actual planetary distances. To Kepler this remarkable system, illustrated in Fig. 7.2, explained the spacings of the planets. Also it indicated the unity he expected between geometry and scientific observations. Kepler's results, published in 1597, demonstrated his imagination and computational ability. Furthermore, it brought him to the attention of Tycho and Galileo. As a result, Kepler was invited to become one of Tycho's assistants at his new observatory in Prague.

There Kepler was given the task of determining more precisely the orbit of Mars. This problem had not been solved by Tycho and his other assistants; the motion of Mars was unusually difficult to explain. Years later, after he had solved the problem, Kepler wrote that ". . . Mars alone

Remind students that all through Chapters 5 and 6, the attempts to design models to explain the motion of heavenly bodies were based on the assumption that uniform circular motions was a necessity.

Fig. 7.1 Johannes Kepler (1571-1630).

The five "perfect solids," taken from Kepler's Harmonices Mundi, are shown on the opposite page. You can distinguish several of these solids in Fig. 7.2. The five regular solids are

1. the tetrahedron: four equilateral triangles;
2. the cube: six squares;
3. the octahedron: eight equilateral triangles;
4. the dodecahedron: twelve pentagons;
5. the icosahedron: twenty equilateral triangles.

Summary 7.1

1. After careful study of Tycho's observations, Kepler found that he could not calculate the orbit of Mars unless he abandoned Plato's assumption of uniform circular motions.

2. The final test of a theory rests upon its fit with observations.

A: The five regular solids

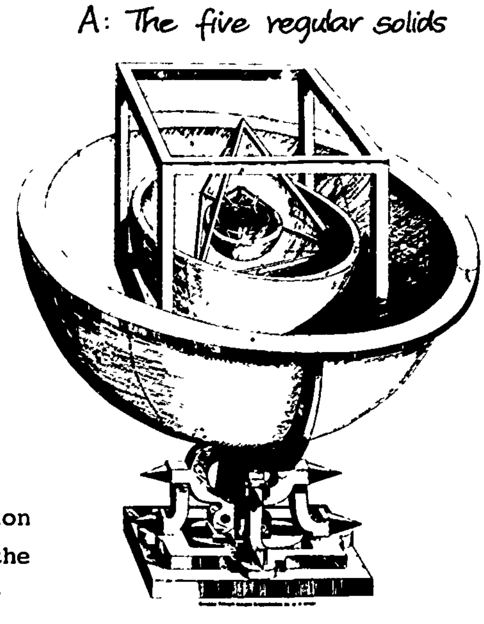


Fig. 7.2 A model of Kepler's explanation of the spacing of the planetary orbits by means of the regular geometrical solids. Notice that the planetary spheres were thick enough to include the small epicycle used by Copernicus.

enables us to penetrate the secrets of astronomy which otherwise would remain forever hidden from us." The motion of Mars, it turned out, was the start from which Kepler could redirect the study of celestial astronomy, just as Galileo used the motion of falling bodies to redirect the study of terrestrial motion.

See "Kepler" in Project Physics Reader 2.

The small difference of 8 minutes of arc between one model and the observations provides a fine opportunity to stress that Kepler, and others, were putting their faith on the observations and recognising that theories, as intellectual constructs, must satisfy the observations within the known limits of observational error.

The change in assumptions is worth attention. Every theory involves assumptions.

Kepler began his study of Mars by trying to fit the observations by means of motions on an eccentric circle and an equant. Like Copernicus, Kepler eliminated the need for the large epicycle by putting the sun motionless at the center and having the earth move around it, as in Fig. 6.5, p. 33. But Kepler made an assumption which differed from Copernicus' You recall that Copernicus had rejected the equant as an improper type of motion, but used small epicycles. Kepler used an equant, but refused to use even a small epicycle. To Kepler the epicycle seemed "unphysical" because the center of the epicycle was empty and empty space could not exert any force on a planet. Thus from the start of his study Kepler was assuming that the orbits were real and that the motions had some causes. Even though Kepler's teacher advised him to stick with "astronomical" (only observational) rather than physical assumptions, Kepler stubbornly stuck to his idea that the motions must have causes. When finally he published his results on Mars, in his book, Astronomia Nova, the New Astronomy, it was subtitled Celestial Physics.

For a year and a half Kepler struggled to fit Tycho's observations of Mars by various arrangements of an eccentric and an equant. When, after 70 trials, success finally seemed near he made a depressing discovery. Although he could represent fairly well the motion of Mars in longitude (along the ecliptic), he failed miserably with the latitude (the positions perpendicular to the ecliptic). However, even in longitude his very best fit still had differences of eight minutes of arc between the predicted and Tycho's observed positions.

See "Kepler on Mars" in Project Physics Reader 2.

Eight minutes of arc, about a fourth of the moon's diameter, may not seem like much of a difference. Others might have been tempted to explain it as observational error. But, with an integrity that has come to be expected of scientists, Kepler did not use that explanation. He knew from his own studies that Tycho's instruments and observations were rarely in error by as much as two minutes of arc. Those eight minutes of arc meant to Kepler that his best system using an eccentric and an equant would not do.

See "Kepler's Celestial Music" in Project Physics Reader 2.

In his New Astronomy Kepler wrote:

Since divine kindness granted us Tycho Brahe, the most diligent observer, by whose observations an error of eight minutes in the case of Mars is brought to light in this Ptolemaic calculation, it is fitting that we recognize and honor this favor of God with gratitude of mind. Let us certainly work it out, so that we finally show the true form of the celestial motions (by supporting ourselves with these proofs of the fallacy of the suppositions assumed). I myself shall prepare this way for others in the following chapters according to my small abilities. For if I thought that the eight minutes of longitude were to be ignored, I would already have corrected the hypothesis found in Chapter 16 (that it, by bissecting the eccentricity). But as it is, because they could not be ignored, these eight minutes alone have prepared the way for reshaping the whole of astronomy, and they are the material which is made into a great part of this work.

Kepler's respect for the observations was new.

The geometrical models of Ptolemy and Copernicus based on uniform circular motions had to be abandoned. Kepler had the finest observations ever made, but now he had no theory by which they could be explained. He would have to start over to account for the difficult questions: what is the shape of the orbit followed by Mars, and precisely how does the speed of the planet change as it moves along the orbit?

Q1 What brought Kepler to the attention of Tycho Brahe?

Q2 Why did Kepler conclude that Plato's problem, to describe the motions of the planets by combinations of circular motions, could not be solved?

7.2 Kepler's Law of Areas. Kepler's problem was immense. To solve it would demand the utmost of his imagination and computational skills, as well as of his persistence and health.

As the basis for his study Kepler had only Tycho's observed directions to Mars and to the sun on certain dates. But these observations were made from a moving earth whose orbit was not well known. Kepler realized that he must first determine more accurately the shape of the earth's orbit so that he would know where it was when the various observations of Mars had been made. Then he might be able to use the observations to determine the shape and size of the orbit of Mars. Finally, to predict positions for Mars he would need some regularity or law that described how fast Mars moved at various points in its orbit.

D31 : Plane motions

Fortunately Kepler made a major discovery which was crucial to his later work. He found that the orbits of the earth and other planets were in planes which passed through the sun. With this simplifying model of planets moving in individual planes Kepler could avoid the old patterns of Ptolemy and Copernicus which required separate explanations for the observed motions of the planets north and south of

Summary 7.2

1. Imagination is necessary for the creation of hypotheses and scientific models which can be tested against observations.

2. Kepler's introduction of orbit planes passing through the sun was a major contribution to clarifying the geometry of planetary motions.

3. Using a geometrical model, Kepler analyzed the observations of Tycho to establish by triangulation points on the orbits of the earth and of Mars.

4. Kepler's second law (law of areas): the line from the sun to a planet sweeps over areas which are proportional to the time intervals.

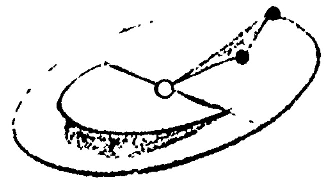


Fig. 7.3 Edge-on view of orbital planes of earth and planet.

the ecliptic. Kepler discovered that the changing positions of a planet could result from the planet's motion in its orbit viewed from the earth moving in its plane (the plane of the ecliptic) as shown in Fig. 7.3.

From his many studies Kepler knew that the earth and Mars moved in continuous paths that differed a bit from circles. His aim was to obtain a detailed picture or plot of the orbits as they might be seen by an observer above the ecliptic plane looking down on these moving bodies. To such an observer the planets would look like marbles rolling along nearly circular paths on the floor. Although the heliocentric idea gave a rough guide to what this system would look like, Kepler's task was to find from the data the general rules, or laws, that precisely fit the observations. As we work through his brilliant analysis, you will see the series of problems that he solved.

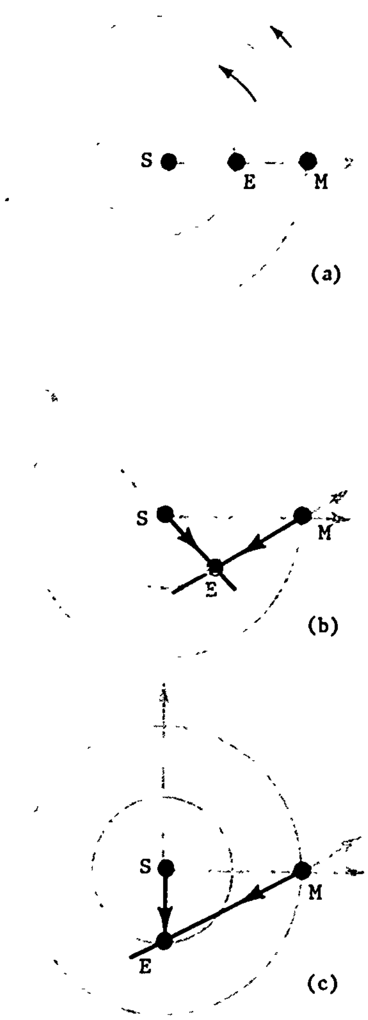


Fig. 7.4 How Kepler determined approximately the shape of the earth's orbit. Initially, (a) Mars is opposite the sun. After 687 days, (b) Mars has returned to the same place in its orbit, but the earth is almost 45° short of being at its initial position. After Mars makes one more cycle, (c) the earth lags by about 90°. Since the directions from the earth to the sun and Mars are known, the directions of the earth as seen from the sun and Mars are also known. Where these pairs of sight-lines cross must be points on the earth's orbit.

To derive the earth's orbit he began by considering the moments when the sun, earth and Mars are essentially in a straight line. After 687 days, as Copernicus had found, Mars would return to the same place in its orbit. Of course, the earth at that time would not be at the same place in its orbit as when the first observation was made. Nevertheless, as Fig. 7.4 indicates, the directions of the earth as seen from the sun and from Mars would be known. The crossing point of the sight-lines from the sun and from Mars must be a point on the earth's orbit. By working with several groups of observations made 687 days apart, Kepler was able to determine fairly accurately the shape of the earth's orbit.

The orbit Kepler found for the earth appeared to be almost a circle, with the sun a bit off center. Kepler also knew, as you have read in Sec. 5.7, that the earth moves around the sun fastest during December and January, and slowest during June and July. Now he had an orbit and timetable for the earth's motion. In Experiment 15 you made a similar plot of the earth's orbit.

With the orbit and timetable of the earth known, Kepler could reverse the analysis. He triangulated the positions of Mars when it was at the same place in its orbit. For this purpose he again used observations separated by one orbital period of Mars (687 days). Because this interval is somewhat less than two earth years, the earth is at different positions in its orbit at the two times (Fig. 7.5). Then the two directions from the earth toward Mars differ, and two sight-lines can be drawn from the earth's positions; where

Kepler's solution for the orbit of the earth was rather rough, but good enough for him to determine the position of the earth in its orbit on various dates.

Mars' Orbit

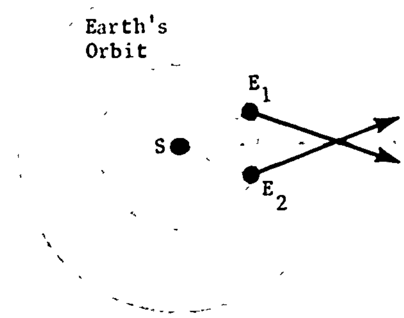


Fig. 7.5 How Kepler determined points on the orbit of Mars by triangulation.

they cross is a point on the orbit of Mars. From such pairs of observations Kepler fixed points on the orbit of Mars. From a curve drawn through such points you can get fairly accurate values for the size and shape of Mars' orbit. Kepler saw at once that the orbit of Mars was not a circle around the sun. You will find the same result from Experiment 18.

Because Mars, like the earth, moves faster when nearer the sun, Kepler began to wonder why this occurred. Perhaps the sun exerted some force which pushed the planets along their orbits. Here we see the beginnings of a major change in interpretation. In the systems of Ptolemy and Copernicus, the sun was not a special object in a mechanical or dynamical sense. Even in the Copernican system each planet moved around its special point near the sun. No physical relation had been assumed, only a geometrical arrangement. Motions in the heavens had been considered as perpetual motions along circles. Now Kepler began to suspect that there was some physical interaction between the sun and the planets which caused the planets to move along their orbits.

F10: Elliptic orbits

A: Measuring irregular areas

While Kepler was studying how the speed of the planet changed along its orbit, he made an unexpected discovery: during equal time intervals a line drawn from the sun to the moving planet swept over equal areas. Figure 7.6 illustrates this for an orbit in which each pair of points is separated by equal time intervals. Between points A and B, the planet moves rapidly; between points G and H it moves slowly. Yet the areas swept over by the line from the sun to the planet are equal. Although Kepler discovered this Law of Areas before he discovered the exact shape of the orbits, it has become known as Kepler's second law. In its general form the Law of Areas states: <u>the line from the sun to the moving planet sweeps over areas that are proportional to the time intervals</u>.

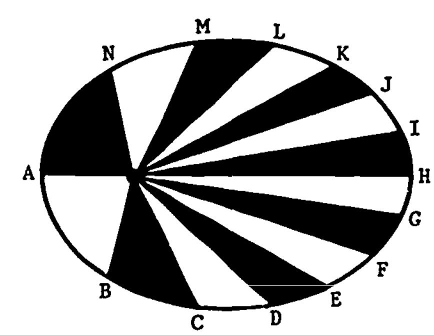
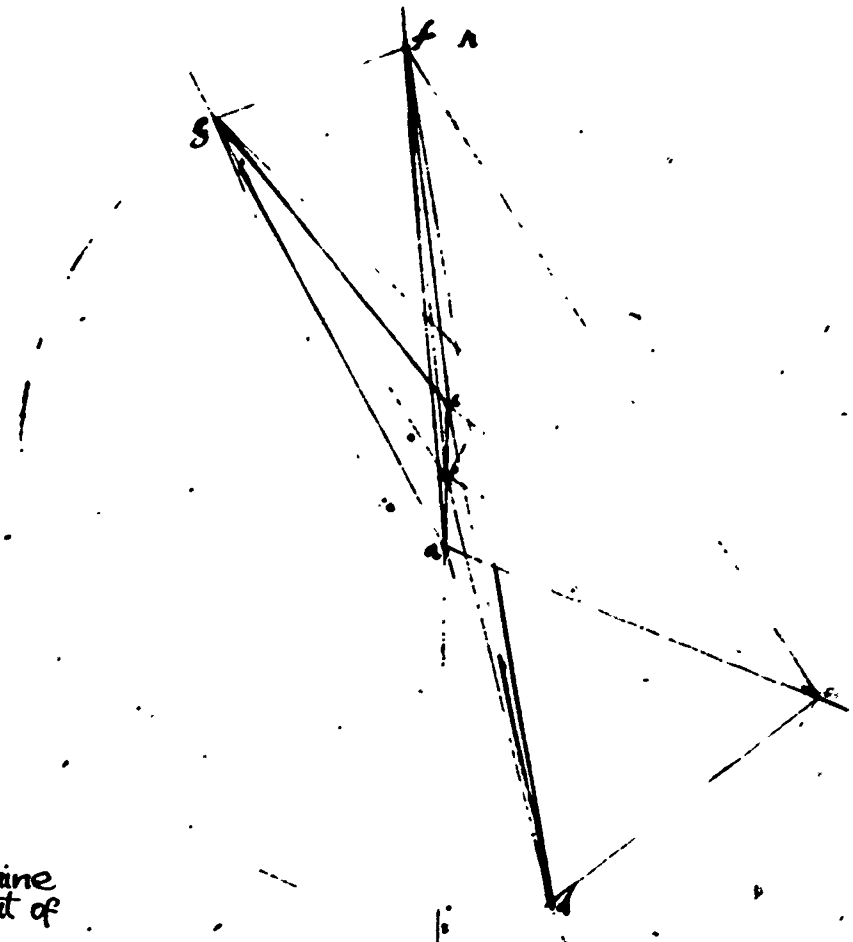


Fig. 7.6 Kepler's second law. A planet moves along its orbit at a rate such that the line from the sun to the planet sweeps over areas which are proportional to the time intervals.

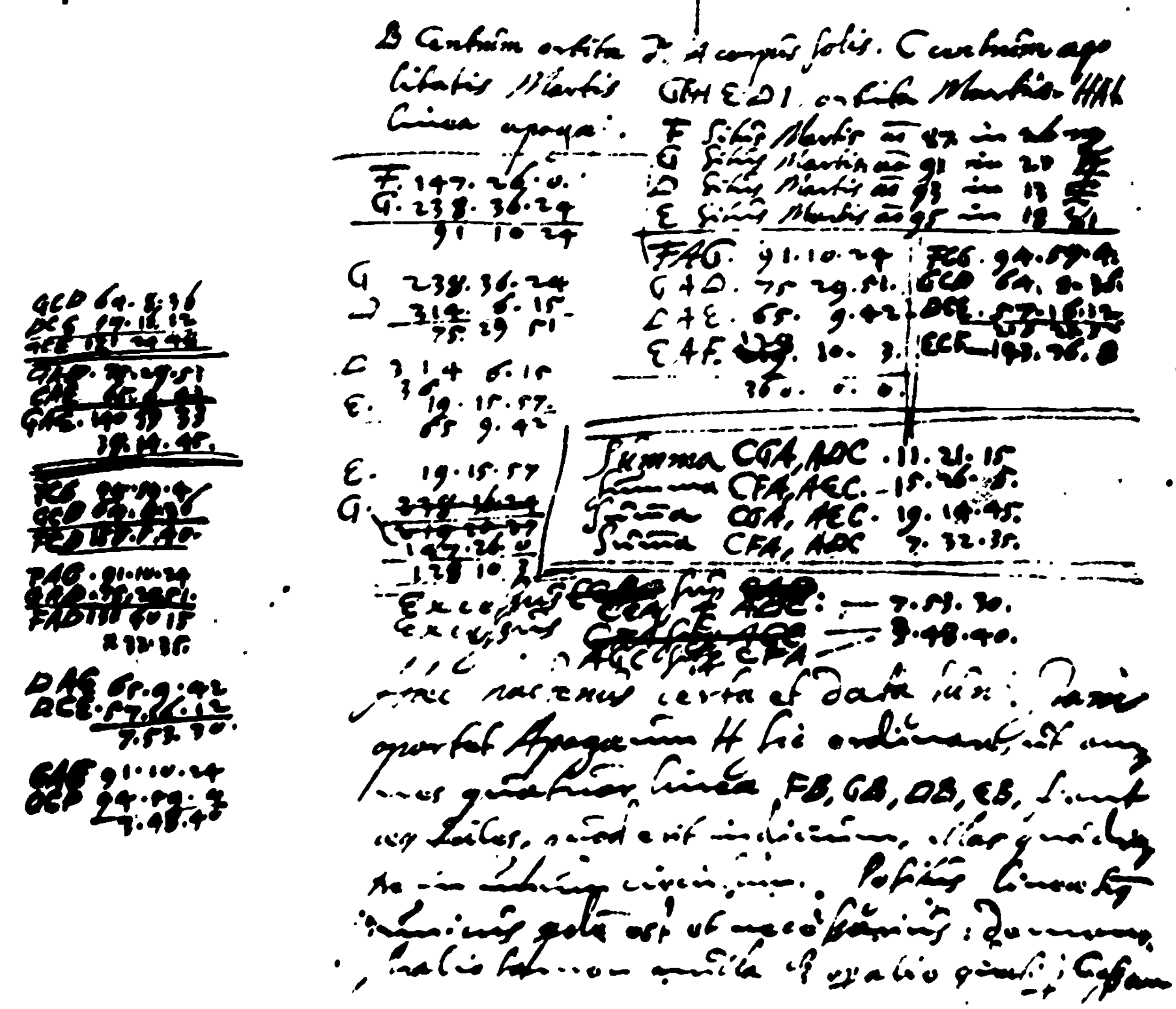
Perhaps you are surprised that the first general law about the motions of the planets is concerned with the areas swept over by the line from the sun to the planet. After we have considered circles, eccentric circles, epicycles and equants to describe the motion, we come upon a quite unexpected property, the area swept over per unit time, as the first property of the orbital motion to remain constant. As we shall see in Chapter 8, this major law of nature applies to all orbits in the solar system and also to double stars. Perhaps you can sympathize with Kepler, who wrote that he was in ecstasy when, after great labor and ingenuity, he finally found this law. At last the problem was beginning to crack.

Are pupils surprised, as was Kepler, that such an unexpected characteristic as the "area swept over per unit time" was the constant factor in orbital motion? (Although this is called Kepler's second law, it was the one he found first.)

E17 on the orbit of Mars, and optional
E19 on the orbit of Mercury provides direct experience with orbital areas



A student might examine the historical development of numerical procedures.



As we shall see, Kepler's other labors would have been of little use without this basic discovery, even though it was an empirical discovery—without any hint why this law should be. The Law of Areas establishes the rate at which Mars (or any other planet or comet) moves at a particular point of its orbit. But to use this for making predictions of positions, viewed from the sun or from the earth, Kepler needed also to know the precise size and shape of Mars' orbit.

Q3 What types of observations did Kepler use for his study of Mars?

Q4 What were the new problems that Kepler had to solve?

Q5 What important simplifying assumption about planetary orbits was added by Kepler?

Q6 State Kepler's Law of Areas.

Q7 Summarize the steps Kepler used to determine the orbit of the earth.

Q8 Describe the velocity changes of a planet as it goes around the sun in an elliptical orbit. (See next section.)

7.3 Kepler's Law of Elliptical Orbits. By using the analysis we have described and illustrated in Fig. 7.5, p. 53, Kepler established some points on the orbit of Mars. But what sort of a path was this? How could he describe it? As Kepler said, "The conclusion is quite simply that the planet's path is not a circle—it curves inward on both sides and outward again at opposite ends. Such a curve is called an oval." But what kind of oval?

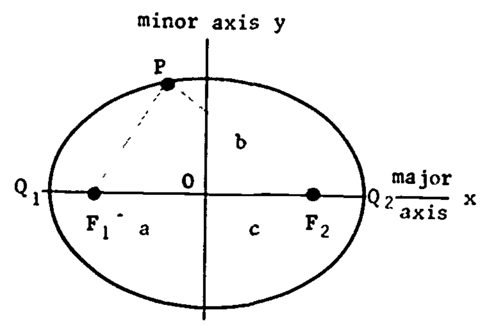


Fig. 7.7 An ellipse showing the semi-major axis a, the semi-minor axis b, and the two foci F_1 and F_2. The shape of an ellipse is described by its eccentricity, e, where $e = FO/OQ$, or $e = c/a$. (7.1)

Many different closed curves can be called ovals. Kepler thought for a time that the orbit was egg-shaped. Because such a shape did not agree with Kepler's ideas of physical interaction between the sun and the planet, he rejected the possibility that the orbit was egg-shaped. Kepler concluded that there must be some better way to describe the orbit and that he could find it. For many months, during which he often was ill and poverty-stricken, Kepler struggled with the question incessantly. Finally he was able to show that the orbit was a simple curve which had been studied in detail by the Greeks two thousand years before. The curve is called an ellipse. It is the shape you see when you view a circle at a slant.

In any ellipse the sum of the distances from the two foci to a point on the curve equals the length of the major axis, or $(F_1P + F_2P) = 2a$.

This property of ellipses allows us to draw them by using a loop of string around two tacks at the foci. Should the length of string tied into the loop have a total length of $(2a + 2c)$?

As Fig. 7.8 shows, ellipses can differ greatly in shape. They also have many interesting properties. For example, you can draw an ellipse by looping a piece of string around two thumb tacks pushed into a drawing board or cardboard at points F_1 and F_2 as shown in Fig. 7.7. Then, with a pencil point pull the loop taut and run the pencil once around the loop. You will have drawn an ellipse. (If the two thumb tacks had been together, what curve would you have drawn? What results do you get as you separate the two tacks more?)

A page from Kepler's notebooks.

Summary 7.3
1. Kepler's first law (law of ellipses): the planets move in orbits which are ellipses with the sun at one focal point.

2. Between the times of Copernicus and Newton a major shift in assumptions occurred; observations became the raw input of science as well as a check on previously stated propositions.

Each of the important points F_1 and F_2 is called a focus of the ellipse.

What Kepler discovered was not merely that the orbit of Mars is an ellipse—a remarkable enough discovery in itself—but also that the sun is at one focus. (The other focus is empty.) Kepler stated these results in his Law of Elliptical Orbits: the planets move in orbits which are ellipses and have the sun at one focus. That this is called Kepler's first law, although discovered after the Law of Areas, is an historical accident.

The long or major axis of an ellipse has a length 2a. The short or minor axis, perpendicular to the major axis, has a length 2b. The point O midway between the two foci, F_1 and F_2, is called the center, and the distance between the foci is called 2c. Thus the distance from the center to either focus is c.

The distance from any point P on the ellipse to the two foci can easily be found. Imagine that the loop is pulled out until your pencil is at the extreme point Q_2. Here the distance F_1Q_2 is $(a + c)$. At the other extreme point Q_1 the distance from F_2 is also $(a + c)$. When we subtract the distance 2c between the foci, the remainder is

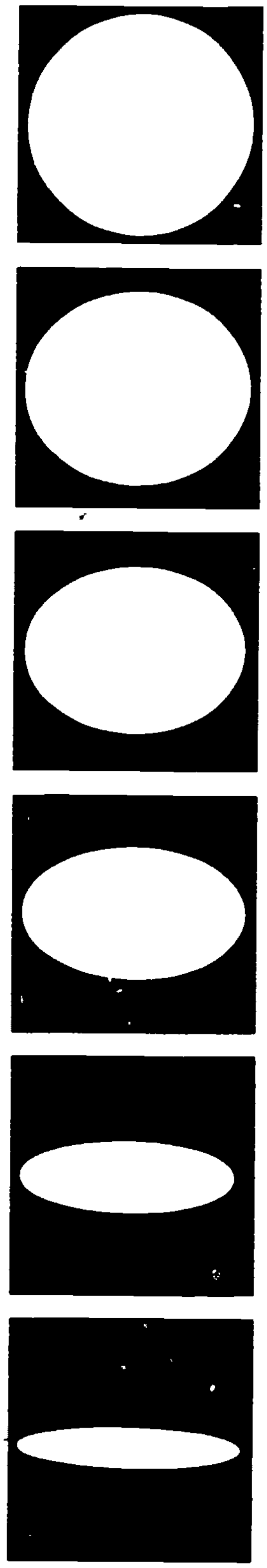
$$F_1P + F_2P = 2a.$$

For astronomical orbits the distance a is called the mean distance of a point P from one focus (the sun).

As you probably discovered, the shape of an ellipse depends upon the distance between the foci. For that reason the shape of an ellipse is described by the ratio c/a, which is called the eccentricity of the ellipse and is denoted by e. Thus $e = c/a$. For a circle, which is an extreme form of an ellipse, the foci are together. Then the distance between foci is zero and the eccentricity is also zero. Other ellipses have eccentricities ranging between 0 and 1.

If Fig. 7.7 were to represent a planetary orbit, the sun would be at one focus, say F_1, with no object at the other focus. The planet would be nearest the sun when it reached point Q_1, and farthest from the sun at point Q_2. The point nearest the sun is called the perihelion point and the point farthest from the sun is called the aphelion point. The distances of these two points from the sun are called the perihelion distance and the aphelion distance respectively.

An example will show how these properties of an ellipse can be used to provide new interesting information. For the planet Mercury the perihelion distance (Q_1F_1 in Fig. 7.7)



e = 0.3

e = 0.5

e = 0.7

e = 0.8

e = 0.94

e = 0.98

Fig. 7.8 Ellipses of different eccentricities. A saucer was photographed at various angles.

The terms perihelion and aphelion come from the Greek, in which helios is the sun, peri means nearest, and apo means farthest. What other words do you know in which the prefixes peri and apo, or ap, have similar meanings?

Some information on the thought and turmoil of the times might be introduced here. During the Thirty Years' War central Europe was a continual battleground. Because the war involved religious groups, persecution was directed at individuals as well as at communities. Kepler's mother was tried as a witch.

has been found to be about 45.8×10^6 kilometers while the aphelion distance F_1Q_2 is about 70.0×10^6 kilometers. What is the eccentricity of the orbit of Mercury?

$$e = c/a$$

$$a = \frac{(Q_1F_1 + F_1Q_2)}{2}, \text{ or}$$

$$= \frac{45.8 \times 10^6 \text{ km} + 70.0 \times 10^6 \text{ km}}{2}$$

$$= 57.9 \times 10^6 \text{ km}$$

$$c = (\text{mean distance - perihelion distance})$$

$$= (OQ_1 - QF_1)$$

$$= 57.9 \times 10^6 \text{ km} - 45.8 \times 10^6 \text{ km}$$

$$= 12.1 \times 10^6 \text{ km.}$$

$$\text{Then } e = \frac{12.1 \times 10^6 \text{ km}}{57.9 \times 10^6 \text{ km}} = 0.21 .$$

F11: Measuring large distances — PSSC

E17: The orbit of Mars

E19: Mercury's orbit

T17: Orbit parameters

E18: The inclination of Mars' orbit

A: Demonstrating satellite orbits

A: Discovery of Neptune and Pluto

As Table 7.1 shows, the orbit of Mars has the largest eccentricity of all the orbits that Kepler could study; those of Venus, earth, Mars, Jupiter and Saturn. Had he studied any planet other than Mars he might never have noticed that the orbit was an ellipse. Even for the orbit of Mars, the difference between the elliptical orbit and an off-center circle is quite small. No wonder Kepler later wrote that "Mars alone enables us to penetrate the secrets of astronomy."

Table 7.1 The Eccentricities of Planetary Orbits

Planet	Orbital Eccentricity	Notes
Mercury	0.206	Too few observations for Kepler to study
Venus	0.007	Nearly circular orbit
Earth	0.017	Small eccentricity
Mars	0.093	Largest eccentricity among planets Kepler could study
Jupiter	0.048	Slow moving in the sky
Saturn	0.056	Slow moving in the sky
Uranus	0.047	Not discovered until 1781
Neptune	0.009	Not discovered until 1846
Pluto	0.249	Not discovered until 1930

The work of Kepler illustrates the enormous change in outlook in Europe that had begun well over two centuries before. Scientific thinkers gradually ceased trying to impose human forms and motivations upon nature. Instead, they were beginning to look for and theorize about mathematical simplicities and mechanical or other models. Kepler rejected the

After Kepler had found that the orbit of Mars was elliptical, and that the other planetary orbits also were ellipses, he asked still another question: what relation exists between the sizes of the various orbits? One answer often opens new questions for further study.

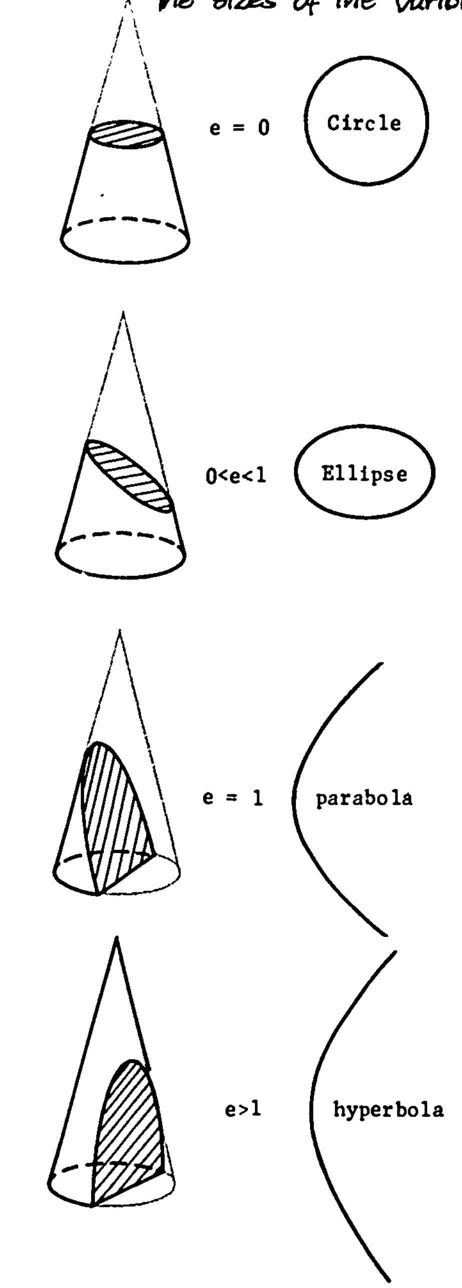


Fig. 7.9 Conic sections. These four figures are formed when a cone is cut at different angles.

D 32: Conic sections from model

Empirical means "based on observations, not on theory."

A: Plotting an analemma

A: Force on a pendulum

ancient idea that each planet had a "soul." Instead he, like Galileo, began to search for physical causes. Where Copernicus and Tycho were willing to settle for geometrical models by which planetary positions could be predicted, Kepler was one of the first to seek causes for the motions. This new desire for causal explanations marks the beginning of modern physical science.

Like Kepler, we must have faith that our observations represent some aspects of a reality that is more stable than the emotions, wishes and behavior of human beings. Like Plato and all scientists, we assume that nature is basically orderly, consistent and therefore understandable in a simple way. This faith has led scientists to devote themselves to careful and sometimes tedious quantitative investigations of nature. As we all know, great theoretical and technical gains have resulted. Kepler's work illustrates one of the scientific attitudes—to regard a wide variety of phenomena as better understood when they can be described by simple, preferably mathematical patterns.

After Kepler's initial joy over the discovery of the law of elliptical paths, he may have asked himself the question: why are the planetary orbits elliptical rather than some other geometrical shape? While we can understand Plato's desire for uniform circular motions, nature's insistence on the ellipse is a surprise.

In fact, there was no satisfactory answer to Kepler's question until Newton showed, almost eighty years later, that these elliptical orbits were required by a much more general law of nature. Let us accept Kepler's laws as rules that contain the observed facts about the motions of the planets. As <u>empirical</u> laws, they each summarize the verifiable data from observations of the motion of any one planet. The first law, which describes the paths of planets as elliptical around the sun, gives us all the possible positions of each planet. That law, however, does not tell us when a planet will be at any one particular position on its ellipse or how rapidly it will then be moving. The second law, the Law of Areas, describes how the speed changes as the distance from the sun changes, but does not involve the shape of the orbit. Clearly these two laws complement each other. With these two general laws, numbers for the size and shape of the orbit, and the date for one position, we can determine both the position and angular speed of a given planet at any time relative to the sun. Since we can also find where the earth is at the same instant, we can derive the position of the planet as seen from the earth.

Q9 What can you do to a circle to have it appear as an ellipse?

Q10 Why was Kepler's decision to study the motion of Mars fortunate?

Q11 Summarize the steps Kepler used to determine the orbit of Mars.

7.4 Using the first two laws. Fig. 7.10 represents the elliptical path of a planet with the sun at one focus. By a short analysis we can determine the ratios of the speeds v_p and v_a of the planet when it is closest to the sun and farthest from the sun. As you note in Fig. 7.10, at these two points the velocity vectors are perpendicular to the radius. In Fig. 7.6, p. 53, the time intervals during which the planet moves from one marked point to the next are equal. In imagination let us make that time interval very small, Δt. Then the orbital speed becomes the instantaneous linear speed v. Also, as Fig. 7.11 suggests, the distances from the sun at the beginning and end of that interval are almost equal, so we may use R for both distances. Since we know that the area of any triangle is ½ base × altitude we may write the area of any such long thin triangle between the sun and a small section of the orbit as: area = $\frac{1}{2} Rv\Delta t$. But by Kepler's Law of Areas, when the time intervals Δt are equal the areas swept over by the radius are also equal. Thus we can equate the area at aphelion, R_a, to the area at perihelion, R_p, and have

$$\tfrac{1}{2} R_a v_a \Delta t = \tfrac{1}{2} R_p v_p \Delta t.$$

After cancelling out the common parts on both sides of the equation, we have

$$R_a v_a = R_p v_p. \qquad (7.2)$$

We can rearrange this equation to obtain the more interesting form:

$$v_a/v_p = R_p/R_a. \qquad (7.2a)$$

Eq. (7.2a) shows that the speeds at perihelion and aphelion are inversely proportional to the radii at these two points. If we return to the example about the orbit of Mercury, p. 57, we can find how the speeds at perihelion and aphelion compare:

$$v_a/v_p = R_p/R_a$$

$$v_a/v_p = 45.8 \times 10^6 \text{ km}/70.0 \times 10^6 \text{ km}$$

$$= 0.65.$$

The speed of Mercury at aphelion is only about 2/3 that at perihelion.

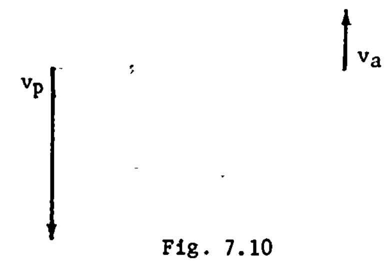


Fig. 7.10

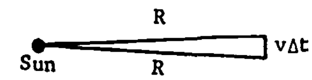


Fig. 7.11

Summary 7.4
1. Statements like Kepler's laws usually evolve from the hard work and long observations of many people over a number of years.

2. Kepler's third law (law of periods): the squares of the periods of the planets are proportional to the cubes of their average distances from the sun.

3. The motive for scientific work is often aesthetic or religious.
SG 7.3

SG 7.5
Kepler's laws, derived from actual observations, i.e., empirical evidence, plus the notion of area, permitted highly accurate predictions.

When Mercury is at perihelion at 0.31 A.U., its orbital speed is 58 km/sec. What is its orbital speed at aphelion at 0.47 A.U.?

Only very rarely are scientific or creative achievements entirely original or independent of the past. Perhaps you could have students list the observations and theories which made Kepler's studies possible.

Does Eq. (7.2a), derived for perihelion and aphelion positions, apply to any point on the orbit, or was some assumption hidden in our analysis? We must be careful not to fool ourselves, whether in words or in mathematical symbols. We did make an assumption: that the displacement, $v\Delta t$, taken as the altitude of the thin triangle, was perpendicular to the radial, sun-planet line. Therefore our results apply only to the two points in the orbit where the velocity vector is perpendicular to the radius. This condition occurs only at the perihelion and aphelion points.

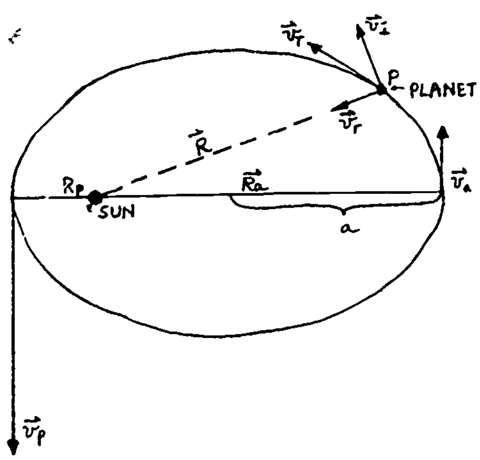


Fig. 7.12

The equation which takes the place of Eq. (7.2a) and holds for <u>any</u> two points along the orbit is $R_1(v_1)_\perp = R_2(v_2)_\perp$, where the $\perp$ means that we consider only the component of the velocity which is perpendicular to the sun-planet line. Can you see how you could use this knowledge to derive the velocity at any point on the orbit? Remember that the velocity vector is always tangent to the orbit. Figure 7.12 shows the relationships.

A major point of the section is made in this paragraph. Here the elegance and simplicity of Kepler's laws are stressed. This illustrates one major aspect of science.

When we know the size and eccentricity of the elliptical orbit and apply Kepler's two simple laws, we can predict for past or future dates where the planet will be along its orbit. The elegance and simplicity of Kepler's two laws are impressive. Surely Ptolemy and Copernicus would have been amazed that the solution to the problem of planetary motions could be given by such short statements. But we must not forget that these laws were distilled from Copernicus' idea of a moving earth, the great labors and expense that went into Tycho's fine observations, and the imagination, devotion and often agony of Kepler's labors.

Summary 7.5
Kepler worked for ten years to find the third law. He was working on other problems as well; but the haunting necessity for <u>some</u> relation describing the spacing of the planets' orbits caused him to keep searching for possibilities.

7.5 <u>Kepler's Law of Periods</u>. Kepler's first and second laws were published in 1609 in his book <u>Astronomia Nova</u>, or <u>New Astronomy</u>. But Kepler was dissatisfied because he had not yet found any relation between the motions of the different planets. So far, each planet seemed to have its own elliptical orbit and speeds, but there appeared to be no overall pattern relating all planets. Kepler, who had begun his career by trying to explain the number of planets and their spacing, was convinced that there must be some regularity, or rule, linking all the motions in the solar system. His conviction was so strong that he spent years examining many possible combinations of factors to find, by trial and error, a third law that would relate all the planetary orbits. His long search, almost an obsession, illustrates a belief that has run through the whole history of science: that nature is simple, uniform and understandable. This belief has

all other planetary theories depended, whether those of Ptolemy, Copernicus or Tycho. With different assumptions and procedures Kepler had at last solved the astronomical problem on which so many great men had worked so hard for centuries. Although he had to abandon the geometrical devices of the Copernican system, Kepler did depend on the Copernican model. None of the earth-centered models could have led to Kepler's three laws.

In 1627, after many troubles with publishers, Kepler finally published the Rudolphine Tables based on Tycho's observations. In these tables Kepler combined Tycho's observations and Kepler's laws in a way that permitted accurate calculations of planetary positions for any times, whether in the past or future. These Rudolphine Tables remained useful for a century until telescopic observations of greater precision replaced Tycho's observations.

L16: Kepler's Laws shows the orbits of two planets as developed by a computer. All of Kepler's laws can be verified.

These tables were also important for a quite different reason. In them Kepler pioneered in the use of logarithms and included a long section, practically a textbook, on the nature of logarithms and their use for calculations. Kepler had realized that logarithms, first described in 1614 by Napier in Scotland, would be very useful in speeding up the tedious arithmetic required for the derivation of planetary positions.

A repeat on an electronic computer of Kepler's computations (Gingerich, Harvard College Obs.) shows that Kepler was working at the limit of his computational ability, often making mistakes, some of which fortuitously cancelled each other.

We honor Kepler for his astronomical and mathematical achievements, but these were only a few of the accomplishments of this great man. As soon as Kepler learned of the development of the telescope, he spent most of a year making careful studies of how the images were formed. These he published in a book titled Dioptrice, which became the standard work on optics for many years. Like Tycho, who was much impressed by the new star of 1572, Kepler similarly observed and wrote about the new stars of 1600 and 1604. His observations and interpretations added to the impact of Tycho's similar observations in 1572 that changes did occur in the starry sky. In addition to a number of important books on mathematical and astronomical problems, Kepler wrote a popular and widely read description of the Copernican system as modified by his own discoveries. This added to the growing interest in and acceptance of the sun-centered model of the planetary system.

Q12 State Kepler's third law, the Law of Periods. Why is it useful?

7.6 The new concept of physical law. One general feature of Kepler's life-long work has had a profound effect on the development of all the physical sciences. When Kepler began his studies he still accepted Plato's assumptions about the importance of geometric models and Aristotle's emphasis on natural place to explain motion. But at the end he stated mathematical laws describing how planets moved, and even attempted to explain these motions in terms of physical forces. His successful statement of empirical laws in mathematical form helped to establish the equation as the normal form of laws in physical science. Thus he contributed to a new way of considering observations and stating conclusions.

Summary 7.6
1. Scientists are rarely willing only to describe phenomena; generally they seek some explanation as to why it occurs.

2. Advances in one science often influence developments in another, e.g., Gilbert's work on magnetism influenced Kepler's astronomical ideas.

3. A mechanistic view of the world became popular and was widely accepted until the early twentieth century.

More than anyone before him, Kepler expected an acceptable theory to agree with precise and quantitative observation. In Kepler's system the planets no longer were considered to move in their orbits because they had some divine nature or influence, or because they had spherical shapes which served as self-evident explanation for their circular motions. Rather, Kepler tried to back up his mathematical descriptions with physical mechanisms. In fact, he was the first to look for a universal physical law based on terrestrial phenomena to describe the whole universe in quantitative detail. In an early letter he expressed his guiding thought:

See "A Search for Life on Earth" in Project Physics Reader 2.

See Galileo's "The Starry Messenger" in Project Physics Reader 2.

> I am much occupied with the investigation of the physical causes. My aim in this is to show that the celestial machine is to be likened not to a divine organism but rather to a clockwork...insofar as nearly all the manifold movements are carried out by means of a single, quite simple magnetic force, as in the case of a clockwork, all motions [are caused] by a simple weight. Moreover, I show how this physical conception is to be presented through calculation and geometry. [Letter to Herwart, 1605.]

Here we encounter for the first time the idea of the universe operating like a clockwork, or a giant machine.

The world as a celestial machine driven by a single force, in the image of a clockwork—this was indeed a prophetic goal! Stimulated by William Gilbert's work on magnetism a few years earlier, Kepler could imagine magnetic forces from the sun driving the planets in their orbits. This was a promising and reasonable hypothesis. As it turned out, the fundamental idea that a single kind of force controls the motions of all the planets was correct; but the force is not magnetism and it is not needed to keep the planets moving.

Use this reference to Gilbert to develop the idea that scientists influence each other through their writings.
Note that Galileo's idea of inertia had not yet been published.

Kepler's statements of empirical laws remind us of Galileo's suggestion, made at about the same time, that we deal first with the how and then with the why of motion in free fall. A half century later Newton used gravitational force

to tie together Kepler's three planetary laws for a heliocentric system with the laws of terrestrial mechanics in a magnificent synthesis (Chapter 8).

Q13 To what did Kepler wish to compare the "celestial machine"?

Q14 Why is Kepler's reference to a "clockwork" model significant?

Summary 7.7
Galileo and his contemporaries are presented in the context of the political and religious tensions of their time.

7.7 Galileo's viewpoint. One of the scientists with whom Kepler corresponded about the latest scientific developments was Galileo. Kepler's contributions to planetary theory were mainly his empirical laws based on the observations of Tycho. Galileo contributed to both theory and observation. As Chapters 2 and 3 reported, Galileo's theory was based on observations of bodies moving on the earth's surface. His development of the new science of mechanics contradicted the assumptions on which Aristotle's physics and interpretation of the heavens had been based. Through his books and speeches Galileo triggered wide discussion about the differences or similarities of earth and heaven. Outside of scientific circles, as far away as England, the poet John Milton wrote, some years after his visit to Galileo in 1638:

...What if earth
Be but the shadow of Heaven, and things therein
Each to the other like, more than on earth is thought?

[Paradise Lost, Book V, line 574, 1667.]

Galileo challenged the ancient interpretations of experience. As you saw earlier, he focused attention on new concepts: time and distance, velocity and acceleration, forces and matter in contrast to the Aristotelian qualities or essences, ultimate causes, and geometrical models. In Galileo's study of falling bodies he insisted on fitting the concepts and conclusions to the observed facts. By expressing his results in concise mathematical form, Galileo paralleled the new style being used by Kepler.

If some of your students want to read about the society in which Galileo lived, you might suggest Autobiography of Benvenuto Cellini.
Also a provocative interpretation of Galileo has been written by Bertolt Brecht in his play Galileo.

The sharp break between Galileo and most other scientists of the time arose from the kind of question he asked. To his opponents, many of Galileo's problems seemed trivial. What was important about watching pendulums swing or rolling balls down inclines when philosophical problems needed clarification? Certainly his procedures for studying the world seemed peculiar, even fantastic, and his statements were strange and his manner haughty.

Another reference is "Galileo Yesterday and Today," Raymond J. Seeger, (American Journal of Physics, Vol. 33 p. 680, 1965).

Although Kepler and Galileo lived at the same time, their lives were quite different. Kepler was struggling to find simplicity among complex astronomical phenomena. He was enchanted by the mysteries of astrology; he lived a hand-to-mouth existence under stingy patrons, and was driven from

city to city by the religious wars of the time. Few people, other than a handful of friends and correspondents, knew of or cared about his studies and results.

Galileo's situation was different. He wrote his numerous papers and books in Italian, which could be read by many people who did not read scholarly Latin. These publications were the work of a superb partisan and publicist. Galileo wanted many to know of his studies and to accept the Copernican theory. He took the argument far beyond a small group of scholars out to the nobles, civic leaders, and religious dignitaries. His reports and arguments, including often bitter ridicule of individuals or ideas, became the subject of dinner-table conversations. In his efforts to inform and persuade he stirred up the ridicule and even violence often poured upon those who have new ideas. In the world of art similar receptions were given initially to Manet and Giacometti, and in music to Beethoven, Stravinsky and Schönberg.

Students interested in music or art might wish to report on the reception by the public of various works now considered famous.

7.8 The telescopic evidence. Like Kepler, Galileo was a Copernican among Ptolemaeans who believed that the heavens were eternal and could not change. Hence, Galileo was interested in the sudden appearance in 1604 of a new star, one of those observed by Kepler. Where there had been nothing visible in the sky, there was now a brilliant star. Galileo, like Tycho and Kepler, realized that such changes in the starry sky conflicted with the old idea that the stars could not change. Furthermore, this new star awakened in Galileo an interest in astronomy which lasted throughout his life.

Consequently, Galileo was ready to react to the news he received four or five years later that a Dutchman "had constructed a spy glass by means of which visible objects, though very distant from the eye of the observer, were distinctly seen as if nearby." Galileo quickly worked out the optical principles involved, and set to work to grind the lenses and build such an instrument himself. His first telescope made objects appear three times closer than when seen with the naked eye. Then he constructed a second and a third telescope. Reporting on his third telescope, Galileo wrote:

> Finally, sparing neither labor nor expense, I succeeded in constructing for myself so excellent an instrument that objects seen by means of it appeared nearly one thousand times larger and over thirty times closer than when regarded with our natural vision" (Fig. 7.13).

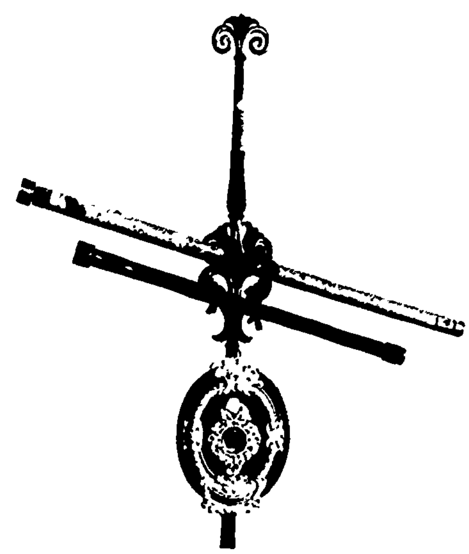


Fig. 7.13 Two of Galileo's telescopes displayed in Florence.

Your students should read a few of Galileo's papers in Discoveries and Opinions of Galileo by Stillman Drake,

In "Sidereal Messenger" Galileo outlines how he determined the heights of mountains on the moon.

Galileo meant that the area of the object was nearly 1000 times larger. The linear magnification was over 30 times.

Summary 7.8 1. Galileo's work with the telescope suggests the impact which an entirely new instrument of measurement and observation can have on knowledge and theory. 2. Galileo's telescopic observations, especially those of the phases of Venus and of the satellites of Jupiter, contradicted the Ptolemaic system.

One might point out that (legend has it) Galileo passed off the telescope as his own invention to the Venetian Senate for increased financial support.

Fig. 7.14 Two of Galileo's early drawings of the moon. (From Galileo's Sidereus Nuncius, which is often translated as The Sidereal Messenger or as The Starry Messenger.)

Discuss with students the impact a new measuring instrument can have upon a society: what can one conclude about a culture, for example the ancient Egyptians or the Mayans, simply from its instruments? (about its artisans, economy, natural resources, aesthetics, intellectual achievements, etc.). How do your students think tools and instruments stimulate man's thinking?

What would you do if you were handed "so excellent an instrument"? Like the men of Galileo's time, you probably would put it to practical uses. "It would be superfluous," Galileo agreed,

> to enumerate the number and importance of the advantages of such an instrument at sea as well as on land. But forsaking terrestrial observations, I turned to celestial ones, and first I saw the moon from as near at hand as if it were scarcely two terrestrial radii away. After that I observed often with wondering delight both the planets and the fixed stars....

What, then, were the findings that Galileo made with his telescope? In the period of a few short weeks in 1609 and 1610 he made several discoveries, each of which is of first rank.

First, Galileo pointed his telescope at the moon. What he saw led him to the conviction that

> ...the surface of the moon is not smooth, uniform, and precisely spherical as a great number of philosophers believe it [and the other heavenly bodies] to be, but is uneven, rough, and full of cavities and prominences, being not unlike the face of the earth, relieved by chains of mountains, and deep valleys. [See Fig. 7.14.]

Galileo did not stop with that simple observation, which was contrary to the Aristotelian idea of heavenly perfection. He supported his conclusions with several kinds of observations, including quantitative evidence. For instance, he worked out a method for determining the height of a mountain on the moon from the shadow it casts. His value of about four miles for the height of some lunar mountains is not far from modern results.

Next he looked at the stars. To the naked eye the Milky Way had seemed to be a continuous blotchy band of light; through the telescope it was seen to consist of thousands of faint stars. Wherever Galileo pointed his telescope in the sky he saw many more stars than could be seen with the unaided eye. This observation was contrary to the old argument that the stars were created to provide light so men could see at night. If that were the explanation, there should not be stars invisible to the naked eye—but Galileo found thousands.

After his observations of the moon and the fixed stars, Galileo turned his attention to the discovery which in his opinion

> ...deserves to be considered the most important of all —the disclosure of four PLANETS never seen from the creation of the world up to our own time; together with the occasion of my having discovered and studied them,

See Student Handbook for student observations of the moons of Jupiter.
An article on observations of Jupiter is in the Scientific American, 1953 May.

their arrangements, and the observations made of their movements during the past two months.

He is here referring to his discovery of four (of the twelve now known) satellites which orbit about Jupiter (Fig. 7.15). Here, before his eyes, was a miniature solar system with its own center of revolution around Jupiter. This was directly opposed to the Aristotelian notion that the earth was at the center of the universe and could be the only center of revolution.

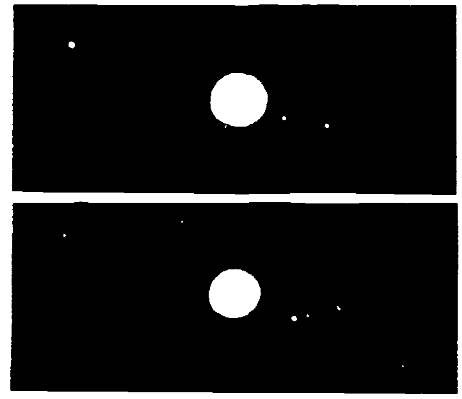


Fig. 7.15 Jupiter and its four brightest satellites. The lower photograph was taken 3 hours later than the upper one. (Photographed at Yerkes Observatory.)

The manner in which Galileo discovered Jupiter's "planets" is a tribute to his ability as an observer. Each clear evening during this period he was discovering dozens if not hundreds of new stars never before seen by man. When looking in the vicinity of Jupiter on the evening of January 7, 1610, he noticed "...that beside the planet there were three starlets, small indeed, but very bright. Though I believe them to be among the host of fixed stars, they aroused my curiosity somewhat by appearing to lie in an exact straight line..." In his notebook he made a sketch similar to that shown in the top line Fig. 7.16. When he saw them again the following night, he saw that they had changed position with reference to Jupiter. Each clear evening for weeks he observed that planet and its roving "starlets" and recorded their positions in drawings. Within days he had concluded that there were four "starlets" and that they were indeed satellites of Jupiter. He continued his observations until he was able to estimate the periods of their revolutions around Jupiter.

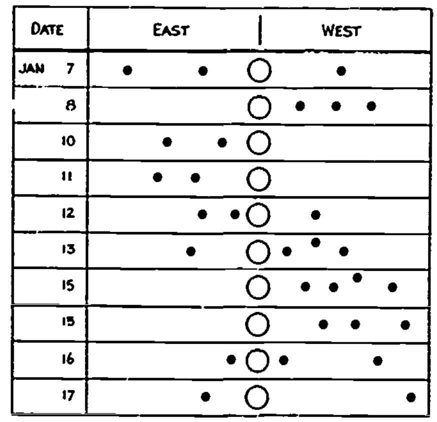


Fig. 7.16 Galileo observed and recorded the relative position of Jupiter's brightest satellites 64 times between January 7 and March 2, 1610. The sketches shown here are similar to Galileo's first ten recorded observations which he published in the first edition of his book Sidereus Nuncius, The Starry Messenger.

L12: Jupiter satellite orbit

Of all of Galileo's discoveries, that of the satellites of Jupiter caused the most stir. His book, The Starry Messenger, was an immediate success, and copies were sold as fast as they could be printed. For Galileo the result was a great demand for telescopes and great public fame.

Galileo continued to use his telescope with remarkable results. By projecting an image of the sun on a screen, he observed sunspots. This was additional evidence that the sun, like the moon, was not perfect in the Aristotelian sense: it was disfigured rather then even and smooth. From his observation that the sunspots moved across the disk of the sun in a regular pattern, he concluded that the sun rotated with a period of about 27 days.

If there are four bright satellites moving about Jupiter, why could Galileo sometimes see only two or three? What conclusions can you draw from the observation (see Figs. 7.15 and 7.16) that they lie nearly along a straight line?

He also found that Venus showed all phases, just as the moon does (Fig. 7.17). Therefore Venus must move completely around the sun as Copernicus and Tycho had believed, rather than be always between the earth and sun as the Ptolemaic astronomers assumed (see again Fig. 5.15). Saturn seemed

See Student Handbook for student observations of the phases of Venus.

Saturn, in Aquarius in 1966, moves eastward more slowly, with a period of 30 years. For some years it will be well placed in the eastern sky for observations during autumn evenings.

Fig. 7.17 Photographs of Venus at various phases taken with a constant magnification.

to carry bulges around its equator (Fig. 7.18), but Galileo's telescopes were not strong enough to show that they were rings. With his telescopes he collected an impressive array of new information about the heavens—and all of it seemed to contradict the basic assumptions of the Ptolemaic world scheme.

Q15 Could Galileo's observation of all phases of Venus support the heliocentric theory, the Tychonic system or Ptolemy's system?

Q16 In what ways did the invention of the telescope provide new evidence for the heliocentric theory?

Q17 What significance did observations of Jupiter have in the development of Galileo's ideas?

Summary 7.9
Until the observations of Galileo there was no new evidence to support the Copernican System. In fact the only new prediction of stellar parallax was not observed. But with Galileo's observations there was new evidence.
The full phases of Venus could be explained only by the Copernican or the Tychonic models; not by the Ptolemaic.

7.9 Galileo's arguments. To Galileo these observations supported his belief in the heliocentric Copernican system, but they were not the cause of them. In his great work, the Dialogue Concerning the Two Chief World Systems (1632), he stressed arguments based on assumptions as much as on observations. The observed motions of planets alone, says Galileo, do not decide uniquely between a heliocentric and a geocentric hypothesis, "for the same phenomena would result from either hypothesis." But Galileo accepted the earth's motion as real because it seemed to him simpler and more pleasing. Elsewhere in this course you will find other cases where a scientist accepted or rejected an idea for reasons arising from his strong belief in particular assumptions.

In the Dialogue Galileo presents his arguments in a systematic and lively way. The Dialogue, like the Discourses Concerning Two New Sciences, which are mentioned in Chapter 2, is in the form of a discussion between three learned men. Salviati, the voice of Galileo, wins most of the arguments. His antagonist is Simplicius, an Aristotelian who speaks for and defends the Ptolemaic system. The third member, Sagredo, represents the objective and intelligent citizen not yet committed to either system. However, Sagredo's role is written so that he usually accepts most of Galileo's arguments.

A small eighteenth-century reflecting telescope, now in the Harvard University collection.

Galileo's arguments in favor of the Copernican system as set forth in the Dialogue Concerning the Two Chief World Systems were mostly those given by Copernicus (see Chapter 6), and Galileo made no use of Kepler's laws. However,

The satellites of Jupiter showed that a system like the Copernican scheme could and did exist.

Yet the argument for and against the Copernican system were still about balanced. Venus showed phases, but stellar parallax was not observed.

Galileo had new evidence from his own observations. After deriving the periods of Jupiter's four moons or satellites, Galileo found that the larger the orbit of the satellite, the longer was the period of revolution. Copernicus had already found that the periods of the planets increased with their average distances from the sun. (Kepler's Third Law stated the relation for the planets in detailed quantitative form.) Now Jupiter's satellite system showed a similar pattern. These new patterns of regularities would soon replace the old assumptions of Plato, Aristotle and Ptolemy.

Students can check this from the data in the appendix.

The Dialogue Concerning the Two Chief World Systems relies upon Copernican arguments, Galilean observations and reasonableness to attack the basic assumptions of the geocentric model. Finally, Simplicius, seemingly in desperation, dismisses all of Galileo's arguments with a characteristic counterargument:

> ...with respect to the power of the Mover, which is infinite, it is just as easy to move the universe as the earth, or for that matter a straw.

This maxim was used to great effect by Wm. of Ockham, and is known as "Ockham's razor" when used in arguments like this.

But to this, Galileo makes a very interesting reply; notice how he quotes Aristotle against the Aristoteleans:

> ...what I have been saying was with regard not to the Mover, but only the movables....Giving our attention, then, to the movable bodies, and not questioning that it is a shorter and readier operation to move the earth than the universe, and paying attention to the many other simplifications and conveniences that follow from merely this one, it is much more probable that the diurnal motion belongs to the earth alone than to the rest of the universe excepting the earth. This is supported by a very true maxim of Aristotle's which teaches that..."it is pointless to use many to accomplish what may be done with fewer."

Summary 7.10 Despite the observation evidence, new theories were accepted slowly.

7.10 The opposition to Galileo. In his characteristic enthusiasm, Galileo thought that his telescopic discoveries would cause everyone to realize the absurdity of the assumptions that prevented a general acceptance of the Copernican system. But men can believe only what they are ready to believe. In their fight against the new Copernicans, the followers of Aristotle were convinced that they were surely sticking to facts and that the heliocentric theory was obviously false and in contradiction with both observation and common sense. The evidences of the telescope could be due to distortions. After all, glass lenses change the path of light rays; and even if telescopes seemed to work for terrestrial observation, nobody could be sure they worked equally well when pointed at these vastly more distant celestial objects.

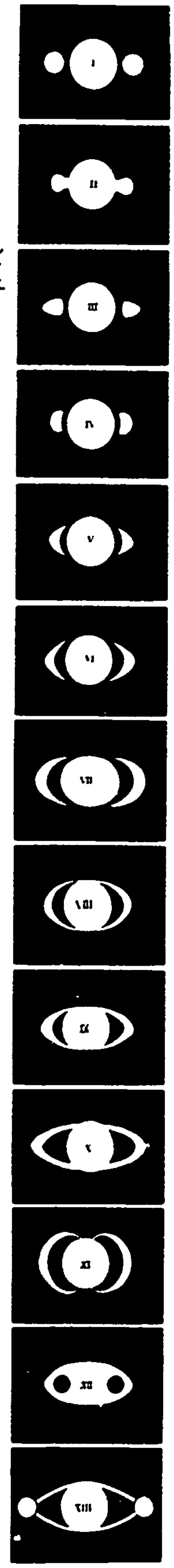


Fig. 7.18 Sketches of Saturn made from telescopic observations during the seventeenth century.

How do your students consider Galileo? As a martyr? A hero? Someone who sold out?

Consider each of Galileo's telescopic observations separately to determine whether it strengthened or weakened the case for (a) the geocentric theory, and (b) the philosophical assumptions underlying the geocentric theory. Give particular attention to the observed phases of Venus. Could these observations of Venus be explained by the Tychonic system? Could the Ptolemaic system be modified to have Venus show all phases?

Furthermore, theological heresies were implied in the heliocentric view. Following Thomas Aquinas, the Scholastics had adopted the Aristotelian argument as the only correct basis for building any physical theory. The Aristotelians could not even consider the Copernican system as a possible theory without giving up many of their basic assumptions, as you read in Chapter 6. To do so would have required them to do what is humanly almost impossible: discard their common-sense ideas and seek new bases for their moral and theological doctrines. They would have to admit that the earth is not at the center of creation. Then perhaps the universe was not created especially for mankind. Is it any wonder that Galileo's arguments stirred up a storm of opposition?

Galileo openly carried his fight to representatives of the established position. He provoked arguments and ridiculed the opposition. As in a Greek tragedy, his clash with the Inquisition was inevitable. His scientific theories and the accepted religious dogmas were incompatible. The standard for the new thinkers had to be silenced. (The Ecumenical Council called by Pope John XXIII finally absolved Galileo in 1965).

Galileo's observations were belittled by the Scholastics. The Florentine astronomer, Francesco Sizzi (1611), argued why there could not, indeed must not be any satellites around Jupiter:

> There are seven windows in the head, two nostrils, two ears, two eyes and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven [including the sun and moon]....Besides, the Jews and other ancient nations, as well as modern Europeans, have adopted the division of the week into seven days, and have named them from the seven planets; now if we increase the number of planets, this whole system falls to the ground.... Moreover, the satellites are invisible to the naked eye and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist.

A year after his discoveries, Galileo wrote to Kepler:

> You are the first and almost the only person who, even after a but cursory investigation, has...given entire credit to my statements....What do you say of the leading philosophers here to whom I have offered a thousand times of my own accord to show my studies, but who with the lazy obstinacy of a serpent who has eaten his fill have never consented to look at the planets, or moon, or telescope?

Summary 7.11
Galileo's troubles with the Inquisition are paralleled by many other examples in which innovators were attacked by those holding contradicting beliefs.

7.11 Science and freedom. The political and personal tragedy that occurred to Galileo is described at length in many books. Here we shall only mention briefly some of the major events. Galileo was warned in 1616 by the Inquisition to cease teaching the Copernican theory as true, rather than as just one of several possible explanations, for it was now held contrary to Holy Scripture. At the same time Copernicus' book was placed on the Index Expurgatorius, and was suspended "until corrected." As we saw before, Co-

Some students may have felt that in this section we are "preaching" to them. Probably we are. Yet many, including the potential scientist, have never looked at the social atmosphere within which creative work is done and received.

pernicus had used Aristotelian doctrine to make his theory plausible. But Galileo had reached the new point of view where he urged acceptance of the heliocentric system on its own merits. While he was himself a devoutly religious man, he deliberately ruled out questions of religious faith and salvation from scientific discussions. This was a fundamental break with the past.

When, in 1623, Cardinal Barberini, formerly a dear friend of Galileo, was elected to be Pope Urban VIII, Galileo talked with him regarding the decree against the Copernican ideas. As a result of the discussion, Galileo considered it safe enough to write again on the controversial topic. In 1632, after making some required changes, Galileo obtained the necessary Papal consent to publish the work, Dialogue Concerning the Two Chief World Systems. This book presented most persuasively the Copernican view in a thinly disguised discussion of the relative merits of the Ptolemaic and Copernican systems. After the book's publication his opponents argued that Galileo seemed to have tried to get around the warning of 1616. Furthermore, Galileo's forthright and sometimes tactless behavior, and the Inquisition's need to demonstrate its power over suspected heretics, combined to mark him for punishment.

Over 200 years after his confinement in Rome, opinions had changed so that Galileo was honored as in the fresco "Galileo presenting his telescope to the Venetian Senate" by Luigi Sabatelli (1772-1850). The fresco is located in the Tribune of Galileo, Florence, which was assembled from 1841 to 1850.

Among the many factors in this complex story, we must remember that Galileo, while considering himself religiously faithful, had become a suspect of the Inquisition. In Galileo's letters of 1613 and 1615 he wrote that to him God's mind contains all the natural laws; consequently he held that the occasional glimpses of these laws which the human investigator may gain were proofs and direct revelations of the Deity, quite as valid and grand as those recorded in the Bible. "From the Divine Word, the Sacred Scripture and Nature did both alike proceed....Nor does God less admirably discover himself to us in Nature's actions than in the Scripture's sacred dictions." These opinions—held by many present-day scientists, and no longer regarded as being in conflict with theological doctrines—could, however, be regarded at Galileo's time as symptoms of pantheism. This was one of the heresies for which Galileo's contemporary, Giordano Bruno, had been burned at the stake in 1600. The Inquisition was alarmed by Galileo's contention that the Bible was not a certain source of knowledge for the teaching of natural science. Thus he quoted Cardinal Baronius' saying: "The Holy Spirit intended to teach us [in the Bible] how to go to heaven, not how the heavens go."

Every subject for theory and creative action has examples of rejection and repression. The late Boris Pasternak of the U.S.S.R. was ordered to refuse the Nobel Prize in literature.

Try to develop a concern for the social atmosphere toward creativity. Tolerance is perhaps all we can hope to achieve.

Students might discuss the motives which forced Galileo and the Inquisition into opposition.

One might see Brecht's play "Galileo" for a discussion of why or how he should have acted during and after his trial.

A: Galileo

Though he was old and ailing, Galileo was called to Rome and confined for a few months. From the proceedings of Galileo's trial, of which parts are still secret, we learn that he was tried, threatened with torture, induced to make an elaborate formal confession of improper behavior and a denial of the Copernican theory. Finally he was sentenced to perpetual house arrest. According to a well-known legend, at the end of his confession Galileo muttered "eppur si muove"—"but it does move." None of Galileo's friends in Italy dared to defend him publicly. His book was placed on the Index where it remained, along with that of Copernicus and one of Kepler's, until 1835. Thus he was used as a warning to all men that the demand for spiritual conformity also required intellectual conformity.

But without intellectual freedom, science cannot flourish for long. Perhaps it is not a coincidence that for 200 years after Galileo, Italy, which had been the mother of outstanding men, produced hardly a single great scientist, while elsewhere in Europe they appeared in great numbers. Today scientists are acutely aware of this famous part of the story of planetary theories. Teachers and scientists in our time have had to face powerful enemies of open-minded inquiry and of free teaching. Today, as in Galileo's time, men who create or publicize new thoughts must be ready to stand up before other men who fear the open discussion of new ideas.

In reference to Plato's recommendation, what alternatives are available in a society which professes to tolerate, if not welcome, differences of opinion?
Recently, civil rights and anti-war demonstrations are examples. Try to keep the discussions concerned with the alternative actions we have as individuals and as responsible groups.

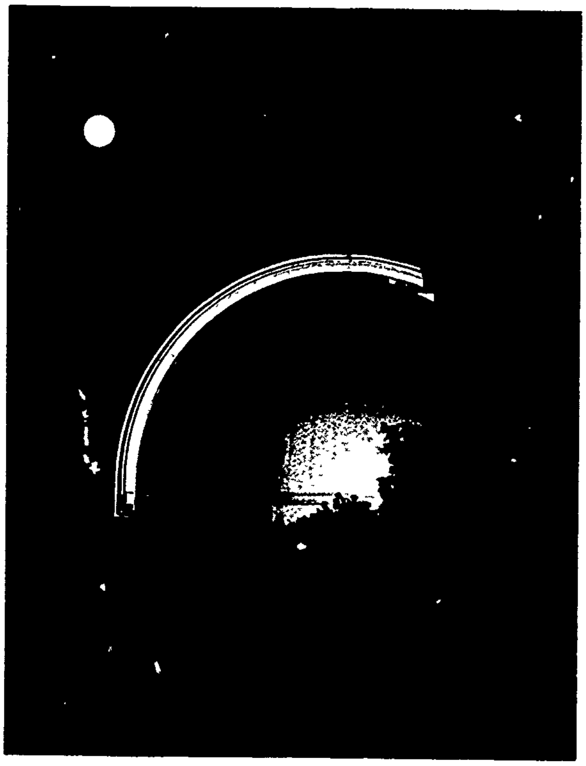
Plato knew that an authoritarian state is threatened by intellectual nonconformists and had recommended for them the well-known treatments: "reeducation, prison, or death." Recently, Russian geneticists have been required to reject well-established theories, not on grounds of persuasive new scientific evidence, but because of conflicts with political doctrines. Similarly, discussion of the theory of relatively was banned from textbooks in Nazi Germany because Einstein's Jewish heritage was said to invalidate his work for Germans. Another example of intolerance was the "Monkey Trial" held during 1925 in Tennessee, where the teaching of Darwin's theory of biological evolution was attacked because it conflicted with certain types of Biblical interpretation.

The warfare of authoritarianism against science, like the warfare of ignorance against knowledge, is still with us. Scientists take comfort from the verdict of history. Less than 50 years after Galileo's trial, Newton's great book, the Principia, brilliantly united the work of Copernicus, Kepler and Galileo with Newton's new statements

of principles of mechanics. Thus the hard-won new laws and new views of science were established. What followed has been termed by historians The Age of Enlightenment. SG 7 6

Q18 What major change in the interpretations of observations was illustrated by the work of both Kepler and Galileo?

Q19 What are some of the reasons that caused Galileo to be tried by the Inquisition?



Palomar Observatory houses the 200-inch Hale reflecting telescope. It is located on Palomar Mountain in southern California.

Study Guide

7.1 If a comet in an orbit around the sun has a mean distance of 20 A.U. from the sun, what will be its period? *89 years*

7.2 A comet is found to have a period of 75 years.

a) What will be its mean distance from the sun? *18 A.U.*

b) If its orbital eccentricity is 0.90, what will be its least distance from the sun? *1.8 A.U.*

c) What will be its velocity at aphelion compared to its velocity at perihelion? *0.053*

7.3 What is the change between the earth's lowest speed in July when it is 1.02 A.U. from the sun and its greatest speed in January when it is 0.98 A.U. from the sun? $v_1 = .96\, v_2$ *(about 4%)*

7.4 The mean distance of the planet Pluto from the sun is 39.6 A.U. What is the orbital period of Pluto? *249 years*

7.5 The eccentricity of Pluto's orbit is 0.254. What will be the ratio of the minimum orbital speed to the maximum orbital speed of Pluto? $v_{max} = 1.680\, v_{min}$

7.6 What are the current procedures by which the public is informed of new scientific theories? To what extent do these news media emphasize any clash of assumptions? *Discussion*

Chapter 8 The Unity of Earth and Sky — The Work of Newton

Chapter 8, the culmination of the unit, deals with Newton's Law of Universal Gravitation. Inasmuch as this is a universal law, it must be considered in an astronomical context. By uniting the laws for terrestrial motion, discussed in Unit 1, and Kepler's laws of heavenly motion, Newton produced a synthesis that could apply to all bodies everywhere. The ways by which such a universal law might be confirmed lead the discussion into various aspects of astronomical study.

Section 8.19 is important as a summary of how scientific theories are made and tested.

Do not overlook the epilogue on page 118.

to go once completely around the orbit; while the mean distance is a. Specifically, the law states that: <u>the squares of the periods of the planets are proportional to the cubes of their mean distances from the sun</u>. In the short form of algebra, this is,

$$T^2 = ka^3, \text{ or } T^2/a^3 = k. \qquad (7.3)$$

For the earth, T is one year. The mean distance of the earth from the sun, a, is one astronomical unit, A.U. Then by Eq. (7.3) we have

$$(1 \text{ year})^2 = k(1 \text{ A.U.})^3, \text{ or } k = 1 \text{ yr}^2/\text{A.U.}^3.$$

Because this relation applies to all the planets, we can use it to find the period or mean distance of any planet when we know either of the quantities. Thus Kepler's third law, the Law of Periods, establishes a beautifully simple relation among the planetary orbits.

Kepler's three laws are so simple that their great power may be overlooked. When they are combined with his discovery that each planet moves in a plane passing through the sun, they let us derive the past and future history of each planet's motion from only six quantities, known as the orbital elements. Two of the elements are the size and shape of the orbit in its plane, three other elements are angles that orient the planet's orbit in its plane and relate the plane of the planet to that of the earth's orbit, while the sixth element tells where in the orbit the planet was on a certain date. These elements are explained more fully in optional Experiment 21 on the orbit of Halley's Comet.

It is astonishing that the past and future positions of each planet can be derived in a simpler and more precise way than through the multitude of geometrical devices on which

SG 7.1

SG 7.2

SG 7.3

• This choice of units gives the simplest value for k.

We know from observation that the orbital period T of Jupiter is about 12 years. What value for a, the mean distance from the sun, is predicted on the basis of Kepler's third law?

Solution:

$$a_j^3 = T_j^2/k$$
$$= 144\ (\text{yr}^2)/1\ (\text{yr}^2/\text{A.U.}^3)$$
$$= 144 \text{ A.U.}^3$$
$$a_j = \sqrt[3]{144 \text{ A.U.}^3}$$
$$= 5.2 \text{ A.U.}$$

In Chapter 8 we consider the factors that go into "k" and eventually derive it from other observed quantities.

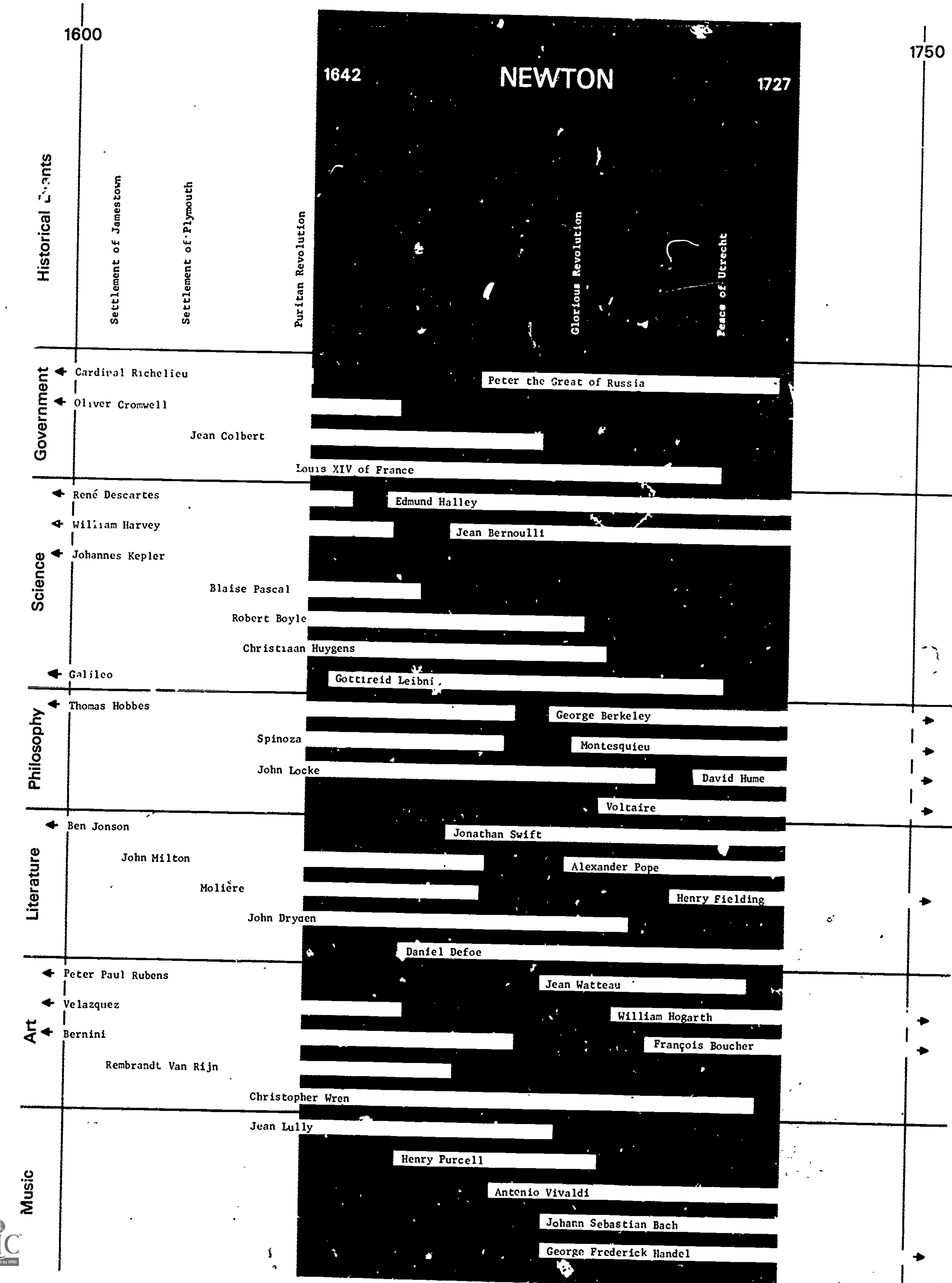
As a brief home activity the 3-d development of the orbit of Halley's comet is enlightening. It can be used to explain Fig. 6.10, page 43.

Introduction: Science in the seventeenth century. In the 44 years between the death of Galileo in 1642 and the publication of Newton's <u>Principia</u> in 1687, major changes occurred in the social organization of scientific studies. The "New Philosophy" of experimental science, applied by enthusiastic and imaginative men, was giving a wealth of new results. Because these men were beginning to work together, they formed scientific societies in Italy, England and France. One of the most famous is the Royal Society of London for Promoting Natural Knowledge, which was founded in 1660. Through these societies the scientific experimenters exchanged information, debated new ideas, published technical papers, and sometimes quarreled heartily. Each society sought support for its work, argued against the opponents of the new experimental activities and published studies in scientific journals, which were widely read. Through the societies scientific activities were becoming well-defined, strong and international.

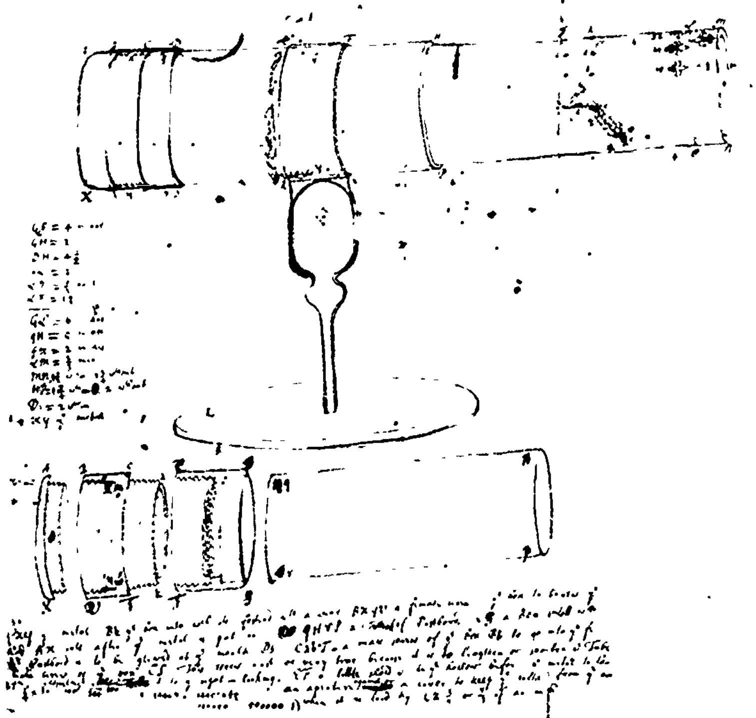
This development of scientific activities was part of the general cultural, political and economic changes occurring in the sixteenth and seventeenth centuries (see the chart). Both craftsmen and men of leisure and wealth became involved in scientific studies. Some sought the improvement of technological methods and products. Others found the study of nature through experiment a new and exciting hobby. But the availability of money and time, the growing interest in science and the creation of organizations are not enough to explain the growing success of scientific studies. Historians agree that this rapid growth of science depended upon able men, well-formulated problems and good mathematical

Summary 8.1 intro.
During the latter half of the seventeenth century scientists began forming societies to discuss new ideas, publish scientific papers and sponsor lectures.

Developments in science had quickened greatly between the time of Kepler and Galileo, and Newton half a century later.



1600
1750
1642
NEWTON
1727
Historical Events
Settlement of Jamestown
Settlement of Plymouth
Puritan Revolution
Glorious Revolution
Peace of Utrecht
Government
Cardinal Richelieu
Oliver Cromwell
Jean Colbert
Louis XIV of France
Peter the Great of Russia
Science
René Descartes
William Harvey
Johannes Kepler
Blaise Pascal
Robert Boyle
Christiaan Huygens
Galileo
Edmund Halley
Jean Bernoulli
Gottfreid Leibni
Philosophy
Thomas Hobbes
Spinoza
John Locke
George Berkeley
Montesquieu
David Hume
Voltaire
Literature
Ben Jonson
John Milton
Molière
John Dryden
Jonathan Swift
Alexander Pope
Henry Fielding
Daniel Defoe
Art
Peter Paul Rubens
Velazquez
Bernini
Rembrandt Van Rijn
Christopher Wren
Jean Watteau
William Hogarth
François Boucher
Music
Jean Lully
Henry Purcell
Antonio Vivaldi
Johann Sebastian Bach
George Frederick Handel



Newton entered Trinity College, Cambridge University, in 1661 at the age of eighteen. He soon was doing experiments and teaching while still a student. This early engraving shows the quiet student wearing a wig and heavy academic robes. Trinity students even today wear the same type of robes to their classes. The drawing of the telescope and its parts was done by Newton, while he was still a student.

each field stimulated developments in the other. Similarly, the instrument-maker and the scientist aided each other.

Another factor of great importance was the rapid accumulation of scientific knowledge. Each study could build on those done previously. Thus from the time of Galileo repeatable experiments woven into testable theories were available for study, modification and application.

Newton, who lived in this new scientific age, is the central person in this chapter. However, before we follow Newton's work, we must recall that in science as in any other field, many men made useful contributions. The whole structure of science depends not only upon those whom we call geniuses, but also upon many lesser-known men. As Lord Rutherford, one of the founders of modern atomic theory, said:

> It is not in the nature of things for any one man to make a sudden violent discovery; science goes step by step, and every man depends upon the work of his predecessors....Scientists are not dependent on the ideas of a single man, but on the combined wisdom of thousands of men.

Properly we should trace in each man's contribution his dependence upon those who worked before him, the influence of his contemporaries and his influence upon his successors. While this would be interesting and rewarding, here we can only briefly hint at these relationships.

Fig. 8.1 Chronological table of Newton's era

Summary 8.1
Unique details of Newton's life and character are sketched.

See "Newton and the Principia" in Project Physics Reader 2.

8.1 A sketch of Newton's life. Isaac Newton was born on Christmas day, 1642, in the small English village of Woolsthorpe in Lincolnshire. He was a quiet farm boy, who, like young Galileo, loved to build and tinker with mechanical gadgets and seemed to have a liking for mathematics. With financial help from an uncle he went to Trinity College of Cambridge University in 1661. There he initially enrolled in the study of mathematics as applied to astrology and he was an eager and excellent student. In 1665, to escape the Black Plague (bubonic plague) which swept through England, Newton went home to the quiet farm in Woolsthorpe. There, by the time he was twenty-four, he had quietly made spectacular discoveries in mathematics (binomial theorem, differential calculus), optics (theory of colors) and mechanics. Referring to this period, Newton once wrote:

> I began to think of gravity extending to the orb of the moon, and...from Kepler's rule [third law]...I deduced that the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since.

Thus during his isolation the brilliant young Newton had developed a clear idea of the first two Laws of Motion and of the formula for gravitational attraction. However, he did not announce the latter until many years after Huygens' equivalent statement.

In Unit 1 a newton was defined as the force needed to give an acceleration of 1 meter per sec^2 to a 1-kilogram mass. In newtons, what approximately is the earth's gravitational attraction on an apple?

The weight of an apple is about one newton.

This must have been the time of the famous and disputed fall of the apple. One of the better authorities for this story is a biography of Newton written in 1752 by his friend William Stukeley, where we can read that on one occasion Stukeley was having tea with Newton in a garden under some apple trees, when Newton recalled that

> he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in a contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre?

The main emphasis in this story should probably be placed on the word contemplative. Moreover, it fits again the pattern we have seen before: a great puzzle (here, that of the forces acting on planets) begins to be solved when a clear-thinking person contemplates a long-known phenomenon. Where others had seen no relationship, Newton did. Similarly

When Newton initially solved many of the questions whose solutions were finally published in the Principia, he was only 21 or 22 years old.

Galileo used the descent of rolling bodies to show the usefulness of mathematics in science. Likewise, Kepler used a small difference between theory and observation in the motion of Mars as the starting point for a new approach to planetary astronomy.

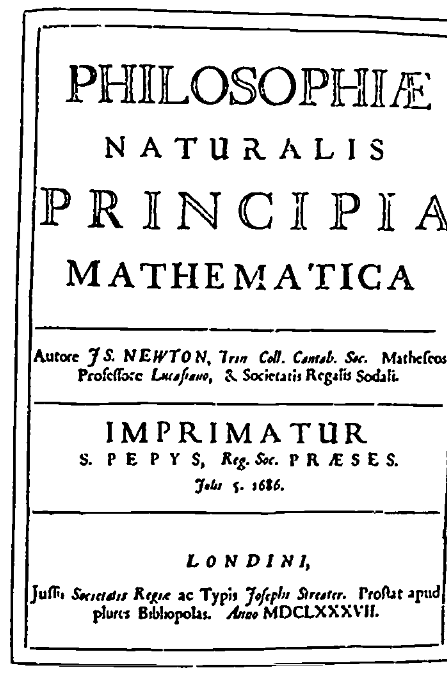
Presumably without the encouragement of Halley, Newton would never have published any of these major results.

Soon after Newton's return to Cambridge, he was chosen to follow his teacher as professor of mathematics. He taught at the university and contributed papers to the Royal Society, at first particularly on optics. His Theory of Light and Colors, when finally published in 1672, involved him in so long and bitter a controversy with rivals that the shy and introspective man resolved never to publish anything more (but he did).

Newton's aversion to publication resulted from his unhappiness about the arguments following his first papers on the theory of light and colors (1672).

Newton made the first reflecting telescope.

In 1684 Newton's devoted friend Halley came to ask his advice in a dispute with Wren and Hooke about the force that would have to act on a body moving along an ellipse in accord with Kepler's laws. Halley was pleasantly surprised to learn that Newton had already derived the rigorous solution to this problem ("and much other matter"). Halley then persuaded his reluctant friend to publish his studies, which solved one of the most debated and interesting scientific questions of the time. To encourage Newton, Halley became responsible for all the costs of publication. In less than two years of incredible labors, Newton had the Principia (Fig. 8.2) ready for the printer. Publication of the Principia in 1687 quickly established Newton as one of the greatest thinkers in history.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Fig. 8.2 Title page of Principia Mathematica. Because the Royal Society sponsored the book, the title page includes the name of the Society's president, Samuel Pepys, whose diary is famous.

A few years afterwards, Newton had a nervous breakdown. He recovered, but from then until his death 35 years later, in 1727, he made no major new scientific discoveries. He rounded out earlier studies on heat and optics, and turned more and more to writing on theological chronology. During those years he received honors in abundance. In 1699 he was appointed Master of the Mint, partly because of his great interest in and knowledge about the chemistry of metals, and he helped to re-establish the British currency, which had become debased. In 1689 and 1701 he represented his university in Parliament, and he was knighted in 1705 by Queen Anne. He was president of the Royal Society from 1703 to his death in 1727, and he was buried in Westminster Abbey.

Check the bibliography for a good source on the life of Newton. Asimov's encyclopedia or Scientific American (Vol. 139, #6, Dec.'55: "Isaac Newton", I.B. Cohen) would be a good starter.

Q1 Why might we conclude that Newton's isolation on the farm during the Plague Years (1665-66) contributed to his scientific achievements?

Q2 What was the important role played by the scientific societies? Do such societies today perform the same functions?

Q3 What was important about Newton's mood when he noticed the apple fall?

8.2 Newton's Principia. In the original preface to Newton's Principia—one of the most important books in the history of science—we find a clear outline of the book:

Summary 8.2
Newton assumed that similar events result from the same cause; properties common to all observed bodies apply to all bodies in general; conclusions based on experiences can be regarded as very nearly true.

> Since the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things, and the moderns, rejecting substantial forms and occult qualities, have endeavored to subject the phenomena of nature to the laws of mathematics, I have in this treatise cultivated mathematics as far as it relates to philosophy [we would say 'physical science']...for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate [induce] the forces of nature, and then from these forces to demonstrate [deduce] the other phenomena, and to this end the general propositions in the first and second Books are directed. In the third Book I give an example of this in the explication of the system of the World; for by the propositions mathematically demonstrated in the former Books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea [tides]....

See Newton's Laws of Motion and Proposition One in Project Physics Reader 2.

The work begins with a set of definitions—mass, momentum, inertia, force. Next come the three Laws of Motion and the principles of composition of vectors (forces and velocities), which were discussed in Unit 1. Newton then included an equally remarkable and important passage on "Rules of Reasoning in Philosophy." The four rules or assumptions, reflecting his profound faith in the uniformity of all nature, were intended to guide scientists in making hypotheses. These are still useful. The first has been called a Principle of Parsimony; the second and third, Principles of Unity. The fourth is a faith without which we could not use the process of logic.

In a short form, with some modern words, these rules are:

These Rules are stated by Newton at the beginning of Book III of the Principia, p. 398 of the Cajori edition, University of California Press.

I. Nature is essentially simple; therefore we should not introduce more hypotheses than are sufficient and necessary for the explanation of observed facts. "Nature does nothing ...in vain, and more is in vain when less will serve." This fundamental faith of all scientists is almost a paraphrase of Galileo's "Nature...does not that by many things, which may be done by few"—and he, in turn, quoted the same opinion from Aristotle. Thus the rule has a long history.

Rule one is similar to "Ockham's Razor."

One might discuss the existence of flying saucers in this context.

II. "Therefore to the same natural effects we must, as far as possible, assign the same causes. As to respiration in a man and in a beast; the descent of stones in Europe and in America; ...the reflection of light in the earth, and in the planets."

A discussion could be developed around these (Newton's) Four Rules of Reasoning in Philosophy.

III. Properties common to all those bodies within reach of our experiments are to be assumed (even if only tentatively) to apply to all bodies in general. Since all physical objects known to experimenters had always been found to have mass, this rule would guide Newton to propose that every object has mass.

Note that these are essentially assumptions and statements of faith.

IV. In "experimental philosophy," those hypotheses or generalizations which are based on experience are to be accepted as "accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined" until we have additional evidence by which our hypotheses may be made more accurate, or revised.

You might wish to have students examine a copy of the two-volume paperbacked edition of the Principia. This was prepared by F. Cajori and is based upon the first English translation from the Latin by Motte in 1729, (University of California Press, # 62, 1962).
If students examine this book, they may realize how differently modern scientific arguments are presented. They might also realize why the Principia never reached a popular audience.

The Principia was an extraordinary document. Its three main sections contained a wealth of mathematical and physical discoveries. But overshadowing everything else in the book is the theory of universal gravitation, with Newton's proofs and arguments leading to it. Newton uses a form of argument patterned after that of Euclid—the type of proofs you encountered in your geometry studies. Because the detailed mathematical steps used in the Principia are no longer familiar, the steps which are given below have often been restated in modern terms.

The central idea of universal gravitation can be very simply stated: every object in the universe attracts every other object. Moreover, the magnitude of these attractions depends in a simple way on the distance between the objects. Such a sweeping assertion certainly defies full and detailed verification—for after all, we cannot undertake to measure the forces experienced by all the objects in the universe!

Q4 Write a brief simple restatement of each of Newton's Rules of Reasoning.

Q5 Why is the Principia difficult for us to read?

8.3 A preview of Newton's analysis. We shall now preview Newton's development of his theory of universal gravitation. Also we shall see how he was able to use the theory to unify the main strands of physical science which had been developing independently. As we proceed, notice the extent to which Newton relied on the laws found by Kepler; be alert to the appearance of new hypotheses and assumptions; and watch for the interaction of experimental observations and theoretical deductions. In short, in this notable and yet typical case, aim for an understanding of the process of the construction and verification of a theory; do not be satisfied with a mere memorization of the individual steps.

Summary 8.3
1. There is a clear relationship between Newton's general laws of motion for all bodies and Kepler's specific laws of motion for planets.

2. It was Newton's unique ability to synthesize these laws into a common set that established the basis of dynamical theory in physics.

A very good article, "A view of Sir Isaac Newton's Philosophy," in the January, 1966 issue of The Physics Teacher.

Emphasize the methodology of the Newtonian synthesis, especially how he ran his arguments backward to clinch his analysis.

Books which can supply excellent ideas for discussion:
(1) The Common Sense of Science, J. Bronowski, Chapter 3 - "Isaac Newton's Model."
(2) The Origins of Modern Science, H. Butterfield, Chapter 8 - "History of the Modern Theory of Gravitation".

What were the known laws about motion that Newton unified? He had his own three laws, which you met in Unit 1. Also he had the three Laws of Planetary Motion stated by Kepler, which we considered in Chapter 7.

Newton (from Chapter 3)	Kepler (from Chapter 7)
1. A body continues in a state of rest, or of uniform motion in a straight line, unless acted upon by a net force. (Law of Inertia.)	1. The planets move in orbits which are ellipses and have the sun at one focus.
2. The net force acting on an object is directly proportional to and in the same direction as the acceleration.	2. The line from the sun to a planet sweeps over areas which are proportional to the time intervals.
3. To every action there is an equal and opposite reaction.	3. The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. $T^2 = k \cdot a^3$ Eq. (7.3).

Now let us preview the argument by which these two sets of laws can be combined.

The curvature of planetary motions requires a net force.

According to Kepler's first law, the planets move in orbits which are ellipses, that is, curved orbits. But according to Newton's first law a change of motion, either in direction or amount, results when a net force is acting. Therefore we can conclude that a net force is continually accelerating all the planets. However, this result does not specify what type of force is acting or whether it arises from some particular center.

Students may restate these arguments as formal syllogisms.

Newton's second law combined with Kepler's first two laws was the basis on which Newton achieved a brilliant solution to a very difficult problem. Newton's second law says that in every case the net force is exerted in the direction of the observed acceleration. But what was the direction of the acceleration? Was it toward one center or perhaps toward many? By a geometrical analysis Newton found that any moving body acted upon by a central force, that is, any force centered on a point, when viewed from that point, will move according to Kepler's Law of Areas. Because the areas found by Kepler were measured around the sun, Newton could conclude that the sun at the focus of the ellipse was the source of the central force.

Motion influenced by any central force will satisfy the Law of Areas.

The net force accelerates the planets toward the sun.

Newton considered a variety of force laws centered on points at various places. Some of his results are quite unexpected. For example, he found that circular motion would result from an inverse-fifth power force law, $F \propto 1/R^5$,

acting from a point on the circle! But, as we have just seen, the center of the force on the planets had to be at the sun, located at one focus of the elliptical orbits. For an elliptical orbit, or actually for an orbit along any of the conic sections discussed in Chapter 7, the central force from the focus had to be an inverse-square force, $F \propto 1/R^2$. In this way Newton found that only an inverse-square force centered on the sun would result in the observed motions of the planets as described by Kepler's first two laws. Newton then clinched the argument by finding that such a force law would also result in Kepler's third law, the Law of Periods, $T^2 = ka^3$.

If the strength of the force varies inversely with the square of the distance, that is,

$$F \propto 1/R^2,$$

the orbits will be ellipses, or some other conic section.

From this analysis Newton concluded that one general Law of Universal Gravitation, that applied to the earth and apples, also applied to the sun and the planets, and all other bodies, such as comets, moving in the solar system. This was Newton's great synthesis. He brought together the terrestrial laws of motion, found by Galileo and others, and the astronomical laws found by Kepler. One new set of laws explained both. Heaven and earth were united in one grand system dominated by the Law of Universal Gravitation. No wonder that the English poet Alexander Pope wrote:

Point out that the same information which was available to Newton was also available to other competent scientists at the time (Halley, Wren, Hooke).

> Nature and Nature's laws lay hid in night:
> God said, Let Newton be! and all was light.

As you will find by inspection, the Principia was filled with long geometrical arguments and was difficult to read. Happily, gifted popularizers wrote summaries, through which many people learned Newton's arguments and conclusions. In Europe one of the most widely read of these popular books was published in 1736 by the French philosopher and reformer Voltaire.

Readers of these books were excited, and perhaps puzzled by the new approach and assumptions. From ancient Greece until well after Copernicus, the ideas of natural place and natural motion had been used to explain the general position and movements of the planets. The Greeks believed that the planets were in their orbits because that was their proper place. Furthermore, their natural motions were, as you have seen, assumed to be at uniform rates in perfect circles, or in combinations of circles. However, to Newton the natural motion of a body was at a uniform rate along a straight line. Motion in a curve was evidence that a net force was continually accelerating the planets away from their natural motion along straight lines. What a reversal of the assumptions about the type of motion which was "natural"!

Emphasize the contrast in assumptions essentially centering on the concept of inertial motion. Compare to U.S. Declaration of Independence—"These truths we hold to be self-evident."

Summary Sec 8.4: In his laws of motion, Newton demonstrated that: (a) the net force on the planets is accelerating them toward the sun, (b) a body acted upon by a central force will move in accordance with Kepler's law of areas.

This sketch of Newton's argument outlines his procedure and the interweaving of earthly physics with astronomical conclusions from Kepler. Now we shall examine the details of Newton's analysis and begin as he did with a study of the motion of bodies accelerated by a central force.

Q6 Complete the following summary of Newton's analysis:

Step 1. Kepler's ellipses + Newton's first law (inertia) → ?

Step 2. Newton's second law (force) + Kepler's area law → ?

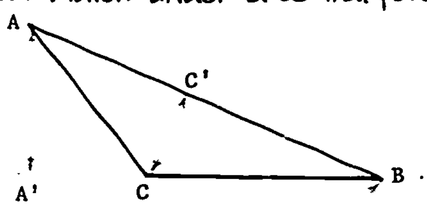
Step 3. Add Kepler's ellipses → ?

Step 4. What observed properties of the planetary orbits required that $F \propto 1/R^2$?

Step 5. Does the result agree with Kepler's Law of Periods?

T18: Motion under a central force.

8.4 Motion under a central force. How will a moving body respond to a central force? Before we proceed with the analysis, we need to review one basic property of triangles. The area of a triangle equals ½ base × altitude. But, as Fig. 8.3 shows, any of the three sides can be chosen as the base with the corresponding vertex being the intersection of the other two sides.



altitude	base
CC'	AB
BB'	AC
AA'	BC

Fig. 8.3 The altitude of a triangle is the perpendicular distance from the vertex of two sides to the third side, which is then the base.

1. Suppose that a body, initially at rest at a point Q, is exposed to a brief force, like a hammer blow, directed toward O. The body will be accelerated toward O. It will begin to move toward O, and after some definite time interval Δt, it will have moved a definite distance to a new point Q', i.e., through the distance QQ'. (Fig. 8.4a.)

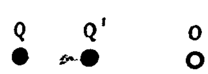


Fig. 8.4(a)

2. Suppose that the body was initially at some point P and moving at a uniform speed v along the straight line through PQ. (See Fig. 8.4b.) In equal intervals of time, Δt, it will move equal distances, PQ, QR, RS, etc. How will its motion appear to an observer at some point O?

To prepare for what follows—and to make incidentally a discovery that may be surprising—consider first the triangles OPQ and OQR. These have equal bases, PQ and QR; and also equal altitudes, ON. Therefore they have equal areas. And therefore, the line from any observer not on the line, say at point O, to a body moving at a uniform speed in a straight line, like PQR will sweep over equal areas in equal times. Strange as it may seem, Kepler's Law of Areas applies even to a body on which there is no net force, and which therefore is moving uniformly along a straight line.

Refer to Fig. 8.4

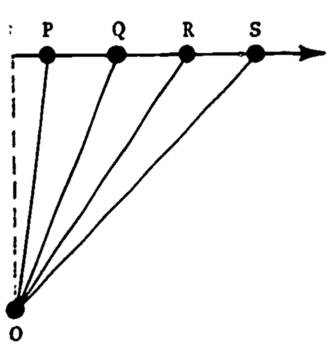


Fig. 8.4(b) A body moving at a uniform rate in a straight line is viewed from point O.

3. How will the motion of the object be changed if at point Q it is exposed to a brief force, such as a blow, directed toward point O? A combination of the constructions in 1 and 2 above can be used to determine the new velocity vector. (See Fig. 8.4c.) As in 1 above, the force applied

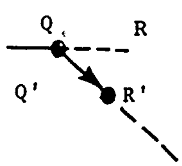


Fig. 8.4(c) A force applied briefly to a body moving in a straight line QR changes the motion to QR'.

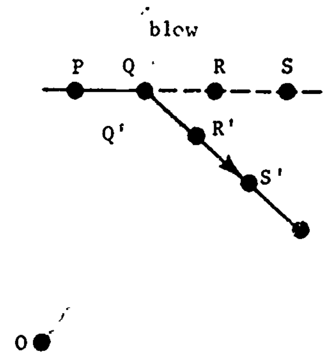


Fig. 8.4(d) The areas of OPQ and OQR' are equal.

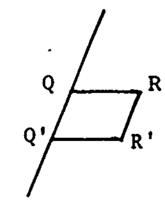


Fig. 8.4(e) The distances of R and R' from QQ' are equal.

E20: Stepwise approximation to an orbit

at point Q accelerates the object toward the center. In the time interval Δt, a stationary object at Q would move to point Q′. But the object was moving and, without this acceleration, would have moved to point R. Then, as Fig. 8.4c indicates, the resultant motion is to point R′. Fig. 8.4d shows Fig. 8.4c combined with Fig. 8.4b.

Earlier we found that the areas of the triangles OPQ and OQR were equal. Are the areas of the triangles OQR and OQR′ also equal? To examine this question we can consider the perpendicular distances of R and R′ from the line Q′Q as the altitudes of the two triangles. As Fig. 8.4e shows, R and R′ are equally distant from the line Q′Q. Both triangles also have a common base, OQ. Therefore, the areas of triangles OQR and OQR′ are equal. Thus we may conclude that the line from the center of force, point O, to the body sweeps over equal areas in equal time intervals.

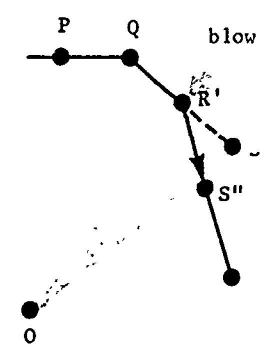


Fig. 8.4(f) A force applied at R' causes the body to move to S''.

If another blow directed toward O, even a blow of a different magnitude, were given at point R′, the body would move to some point S′′, as indicated in Fig. 8.4f. By a similar analysis you can find that the areas of triangles OR′S′′ and OR′S′ are equal. Their areas also equal the area of triangle OPQ.

L 13: Program orbit I
L 14: Program orbit II
A: Three-dimensional model of two orbits

SG 8.1

In the geometrical argument above we have considered the force to be applied at intervals Δt. What motion will result if each time interval Δt is made vanishingly small and the force is applied continuously? As you would suspect, and as can be shown by rigorous proof, the argument holds for a continuously acting central force. We then have an important conclusion: if a body is continuously acted upon by any central force, it will move in accordance with Kepler's Law of Areas. In terms of the planetary orbits from which the Law of Areas was found, the accelerating force must be a central force. Furthermore, the sun is at the center of the force. Notice that the way in which the strength of the central force depends on distance has not been specified.

Does this conclusion apply if the central force is one of repulsion rather than attraction?

Any student who is interested in the impact of Newtonian dynamics on 18th century writers should see Marjorie Nicholson's Science and Imagination Also the Prologue to Unit 3.

Q7 What types of motion satisfy Kepler's Law of Areas?

Summary 8.5
Kepler's third law, $T^2 = kR^3$, describes motion in which the central attracting force varies inversely with the square of the distance.

8.5 The inverse-square law of planetary force. In the last section we found that the force influencing the motion of a planet had to be a central force toward the sun. But many questions remained to be considered. How did the strength of the force change with the planet's distance from the center of motion? Would one general force law account for the motions of all the planets? Or was a different law needed for each planet? Would the force law be consistent with Kepler's third law, $T^2 \propto R^3$? How could this be tested? And from what observations could the amount of the force be determined?

L16: Kepler's Laws

Clearly, the nature of the force law should now be found; but how? The Law of Areas will not be useful, for we have seen that it is satisfied by any central force. But Kepler's first law (elliptical orbits) and third law (the relation between distance and period) remain to be used. We could start by considering what the relation between force and distance must be in order to satisfy the third law. Newton preferred to assume a force law and tested it against Kepler's third law.

See "Newton's Laws of Dynamics" in Project Physics Reader 1.

The argument here is a reasonable one. Students might discuss what alternatives seem possible and whether they seem reasonable. The effort here is to develop an intuitive feeling for the kind of problem to be solved and the general nature of the force law.

Some clues suggest what force law to consider first. Because the orbits of the planets are curved toward the center of motion at the sun, the force must be one of attraction toward the sun. While the force might become greater with distance from the sun, we could expect the force to decrease with distance. Possibly the force varies inversely with the distance: $F \propto 1/R$. Or perhaps, like the brightness of a light, the force decreases with the inverse square of the distance: $F \propto 1/R^2$. Possibly the force weakens very rapidly with distance: $F \propto 1/R^3$, or even $F \propto 1/R^n$, where n is some large number. One could try any of these possibilities. But here let us follow Newton's lead and test whether $F \propto 1/R^2$ agrees with the astronomical results.

Newton assumed that:

$$F_{grav} \propto \frac{1}{R^2}.$$

Note here that others besides Newton were asking about the nature of the force law.

Actually Newton's choice of this inverse-square law was not accidental. Others such as Wren, Halley and Hooke were attempting to solve the same problem and were considering the same force law. In fact, Halley came to Newton in 1684 specifically to ask if he could supply a proof of the correctness of the inverse-square law which the others were seeking in vain. Even if Newton had not already derived the proof, it is likely that someone would have soon. At any one time in the development of science, a rather narrow range of interesting and important problems holds the attention of many scientists in a given field, and often several solutions are proposed at a the same time.

How can you explain the historical fact that many scientific discoveries have been made independently and almost simultaneously by two or more men? What examples can you list?

Because Kepler's third, or Harmonic, law relates the periods of different planets to their distances from the sun, this law should be useful in our study of how the gravitational attraction of the sun changes with distance. We have another clue from Galileo's conclusion that the distance d through which a body moves as a result of the earth's gravitational attraction increases with the square of the time, t^2. The full relation is

Note that by using ratios we have avoiding needing any scale factors for d or R. The argument is based on the geometry of Fig. 8.5. When applied to two planets, the relation of their periods and solar distances can be compared.

$$d = \tfrac{1}{2}at^2,$$

For classroom discussion, you may wish to draw the full circle of which Fig. 8.5 gives only a section

where a is a constant acceleration.

We wish to compare the central forces, or accelerations, acting on two planets at different distances from the sun. Fig. 8.5 indicates the geometry for two planets moving in circular orbits. For convenience we can consider one of the planets to be the earth, E. The other planet P can be at any distance from the sun. According to Newton's Law of Inertia, any planet continually tends to move in a straight line. But we observe that it actually moves in an orbit which is (nearly) a circle. Then, as Fig. 8.5 indicates, in a small interval of time t the planet moves forward, and also falls a distance d toward the sun. No matter how large the orbit is, the motion of each planet is similar to that of the others, though each has its own period T.

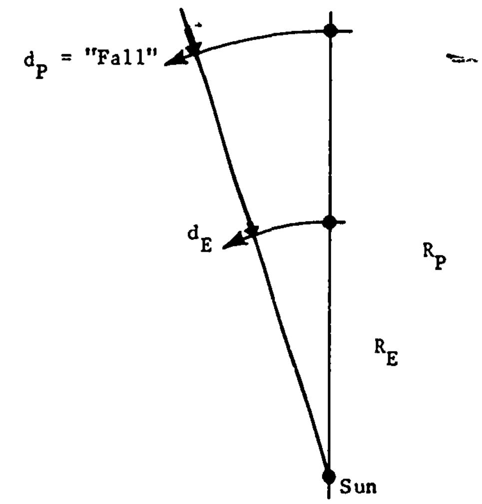


Fig. 8.5 The "fall" of the earth and a planet toward the sun.

This time t taken for either planet to move through the portion of the orbit indicated in Fig. 8.5 is a fraction of the total time T required for the planet to make one revolution around the sun. No matter how large the orbit, the fraction $\frac{t}{T}$ required to move through this angle, will be the same for any planet. But while the planet moves through this arc, it also falls toward the sun through a distance d. From Fig. 8.5 we can also see that the distance d increases in proportion to the distance R.

L 15: Central forces - iterated blows

Now let us use Galileo's equation to compare the "falls" of two planets toward the sun:

$$\frac{d_P}{d_E} = \frac{\tfrac{1}{2}a_P\ t_P{}^2}{\tfrac{1}{2}a_E\ t_E{}^2}, \text{ or } \frac{d_P}{d_E} = \frac{a_P\ t_P{}^2}{a_E\ t_E{}^2}$$

But because the values of d are proportional to R, we can replace the ratio (d_P/d_E) by (R_P/R_E) and have:

The mathematical argument that begins here is in two parts. The value of the acceleration a is identified (top of page 88) and inserted. The result then leads to Kepler's Third or Harmonic Law.

$$\frac{R_P}{R_E} = \frac{a_P\ t_P{}^2}{a_E\ t_E{}^2}$$

Now, if we can express the a's in terms of R's or t's, we can see whether the result agrees with Kepler's third law.

Assume that $a \propto 1/R^2$.

Like Newton, let us assume that a varies with $1/R^2$. By rearranging the equation we have:

$$\frac{R_P}{R_E} = \frac{\frac{1}{R_P^2}\, t_P^{\,2}}{\frac{1}{R_E^2}\, t_E^{\,2}}, \text{ or } \quad \frac{R_P^{\,3}}{R_E^{\,3}} = \frac{t_P^{\,2}}{t_E^{\,2}} .$$

This has the same form as Kepler's third law, but involves the times t for the planet to move through a small arc, rather than the full period T. However, we have seen that the times t required for planets to move through the same arcs of circular orbits were the same fractions $\frac{t}{T}$ of their total period. Thus we can replace t by T and have

$$\frac{R_P^{\,3}}{R_E^{\,3}} = \frac{T_P^{\,2}}{T_E^{\,2}}, \text{ or } \quad R_P^{\,3} = k\, T_P^{\,2} ,$$

where the constant k adjusts the equality for the units in which R and T are expressed. This last equation is Kepler's Law of Periods which we saw as Eq. (7.3).

In this derivation we have made a number of assumptions: 1) that Galileo's law of acceleration relating times and distances of falling bodies on the earth applies to the acceleration of the planets toward the sun, and 2) that the acceleration toward the sun changes as $1/R^2$. As a result we have found that the planets should move according to Kepler's Law of Areas. Since they do, we can (by Newton's fourth rule) accept our assumption that the sun's gravitational attraction does change with $1/R^2$.

A: Demonstrating conic sections, graphical construction of conic sections

A: Conic sections model

In the analysis above we substituted $1/R^2$ for a in the general relation $R = at^2$. What results do you get if you set $a = 1/R^n$, where n is any number? What value must n have to satisfy Kepler's Third Law?

We assumed that the orbits were circles. However, Newton showed that any object moving in an orbit that is a conic section (circle, ellipse, parabola or hyperbola) around a center of force is being acted upon by a net force which varies inversely with the square of the distance from the center of force.

Q8 Why is Kepler's third law ($R^2 \propto T^3$) useful for testing how the gravitational acceleration changed with distance from the sun?

Q9 What simplifying assumptions were made in the derivation here?

8.6 Law of Universal Gravitation. Evidently a central force is holding the planets in their orbits. Furthermore, the strength of this central force changes inversely with the square of the distance from the sun. This strongly suggests that the sun is the source of the force—but it does not necessarily require this conclusion. Newton's results might be fine geometry, but so far they include no physical mech-

Direct attention to the differences between a geometric description in which there are forces between objects, rather than motion around points.

anism. The French philosopher Descartes (1596-1650) had proposed an alternate theory that all space was filled with a subtle, invisible fluid which carried the planets around the sun in a huge whirlpool-like motion. This was an extremely attractive idea, and at the time was widely accepted. However, Newton was able to prove that this mechanism could not account for the quantitative observation of planetary motion summarized in Kepler's laws.

Kepler, you recall had still a different suggestion. He proposed that some magnetic force reached out from the sun to keep the planets moving. To Kepler this continual push was necessary because he had not realized the nature of inertial motion. His model was inadequate, but at least he was the first to regard the sun as the controlling mechanical agent behind planetary motion. And so the problem remained: was the sun actually the source of the force? If so, on what characteristics of the sun did the amount of the force depend?

At this point Newton proposed a dramatic solution: the force influencing the planets in their orbits is nothing other than a gravitational attraction which the sun exerts on the planets. This is a gravitational pull of exactly the same sort as the pull of the earth on an apple. This assertion, known as the Law of Universal Gravitation, says:

<u>every object in the universe attracts every other object with a gravitational force</u>.

If this is so, there must be gravitational forces between a rock and the earth, between the earth and the moon, between Jupiter and its satellites—and between the sun and each of the planets.

But Newton did not stop by saying only that there is a gravitational force between the planets and the sun. He further claimed that the force is just exactly the right size to account <u>completely</u> for the motion of every planet. No other mechanism is needed...no whirlpools in invisible fluids, no magnetic forces. Gravitation, and gravitation alone, underlies the dynamics of the heavens.

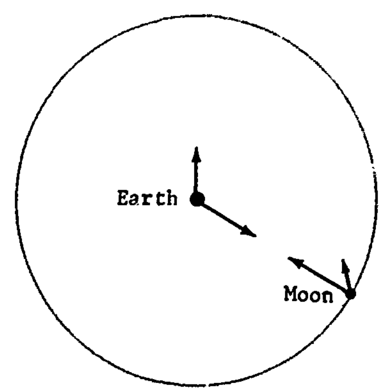
Because this concept is so commonplace to us, we are in danger of passing it by without really understanding what it was that Newton was claiming. First, he proposed a truly universal physical law. He excluded no object in the universe from the effects of gravity. Less than a century before it would have been impious or foolish even to suggest that terrestrial laws and forces were the same as those that regulated the whole universe. But Kepler and Galileo had

Summary 8.6

1. Newton's work forced scientists to consider the planets as substantial moving bodies, not as geometric points.

2. Newton did not attempt to explain the cause of gravitation; in fact, how gravitation acts over large distances has ...en and continues to be an ...eresting problem in physics.

A: Demonstrating vortices



The sun, moon and earth each pull on the other. The forces are in matched pairs in agreement with Newton's third law.

begun the unification of the physics of heaven and earth. Newton was able to carry a step further what had been well begun. Because Newton was eventually able to unite the mechanics of terrestrial objects and the motion of celestial bodies through one set of propositions, his result is called the Newtonian synthesis.

A second feature of Newton's claim, that the orbit of a planet is determined by the gravitational attraction between it and the sun, was to move physics away from geometrical explanations toward physical ones. He shifted the question from "what are the motions?", which Kepler had answered, to "what force effects the motion?" In both the Ptolemaic and Copernican systems the planets moved about points in space rather than about objects, and they moved as they did because they had to by their nature or geometrical shape, not because forces acted on them. Newton, on the other hand, spoke not of points, but of things, of objects, of physical bodies. Without the gravitational attraction of the sun to deflect them continually from straight-line paths, the planets would fly out into the darkness of space. Thus, it was the physical sun which was important rather than the point at which the sun happened to be located.

Newton postulated a specific force. By calling it a force of gravity he was not, however, explaining why it should exist. He seems to be saying essentially this: hold a stone above the surface of the earth and release it. It will accelerate to the ground. Our laws of motion tell us that there must be a force acting on the stone driving it toward the earth. We know the direction of the force and we can find the magnitude of the force by multiplying the mass of the stone by the acceleration. We can give it a name: weight, or gravitational attraction to the earth. Yet the existence of this force is the result of some unexplained interaction between the stone and the earth. Newton assumed, on the basis of his Rules of Reasoning, that the same kind of force exists between the earth and moon, or any planet and the sun. The force drops off as the square of the distance, and is of just the right amount to explain the motion of the planet. But why such a force should exist remained a puzzle, and is still a puzzle today.

Newton's claim that there is a mutual force between a planet and the sun raised a new question. How can a planet and the sun act upon each other at enormous distances without any visible connections between them? On earth you can exert a force on an object by pushing it or pulling it. We

are not troubled when we see a cloud or a balloon drifting across the sky even though nothing seems to be touching it. Although air is invisible, we know that it is actually a material substance, which we can feel when it blows against us. Objects falling to the earth and iron objects being attracted to a magnet are more troublesome examples. Still, the distances are usually short, and one could imagine that there is some sort of tenuous contact or influence involved. But the earth is over 90 million miles and Saturn more than 2 billion miles from the sun. How could there possibly be any physical contact between such distant objects? How can we account for such "action at a distance"?

If you have a copy of the Principia (see Bibliography), ask the students to contrast the Descartes theory of vortices with the kind of mathematical analysis used by Newton.

How can forces act at a great distance through space that seems to be empty?

There were in Newton's time, and for a long time afterward, a series of suggestions to explain how mechanical forces could be exerted at such distances. Newton himself, however, did not claim to have discovered how the gravitational force he had postulated was transmitted through space. At least in public, Newton refused to speculate on how the postulated gravitational force was transmitted through space. He saw no way to reach any testable answer which would replace the unacceptable whirlpools of Descartes. As he said in a famous passage in the General Scholium added to his second edition of the Principia (1713):

The theory of vortices was based on the assumption that natural motion can occur in circles.

> ...Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

This quotation is part of an important comment, or General Scholium, beginning on page 543 of the Cajori edition, University of California Press. The quotation is on page 547.

Some of your good students might be especially interested in the last ten pages of book II (end of Vol. I of the edition in the Bibliography) of the Principia, where Newton worked out the physics of fluids in order to demolish the theory of vortices.

We quoted Newton at length because this particular passage is frequently misquoted and misinterpreted. The original Latin reads: hypotheses non fingo. This means: "I frame no hypotheses," "I do not feign hypotheses," or "I do not make false hypotheses."

Newton did make numerous hypotheses in his numerous publications, and his letters to friends contain many other speculations which he did not publish. More light is shed on his purpose in writing the General Scholium by his manuscript first draft (written in January, 1712-13). Here Newton very plainly confessed his inability to unite the

Your students ought to be aware of the shift from a geometric argument of planetary motion to a physical one.

Summary Sec 8.7
1. The gravitational force between two bodies is proportional to the product of their masses.

2. The mathematical statement of the law of universal gravitation can be written

$$F_{grav.} = G\frac{m_1 m_2}{R^2}$$

hypothesis of gravitational forces with observed ph-nomena. He wrote:

> I have not yet disclosed the cause of gravity, nor have I undertaken to expla.n it, since I could not understand it from the ph nomena.

For us as for Newton the questions which we can answer are those specifying the quantitative nature of this force. We can test the results by applying them to a few selected cases.

Q10 Why do the quantitative predictions from Newton's Law of Universal Gravitation cause us to accept this universal law?

Q11 Why was the explanation of "action at a distance" of interest to scientists?

Q12 Why didn't Newton explain gravitation?

8.7 The magnitude of planetary force. We want to build a physics of the heavens which explains the motion of bodies by the forces between them. Therefore the general statement that a universal gravitational force exists has to be turned into a quantitative one that gives an expression for both the magnitude and direction of the force between any two objects. It was not enough to assert that a mutual gravitational attraction exists between the sun and say Jupiter. Newton had to specify what quantitative factors determine the size of that mutual force, and how it could be measured, either directly or indirectly.

While studying Newton's Laws of Motion in Unit 1, you first encountered the concept of mass. Indeed, Newton's second law, $F_{net} = ma$, states that the acceleration of any object depends upon the net force and the mass. Consider the two spheres in Fig. 8.6. Let us say that body B has double the mass of body A. Newton's second law tells us that if a net force causes body A to be accelerated a certain amount, double that force will be needed to accelerate body B by the same amount. If we drop bodies A and B, the earth's gravitational attraction causes them to accelerate equally: $a_A = a_B = g$, so the force on B must be twice the force on A. Twice the mass results in twice the force—which suggests that mass itself is the key factor in determining the magnitude of the gravitational force.

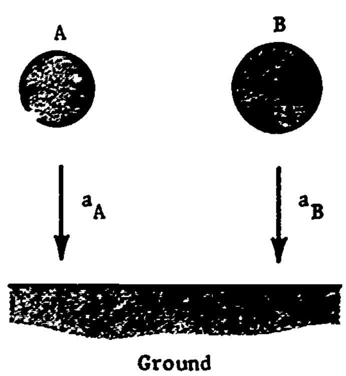


Fig. 8.6. Because falling spheres of unequal mass accelerate at the same rate, the gravitational forces on them must be proportional to their masses.

Like Newton, let us propose that the gravitational force on a planet, due to the pull of the sun, must be proportional to the mass of the planet itself. It immediately follows that this force is also proportional to the mass of the sun. We can see that the second proportionality follows, if we consider the problem in terms of a stone and the earth. We saw before that the downward force of gravity on the stone

You may find that some students have difficulty in understanding equal and opposite forces acting between two bodies of differing masses. Figures 8.7 and 8.8 are designed to explain very simply how that works.

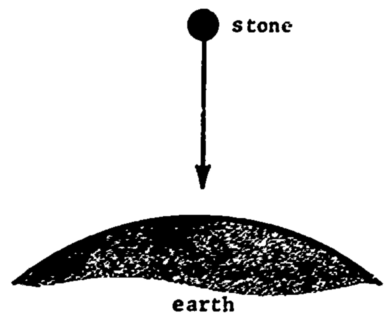
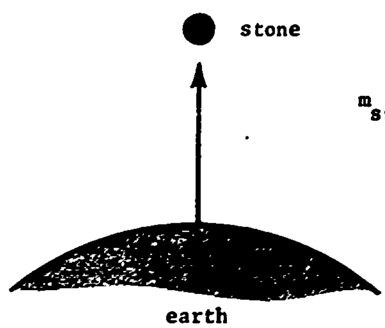
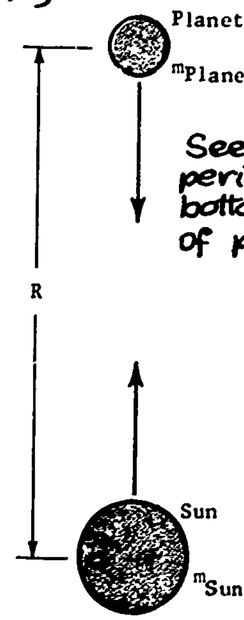


Fig. 8.7(a) The gravitational force on the earth is equal and opposite to the gravitational force on the stone.



$$m_{stone} \times a_{stone} = \text{force}\left\{\begin{array}{l}\text{of stone}\\\text{on earth}\\\text{or}\\\text{of earth}\\\text{on stone}\end{array}\right\}$$

$$= m_{earth} \times a_{earth}$$



See "thought experiment" in the bottom left column of page 94.

Fig. 8.7(b) The gravitational force on a planet is equal and opposite to the gravitational force, due to the planet, on the sun.

is proportional to the mass of the stone. Then by Newton's third law the force experienced by the earth owing to the stone is equally large and opposite, or upward. Thus while the stone falls, the earth rises. If the stone were fixed in space and the earth free to move, the earth would rise toward the stone until they collided. As Fig. 8.7 indicates, "the forces are equal and opposite," and the accelerations are inversely proportional to the masses.

The conclusion that the forces are equal and opposite, even between a very large mass and a small mass, may seem contrary to common sense. Therefore, let us consider the force between Jupiter and the sun, whose mass is about 1000 times that of Jupiter. As Fig. 8.8 indicates, we could consider the sun as a globe containing a thousand Jupiters. Let us call the force between two Jupiter-sized masses, separated by the distance between Jupiter and the sun, as one unit. Then Jupiter pulls on the sun (a globe of 1000 Jupiters) with a total force of 1000 units. Because each of the 1000 parts of the sun pulls on the planet Jupiter with one unit the total pull of the sun on Jupiter is also 1000 units. Remember that each part of the massive sun not only pulls <u>on</u> the planet, but is also pulled upon <u>by</u> the planet. The more mass there is to attract, the more there is to be attracted.

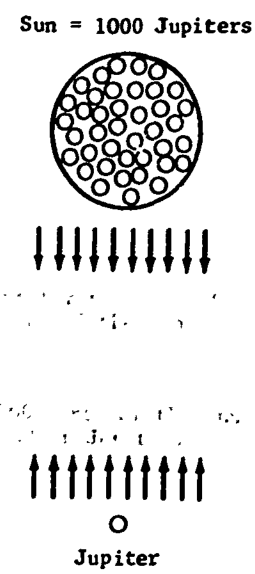


Fig. 8.8 The gravitational forces between the sun and a planet are equal and opposite.

SG 8.3

Force depends on mass of sun.

Force depends on mass of planet.

The gravitational attraction is proportional to the mass of the large body and the attraction is also proportional to the mass of the small body. How do we combine these two proportionalities to get one expression for the total force? Do we multiply the masses, add them, or divide one by the other?

Figure 8.8 already suggests the answer. If we replaced the sun (1000 Jupiters) by only one Jupiter, the force it would exert on the planet would be only one unit. Likewise, the planet would attract this little sun with a force of only one unit. But because the sun is a thousand times larger than Jupiter, each pull is 1000 units. If we could make the planet Jupiter three times more massive, what would the force be? You probably answered immediately: 3000 units. That is, the force would be multiplied three times. Therefore we conclude that the attraction increases in direct proportion to increases in the mass of <u>either</u> body, and that the total force depends upon the <u>product</u> of the two masses. This conclusion should not be surprising. If you put one brick on a scale, it has a certain weight (a measure of the earth's gravitational attraction on the brick). If you put three bricks on the scale, what will they weigh? That is, how much more will the earth attract the three bricks compared to one brick?

SG 8.7

F_{total} varies with $(m_P \times m_S)$. (8.1)

$$F_{grav} \propto \frac{m_P \times m_S}{R^2} \quad (8.2)$$

$$F_{grav} = G\frac{m_P \times m_S}{R^2} \quad (8.3)$$

Thus far we have concluded that the force between the sun and a planet will be proportional to the product of the masses [Eq. (8.1)]. Earlier we concluded that this force also depends upon the inverse square of the distance between two bodies. Once again we multiply the two parts to find one force law [Eq. (8.2)] that relates both masses and distance.

Such a proportionality as (8.2) can be changed into an equation by introducing a gravitational constant, G, to allow for the units of measurement used. Equation (8.3) is a bold assertion that the force between the sun and any planet depends only upon the masses of the sun and planet and the distance between them. We should notice what the equation omits. The force does not depend upon the name of the planet or its mythological identifications. Furthermore, it does not consider the mass of sun or earth as being in any way special compared with the mass of some other planet.

Try a thought experiment. Consider the possibilities that the force could depend upon the masses in either of two other ways:

(a) total force depends on $(m_{sun} + m_{planet})$, or

(b) total force depends on (m_{sun}/m_{planet}).

Now in imagination let one of the masses become zero. On the basis of these choices, would there still be a force even though there were only one mass left? Could you speak of a gravitational force when there was no body there to be accelerated?

According to Eq. (8.3) the gravitational force is determined <u>by and only by</u> the masses of the bodies and the distance separating them. This equation seems unbelievably simple when we remember the observed complexity of the planetary motions. Yet every one of Kepler's Laws of Plane-

Thought experiment will help get across the reason for multiplying the masses, instead of adding or dividing. Students often find this difficult to understand; but they should understand the <u>reductio ad absurdum</u> argument.

tary Motion is consistent with this relation, and this is the real test whether or not Eq. (8.3) is useful.

The argument for the development of gravitational law formulae has been spelled out in easy steps, so that the student may follow them from Eq. (8.1) to Eq. (8.4).

Moreover, Newton's proposal that such a simple equation defines the forces between the sun and planets is not the final step. He believed that there was nothing unique or special about the mutual force between sun and planets, or the earth and apples: a relation just like Eq. (8.3) should apply universally to any two bodies having masses m_1 and m_2 separated by a distance R that is large compared to the diameters of the two bodies. In that case we can write a "general law of universal gravitation" [Eq. (8.4)]. The numerical constant G, called the Constant of Universal Gravitation, is assumed to be the same everywhere, whether the objects are two sand grains, two members of a solar system, or two galaxies separated by half a universe. As we shall see, our faith in this simple relationship has become so great that we assume Eq. (8.4) applies everywhere and at all times, past, present and future.

The general Law of Universal Gravitation:

$$F_{grav} = \frac{Gm_1m_2}{R^2} \quad (8.4)$$

SG 8.2

Even before we gather the evidence supporting Eq. (8.4), the sweeping majesty of Newton's theory of universal gravitation commands our wonder and admiration. You may be curious as to how such a bold universal theory can be tested. The more diverse these tests are, the greater will be our growing belief in the correctness of the theory.

Your students should be aware of some of the problems that arose for Newton, and which still lie at the heart of research in physical science. (See Development Section.)

Q13 According to Newton's Law of Action and Opposite Reaction, the earth should rise toward a falling stone. Why don't we observe the earth's motion toward the stone?

Q14 What meaning do you give to G, the Constant of Universal Gravitation?

8.8 Testing a general law. To make a general test of an equation such as Eq. (8.4) we would need to determine the numerical value of all quantities represented by the symbols on both the left and right side of the equality sign. Also we should do this for a wide variety of cases to which the law is supposed to apply and check to see if the values always come out equal on both sides. But we surely cannot proceed that way. How would we determine the magnitude of F_{grav} acting on celestial bodies, except through the application of this equation itself? Newton faced this same problem. Furthermore, he had no reliable numerical values for the masses of the earth and the sun, and none for the value of G.

Summary 8.8
1. A general law has to be verified by measuring all the physical quantities concerned.

2. For spherical bodies that are far apart, the gravitational force can be considered to act between their centers.

But worst of all—just what does R represent? As long as we deal with particles or objects so small that their size is negligible compared with the distance between them,

The concept of the mass point is an extremely important one; Newton's argument for considering the mass of a sphere to be concentrated at the center provided a beautifully simple way of working out problems in gravitational force.

there is no ambiguity. But even between the sun and earth, it is not self-evident whether R should refer to the distance from surface to surface, or center to center, or some other distance.

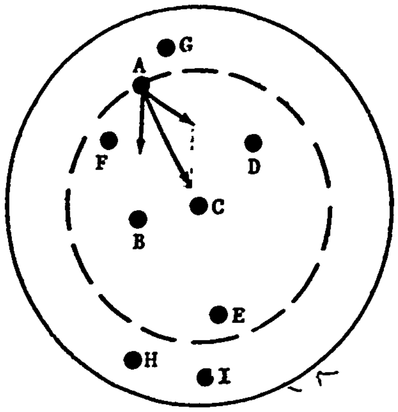


Fig. 8.9 The net gravitational force on a point, A, in a spherical body is towards the center, C.

To answer this question, Newton turned to a model—and we can follow his argument in general terms. Let us imagine a rigid homogeneous spherical body, or one composed of spherical shells, such as the body in Fig. 8.9. Within it, any small particle, A, is attracted by all the other particles, near and distant. For each attracting particle B on one side of a line to the center C, there is a symmetrically placed particle D exerting an equal attraction on the other side of the center line. Therefore the net force on particle A is in the direction of the center C.

The amount of force on A is, however, more difficult to determine. Some points, like G, are relatively near to A, while many more, like H and I are farther away. To sum up all the small attractions Newton developed a new mathematical procedure called the infinitesimal calculus. With it he concluded that the the force on a particle at a point A depended only upon the mass of material closer than A to the center. The attractions of the material located further from the center than point A canceled out.

The important effects of small differences in gravitational force will become apparent in Sec. 8.11, where the effect of both sun and moon on the earth's tides is discussed.

A similar analysis shows that on an external spherical object—an apple on a branch, or the moon, or a planet—the total gravitational attraction acts as if it originated at the center of the attracting spherical body. Then the distance R in the Law of Universal Gravitation is the distance between centers.

Assume that a planet is a sphere of uniform density (which the earth is not). Compute the gravitational attraction at points 3/4, 1/2, 1/4 of the radius R from the center. The volume of a sphere is given by: Volume = $4\pi R^3/3$.

This is a very critical discovery. Now we can consider the gravitational attraction from a rigid spherical body as though its mass was concentrated at a point, called a mass-point.

If some of your students seem worried by the mention of Newton's infinitesimal calculus (middle of page), you might remind them that this is just a name for a very imaginative method of handling quantities that are continually changing.

Q15 What difficulties do we have in testing a universal law?

Q16 What is meant by the term "mass point"?

Q17 How did Newton go about devising some tests of his Theory of Universal Gravitation even though he did not know the value of G? (See next section.)

Summary 8.9
By using the technique of comparison and the concept of universal gravitation, Newton was able to predict the quantity of the moon's acceleration toward the sun.

8.9 The moon and universal gravitation. Before we digressed to discuss what measurements we should use for distance R, we were considering how Newton's equation for universal gravitational attraction could be tested experimentally. To use Eq. (8.4) to find any masses the value of G must be known. However, the value of G was not determined for more than a hundred years after Newton proposed his equation.

Newton had to test Eq. (8.4) indirectly. Since he could

not measure any of the attractions directly, he had to be satisfied with <u>comparing</u> various similar motions. This is a procedure often used in scientific studies. Newton began by comparing the gravitational pull of the earth on a terrestrial object (like an apple) and on the moon. From the data available to him, Newton knew that the distance between the center of the earth and the center of the moon was nearly 60 times the radius of the earth. Since the attractive force was believed to vary as $1/R^2$, the gravitational acceleration of the earth upon the moon should be only $1/60^2$ or 1/3600 of that upon an apple at the surface of the earth. From observations of falling bodies it was known that the gravitational acceleration at the earth's surface was nearly 9.80 meters per second per second. Therefore, because of the earth's attraction, in one second the moon should, as Fig. 8.10 indicates, "fall" toward the earth at the rate of 9.80/3600 meters per second^2, or 2.72×10^{-3} m/sec^2. Does it?

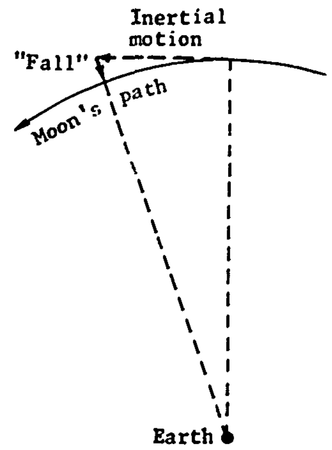


Fig. 8.10 As the moon moves through space, the gravitational attraction of the earth causes the moon to "fall" toward the earth. The continuous combination of straightline inertial motion and "fall" produce the curved orbit.

As starting information Newton knew that the orbital period of the moon was very nearly $27\frac{1}{3}$ days. Also, he knew that the moon's average distance from the earth is nearly 240,000 miles. But most important was the equation [Eq. (8.5)] for centripetal acceleration, a_c, that you first saw in Unit 1. That is, the acceleration, a_c, toward the center of attraction equals the square of the speed along the orbit, v, divided by the distance between centers, R. To find the average linear speed, v, we divide the total circumference of the moon's orbit, $2\pi R$, by the moon's period, T. Then upon substitution for v^2 in Eq. (8.5) we find Eq. (8.6) for the centripetal acceleration in terms of the radius of the moon's orbit and the orbital period of the moon. When we substitute the known quantities and do the arithmetic we find

$$a_c = 2.74 \times 10^{-3} \text{ m/sec}^2.$$

Predicted value of the earth's gravitational acceleration at the distance of the moon: 2.72×10^{-3} m/sec^2.

$$a_c = v^2/R \qquad (8.5)$$

$v = 2\pi R/T$, and $v^2 = 4\pi^2R^2/T^2$

$$a_c = 4\pi^2R/T^2 \qquad (8.6)$$

Observed rate of moon's acceleration toward the earth: 2.74×10^{-3} m/sec^2.

From Newton's values, which were about as close as these, he concluded that he had

> compared the force requisite to keep the moon in her orbit with the force of gravity at the surface of the earth, and found them to answer pretty nearly.
>
> Therefore the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by Rules [of Reasoning] 1 and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity....

You might ask your students to verify the prediction of 2.74×10^{-3} m/sec^2 for the moon's acceleration by actually working out the arithmetic of Eq. (8.6). R for the moon is 3.84×10^5 km, and you can use 27.3 days for T. Have them see what happens to the final answer when various quantities are "rounded off".

By this one comparison Newton had not yet proved his law of universal gravitational attraction. However, Newton was

This might be a good place to emphasize the thought that Newton was using mathematical statements based upon the motion of bodies on earth (that is, Galileo's laws of moving bodies) to make predictions about the motions of the planets, thus fusing two sets of laws about two different sets of physical phenomena into a single theory.

SG 8.4 able to show, as we did in Section 8.5, that the general statement of Eq. (8.4) applied also to the motion of planets around the sun along orbits that are conic sections (in this case, ellipses). SG 8.8

The Moon's Irregular Motion
See Article section.

Q18 In what way did Newton compare the motions of a falling apple and of the moon?

Q19 Explain the significance of the numerical results of Newton's computation of the moon's acceleration.

Summary 8.10
Follows the derivation of Keplers third law from the combined forms of the second law of motion and the law of universal gravitation.
(For students weak in algebra, a qualitative treatment would be sufficient.)

See "Gravity Experiments" in Project Physics Reader 2.

$$F_c = \text{mass} \times \text{acceleration} = m_P a_c = m_P\,(4\pi^2 R/T^2) \qquad (8.7)$$

$$F_{grav} = G\frac{m_P m_S}{R^2} \qquad (8.3)$$

but $F_c = F_{grav}$

so $\cancel{m_P}\dfrac{4\pi^2 R}{T^2} = G\dfrac{\cancel{m_P} m_S}{R^2}$

$$T_P{}^2 = \left[\frac{4\pi^2}{Gm_S}\right]R^3 \qquad (8.8)$$

$T_P{}^2 = kR^3$ is Kepler's Third Law.

In Chapter 7 we wrote $T^2 = ka^3$, where a was the mean distance in the orbit. However, in Chapter 8 we have used a for acceleration. To lessen confusion we use R here to describe the mean distance of an orbit.

It is important to remind the students that the equations are saying something like this: we are looking at the motion of a planet from two different points of view, (1) as acceleration caused by a centripetal force — this is a purely mechanical point of view, as though the planet were a ball being whirled around at the end of a string, and (2) from a gravitational point of view — as though we have replaced the string by an invisible one called gravitation.

$$F_{grav} = G\frac{m_1 m_2}{R^2} \qquad (8.4)$$

8.10 Gravitation and planetary motion. The line of discussion used to consider the force of the earth's gravitational attraction upon the moon opens up a further opportunity to check Newton's theory. Newton made a very important assumption: the gravitational attraction must be the cause of centripetal acceleration.

Equation (8.6) related the centripetal acceleration, a_c, to the distance and period of a body moving in a circle. Therefore, for a planet going in a circular orbit—a good approximation for this purpose—the centripetal force F_c is just the acceleration multiplied by the mass of the planet [Eq. (8.7)]. Earlier we found that the gravitational force on a planet in its orbit around the sun was given by Eq. (8.3). Now we can equate the relations, Eqs. (8.3) and (8.7), for F_{grav} and F_c. After canceling the quantity m_P on both sides and rearranging the symbols, we have Eq. (8.8) which has the form of Kepler's third law, $T^2 = kR^3$. The quantity within the bracket of Eq. (8.8) occupies the same place as the constant k which we earlier found to be the same for all planets. The brackets contain only the terms: G, which is supposed to be a universal constant; m_S, the mass of the sun; and the numbers $4\pi^2$. Since none of the terms in the brackets depends upon the particular planet, the bracket gives a value which applies to the sun's effect on any planet.

This does not prove that $k = 4\pi^2/Gm_S$. If, however, we tentatively accept the relation, it follows that $Gm_S = 4\pi^2/k$. Therefore, while we have not seen how one could determine either G or m_S separately, the determination of G would allow us to calculate the mass of the sun, m_S.

We must still question whether G is really a universal constant, i.e., one with the same value for all objects that interact according to Eq. (8.4). Although Newton knew in principle how one might measure the numerical value of G, he lacked the precision equipment necessary. However, he did provide a simple argument in favor of the constancy of G. Consider a body of mass m_1 on the surface of the earth (of mass m_E), at a distance R_E from the earth's center. The

body's weight, which we call F_{grav}, is given by m_1g. Then we can again equate the earthly gravitational force, m_1g, with the force predicted by the Law of Universal Gravitation. The result is Eq. (8.9). At any one position on earth, (R_E^2/m_E) is constant (because R_E and m_E are each constant) regardless of what the numerical value of this ratio may be. Then, if all substances at that place show precisely the same value for g, the gravitational acceleration in free fall, the quantity G also must be constant there. This conclusion should hold regardless of the chemical composition, texture, shape and color of the bodies involved in free fall. That g is constant at a given location is just what Newton showed experimentally. His measurements were made not by just dropping small and large bodies, from which Galileo had previously concluded that g cannot vary significantly. Instead Newton used the more accurate method of timing pendulums of equal lengths but of different materials such as wood and gold. After exhaustive experiments, all pointing to the constancy of g, and therefore of G at a given location, Newton could write:

> This [constancy] is the quality of all bodies within the reach of our experiments; and therefore (by Rule 3) to be affirmed of all bodies whatsoever.

Thus G attained the status of the Universal Constant of Gravitation—one of the very few universal constants in nature.

Weight = $F_{grav} = m_1g$, but also

$$F_{grav} = G\frac{m_1m_E}{R_E^2}\text{, then}$$

$$\cancel{m}_1g = G\frac{\cancel{m}_1m_E}{R_E^2}\text{, and}$$

$$G = \left[\frac{R_E^2}{m_E}\right]g \qquad (8.9)$$

SG 8.6

SG 8.9

The discovery that the k of Kepler's third law is the quantity $4\pi^2/Gm_S$ gives the students an insight into the constancy of the quantity G: since the k of Kepler's law has been determined as a constant for all planets by observation, and since 4, π, and m_S are all constants, it follows that G ought to be a constant also. Of course this does not mean that G is a universal constant.

SG 8.10

Q20 How did Newton use the centripetal force in his analysis of the motions of the moon and planets?

Q21 On what basis did Newton conclude that G was a universal constant?

Q22 Since the value of g is not the same at all places on the earth, does this mean that perhaps G is not really constant?

8.11 The tides. The flooding and ebbing of the tides, so important to navigators, tradesmen and explorers through the ages, had remained a mystery despite the studies of such men as Galileo. Newton, however, through the application of the Law of Gravitation, was able to explain the main features of the ocean tides. These he found to result from the attraction of the moon and sun upon the fluid waters of the earth. Each day two high tides normally occur. Also, twice each month, when the moon, sun and earth are in line, the tides are higher than the average. Near quarter moon, when the directions from the earth to the moon and sun differ by about 90°, the tidal changes are smaller than average.

Summary 8.11
The principle of universal gravitation was able, in general, to account for the phenomenon of ocean tides.

What are the phases of the moon when the moon, sun and earth are in line?

The diameter of the earth, compared to the earth-sun distance, is a very small part of the R that is measured for the gravitational equation. You might ask your students to calculate what percentage the value of 12.8×10^6 m is of 1.5×10^{11} m.

Two questions about tidal phenomena demand special attention. First, why do high tides occur on both sides of

When it comes to the earth-moon distance, however, 12.8×10^6 m becomes a significant part of 3.8×10^8 m (the earth-moon distance).

You might wish to work out Study Guide 8.16 on the board. This problem serves better as a demonstration than as a student exercise.

the earth, including the side away from the moon? Second, why does the time of high tide occur some hours after the moon has crossed the north-south line (meridian)?

There is an excellent example in this section of how deduction from a general theory compares with induction from observation. (See Development Sec.)

Newton realized that the tide-raising force would be the difference between the pull of the moon on the whole solid earth and on the fluid waters at the earth's surface. The moon's distance from the solid earth is 60 earth radii. On the side of the earth nearer the moon, the distance of the water from the moon is only 59 earth radii. On the side of the earth away from the moon the water is 61 earth radii from the moon. Then the accelerations would be those shown in Fig. 8.11. On the side near the moon the acceleration is greater than that on the rigid earth as a whole, so the fluid water on the surface of the earth has a net acceleration toward the moon. On the far side of the earth, the acceleration is less than it is on the earth as a whole, so the water on the far side has a net acceleration away from the moon. We could say that the rigid earth is pulled away from the water.

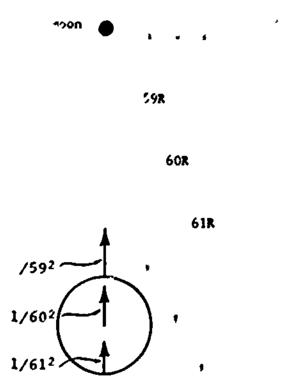


Fig. 8.11 Tidal forces. The earth-moon distance indicated in the figure is greatly reduced because of the space limitations.

A gravitational-force graph provides an excellent way of getting across the concept of differing gravitational force on either side of a significant diameter.

If you have been to the seashore or examined tide tables, you know that the high tide does not occur when the moon crosses the north-south line, but some hours later. To explain this even qualitatively we must remember that the oceans are not very deep. As a result, the waters moving in the oceans in response to the moon's attraction encounter friction from the ocean floors, especially in shallow water, and consequently the high tide is delayed. In any particular place the amount of the delay and of the height of the tides depends greatly upon the ease with which the waters can flow. No general theory can be expected to account for all the particular details of the tides. Most of the local predictions in the tide tables are based upon the cyclic variations recorded in the past.

SG 8.16

Since there are tides in the fluid seas, you may wonder if there are tides in the fluid atmosphere and in the earth itself. There are. The earth is not completely rigid, but bends about like steel. The tide in the earth is about a foot high. The atmospheric tides are generally masked by other weather changes. However, at heights near a hundred miles where satellites have been placed, the thin atmosphere rises and falls considerably.

Q23 Why do we consider the acceleration of the moon on the ground below a high tide to be the acceleration of the moon on the earth's center?

Q24 Why is there a high tide on the side of the earth away from the moon?

Summary 8.12

1. As a result of the work of Newton and Halley, comets were recognized as actual astronomical bodies, rather than portents from heaven warning of some catastrophe on earth.

2. Newton used the law of universal gravitation to account for cometary orbits, which are generally quite unlike the orbits of planets

8.12 Comets. Comets, whose unexpected appearances had through antiquity and the Middle Ages been interpreted as omens of disaster, were shown by Halley and Newton to be nothing more than some sort of cloudy masses that moved around the sun according to Newton's Law of Gravitation. They found that most comets were visible only when closer to the sun than the distance of Jupiter. Several of the very bright comets were found to have orbits that took the ... ide the orbit of Mercury to within a few milli... es of the sun, as Fig. 8.12 indicates. Some of the orbits have eccentricities near 1.0 and are almost parabolas, and those comets have periods of thousands or even millions of years. Other faint comets have periods of only five to ten years. Unlike the planets, whose orbits are nearly in the plane of the ecliptic, the planes of comet orbits are tilted at all angles to the ecliptic. In fact, about half of the long-period comets move in the direction opposite to the planetary motions.

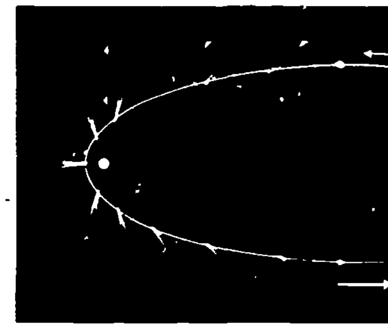


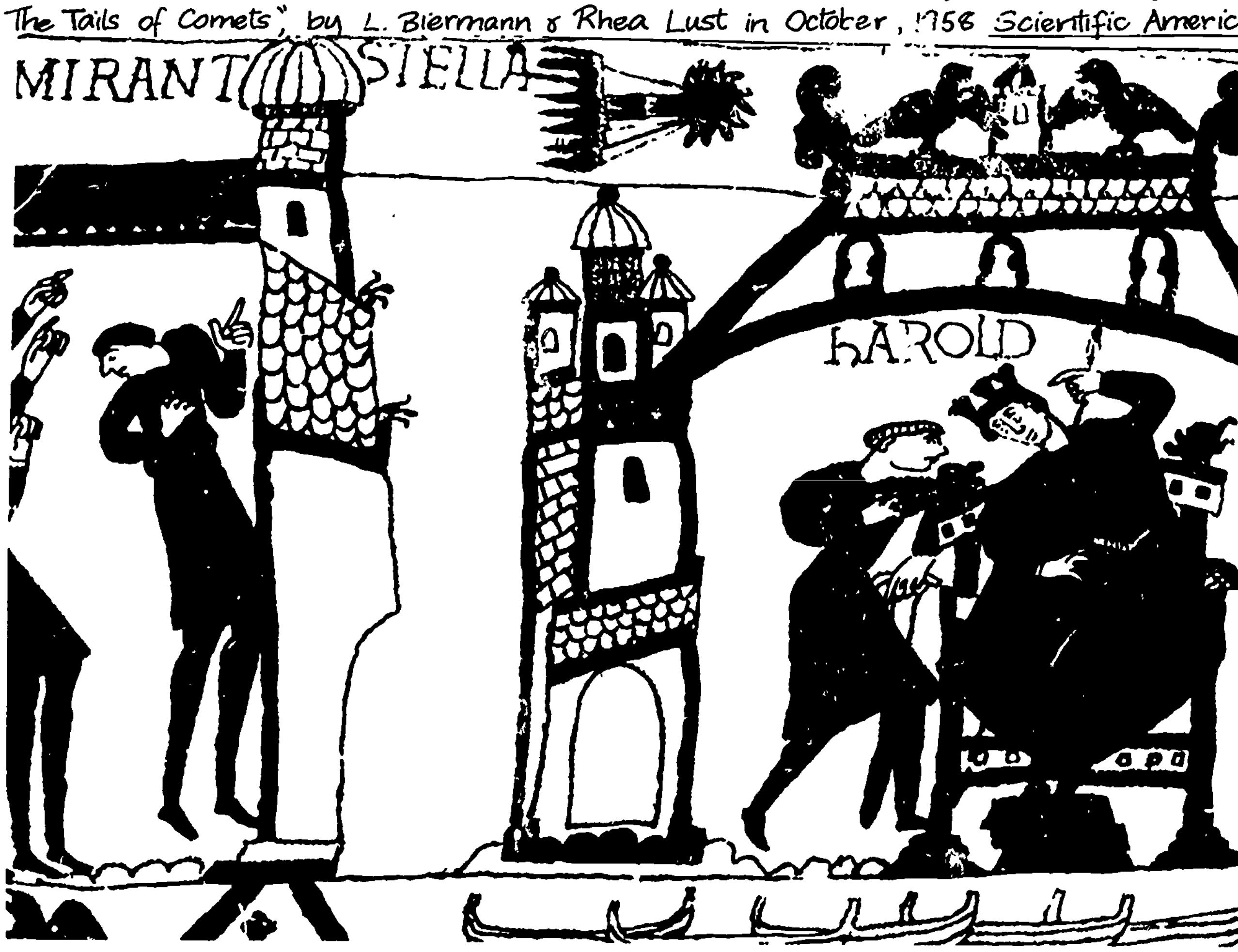
Fig. 8.12 Schematic diagram of the orbit of a comet projected onto the ecliptic plane; comet orbits are tilted at all angles.

More information can be found in Between the Planets, F. G. Watson (see Bibliography).

Edmund Halley applied Newton's gravitational theory to the motion of bright comets. Among those he studied were the comets of 1531, 1607 and 1682 whose orbits he found to be very nearly the same. Halley suspected that these might be the same comet seen at intervals of about 75 years. If this were the same comet, it should return in about 1757—as

Fig. 8.13 A scene from the Bayeux tapestry, which was embroidered about 1070. The bright comet of 1066 can be seen at the top of the figure. This comet was later identified as being Halley's comet. At the right, Harold, pretender to the throne of England, is warned that the comet is an ill omen. Later that year at the Battle of Hastings Harold was defeated by William the Conqueror.

"The Tails of Comets", by L. Biermann & Rhea Lust in October, 1958 Scientific American.



Ask students what tangential velocity a comet would have at very large distances from the sun.

E: Model of a comet orbit
A: Other comet orbits.

Point out that the sun is a very small target when viewed from a distance of several hundred AU s. Therefore, even a slight tangential velocity will be enough to cause the body to miss the sun. Actually, very unusual conditions would be required for a body to hit the sun after falling in from a great distance.

See "Boy Who Redeemed His Father's Name" in Project Physics Reader 2.

See "Great Comet of 1965" in Project Physics Reader 2.

it did—after moving out to 35 times the earth's distance from the sun. This is Halley's comet, due to be near the sun and bright again in 1985 or 1986.

With the period of this bright comet known, its approximate dates of appearance could be traced back in history. In the records kept by Chinese and Japanese this comet has been identified at every expected appearance except one since 240 B.C. That the records of such a celestial event are incomplete in Europe is a sad commentary upon the interests and culture of Europe during the so-called "Dark Ages." One of the few European records of this comet is the famous Bayeux tapestry, embroidered with 72 scenes of the Norman Conquest of England in 1066, which shows the comet overhead and the populace cowering below (Fig. 8.13). A major triumph of Newtonian science was its use to explain comets, which for centuries had been frightening events.

Q25 In what way did Halley's study of comets support Newton's theory?

$$T^2 = \left[\frac{4\pi^2}{Gm_S}\right] R^3 \qquad (8.8)$$

By rearrangement, this becomes

$$m_S = \left[\frac{4\pi^2}{G}\right] \frac{R^3}{T^2}\,. \qquad (8.10)$$

Summary 8.13
One of the more exciting discoveries that comes out of this chapter is that Kepler's laws seemed to hold for any system where small bodies revolve about a central body.

$$\frac{m_{sun}}{m_P} = \left[\frac{R_P}{R_{Sat}}\right]^3 \left[\frac{T_{Sat}}{T_P}\right]^2 \qquad (8.11)$$

$$\left[\frac{R_P}{R_{Sat}}\right] = \frac{483{,}000{,}000 \text{ mi}}{1{,}170{,}000 \text{ mi}} = 413$$

$$\left[\frac{T_{Sat}}{T_P}\right] = \frac{16.71 \text{ d}}{4{,}332 \text{ d}} = \frac{1}{259}$$

then

$$\frac{m_{sun}}{m_P} = (413)^3 \times \left[\frac{1}{259}\right]^2$$

$$\frac{m_{sun}}{m_P} = \frac{7.05 \times 10^7}{6.70 \times 10^4} = 1050.$$

8.13 Relative masses of the planets compared to the sun. The masses of the planets having satellites can be compared to the mass of the sun by the use of Eq. (8.8) even though we do not know the value of the Universal Gravitational Constant, G. This we can do by forming ratios involving the periods and distances of the planets around the sun and of satellites around the planets. As a start we rearrange Eq. (8.8) into the form of Eq. (8.10). This relates the period of a planet to its distance from the sun and the total mass of the sun plus planet, but the mass of the planet can be neglected. If we change the subscripts, this relation applies equally well to a planet P and one of its satellites, Sat. Then we may find the ratio of the mass of the sun to the mass of the planet:

$$\frac{\text{mass of the sun-planet pair}}{\text{mass of the planet-satellite pair}} = \frac{m_{sun}}{m_P} = \frac{\left[\frac{4\pi^2}{G}\right] \frac{R_P^{\,3}}{T_P^{\,2}}}{\left[\frac{4\pi^2}{G}\right] \frac{R_{Sat}^{\,3}}{T_{Sat}^{\,2}}}\,.$$

The brackets cancel. After some rearrangement we have

$$\frac{m_{sun}}{m_P} = \left[\frac{T_{Sat}^{\,2}}{T_P^{\,2}}\right] \left[\frac{R_P^{\,3}}{R_{Sat}^{\,3}}\right]. \qquad (8.11)$$

Now, for example, we can determine the relative mass of Jupiter and the sun. We know the period T_S and the distance R_S between Jupiter and one of its satellites—Callisto, one

of the Galilean satellites. Also we know the orbital period T_P and the distance R_P of Jupiter from the sun. Table 8.1 presents the modern data. In the margin we have worked out the arithmetic.

SG 8 11

SG 8 12

Table 8.1. Data on the Motion of Callisto around Jupiter, and on the Motion of Jupiter around the Sun.

Object	Period (T), days	Distance, R, miles
Callisto	16.71	1,170,000
Jupiter	4,332	483,000,000

L12: Jupiter satellite orbit

In this way Newton found the masses of Jupiter, Saturn and the earth compared to the sun's mass to be: 1/1067, 1/3021 and 1/169,282. (The modern values are: 1/1048, 1/3499 and 1/331,950.) Thus the application of gravitational theory permitted for the first time a determination of the relative masses of the sun and planets.

Newton's relative value for the mass of the earth was in error because the distance from the earth to the sun was not accurately known.

F14: Harlow Shapley

Q26 Even though Newton did not know the value of G, how could he use Eq. (8.10),

$$m_S = \left[\frac{4\pi^2}{G}\right]\frac{R^3}{T^2},$$

to derive relative masses of some planets compared to the sun? For which planets could he find such masses, in terms of the sun's mass?

Q27 If the period of a satellite of Uranus is 1 day 10 hours and its mean distance from Uranus is 81,000 miles, what is the mass of Uranus compared to the sun's mass?

8.14 The scope of the principle of universal gravitation. Although Newton made numerous additional applications of his Law of Universal Gravitation, we cannot consider them in detail here. He investigated the causes of the irregular motion of the moon and showed that its orbit would be continually changing. As the moon moves around the earth, the moon's distance from the sun changes continually. This changes the net force of the earth and sun on the orbiting moon. Newton also showed that other changes in the moon's motion occur because the earth is not a perfect sphere, but has an equatorial diameter 27 miles greater than the diameter through the poles. Newton commented on the problem of the moon's motion that "the calculation of this motion is difficult." Even so, he obtained predicted values in reasonable agreement with the observed values available at his time.

Summary 8.14
Relative to this rather short section, it should be noted that the success of Newtonian dynamics as a "predicting machine" became enhanced during the following two centuries by the development of new mathematical techniques and methods of analysis, and also by the invention of instruments capable of making finer measurements.

Newton investigated the variations of gravity at different latitudes on the bulging and spinning earth. Also, from the differences in the rates at which pendulums swung at different latitudes he was able to derive an approximate shape for the earth.

The chief source of changes of "g" with latitude is the centripetal acceleration, not the shape of the earth.

What Newton had done was to create a whole new quantitative apprcach to the study of astronomical motions. Because some of his predicted variations had not been observed, new improved instruments were built. These were needed anyway to improve the observations which could now be fitted together under the grand theory. Numerous new theoretical problems also needed attention. For example, what were the predicted and observed interactions of the planets upon their orbital motions? Although the planets are small compared to the sun and are very far apart, their interactions are enough so that masses can be found for Mercury, Venus and Pluto, which do not have satellites. As precise data have accumulated, the Newtonian theory has permitted calculations about the past and future states of the planetary system. For past and future intervals up to some hundreds of millions of years, when the extrapolation becomes fuzzy, the planetary system has been and will be about as it is now.

Film: <u>Universal Gravitation</u>

See "Universal Gravitation" in <u>Project Physics Reader 2</u>.

Summary Sec 8.15
1. The major problem faced in the determination of the gravitational constant G was the measurement of the gravitational force between two small bodies on earth.

2. Henry Cavendish solved the technical difficulties of such an apparatus at the end of the eighteenth century.

3. After the value of G had been determined, the actual masses of the earth, sun, moon, and the planets could be determined.

What amazed Newton's contemporaries and increases our own admiration for him was not only the range and genius of his work in mechanics, not only the originality and elegance of his proofs, but also the detail with which he developed each idea. Having satisfied himself of the correctness of his principle of universal gravitation, he applied it to a wide range of terrestrial and celestial problems, with the result that it became more and more widely accepted. Remember that a theory can never be completely proven; but it becomes increasingly accepted as its usefulness is more widely shown.

The great power of the theory of universal gravitation became even more apparent when others applied it to problems which Newton had not considered. It took almost a century for science to comprehend, verify and round out his work. At the end of a second century (the late 1800's), it was still reasonable for leading scientists and philosophers to claim that most of what had been accomplished in the science of mechanics since Newton's day was but a development or application of his work. Thus, due to the work of Newton himself and of many scientists who followed him, the list of applications of the principle of universal gravitation is a long one.

Q28 What are some of the reasons which caused Newton to comment that "the calculation of the moon's motion is difficult"? (See Fig. 8.10.)

Q29 What were some of the problems and actions that needed further effort as a result of Newton's theory?

8.15 The actual masses of celestial bodies. The full power of the Law of Universal Gravitation could be applied only after the numerical value of the proportionality constant G had been determined. As we noted earlier, although Newton understood the process for determining G experimentally, the actual determination had to await the invention of delicate instruments and special techniques.

Film : Forces

The procedure is simple enough: in the laboratory, measure all of the quantities in the equation

$$F_{grav} = G\frac{m_1 m_2}{R^2} \qquad (8.4)$$

except G, which can then be computed. The masses of small solid objects can be found easily enough from their weights. Furthermore, measuring the distance between solid objects of definite shape is not a problem. But how is one to measure the small gravitational force between relatively small objects in a laboratory when they are experiencing the large gravitational force of the earth?

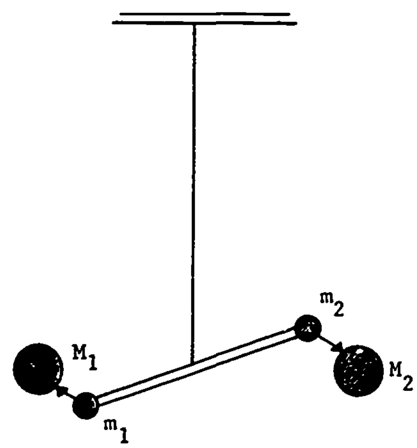


Fig. 8.14 Schematic diagram of the device used by Cavendish for determining the value of the gravitational constant, G. The large lead balls of masses M_1 and M_2 were brought close to the small lead balls of masses m_1 and m_2. The mutual gravitational attraction between M_1 and m_1 and between M_2 and m_2 caused the vertical wire to be twisted by a measurable amount.

This serious technical problem of measurement was eventually solved by the English scientist Henry Cavendish (1731-1810). As a device for measuring gravitational forces he employed a torsion balance (Fig. 8.14), in which the force to be measured twists a wire. This force could be measured separately and the twist of the wire calibrated. Thus, in the Cavendish experiment the torsion balance allowed measurements of the very small gravitational forces exerted on two small masses by two larger ones. This experiment has been progressively improved, and today the accepted value of G is

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

$$= 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot sec^2}.$$

The gravitational forces between two 1-kg masses such as two quarts of milk one meter apart is less than one ten-billionth of a newton! Evidently gravitation is a weak force which becomes important only when the mass is large.

To derive the gravitational force between two kilogram masses, one meter apart:

$$F = G\left[\frac{m_1 m_2}{R^2}\right]$$

$$= 6.67 \times 10^{-11}\left[\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}\right]\left[\frac{1\ \mathrm{kg} \times 1\ \mathrm{kg}}{1\ \mathrm{m^2}}\right]$$

$$= 6.67 \times 10^{-11}\ \mathrm{N}$$

Once the value of G had been determined, the actual masses of the earth, planets and sun could be found. Rearrangement of Eq. (8.9) leads to Eq. (8.12). Since the values of g, R_E and G are now known, the mass of the earth, m_E, can be calculated. Substitute the numbers given in the margin into Eq. (8.12) and find the mass of the earth.

$$G = \left[\frac{R_E^2}{m_E}\right]g \qquad (8.9)$$

$$m_E = \left[\frac{R_E^2}{G}\right]g \qquad (8.12)$$

Like other large numbers we have encountered, this one is difficult to comprehend. Since at most we can lift but a few kilograms, we have nothing in our experience with which to compare the mass of the earth—or for that matter, the

$g = 9.80\ \mathrm{m/sec^2}$

$G = 6.67 \times 10^{-11}\ \mathrm{m^3/kg\ sec^2}$

$R_E = 6.37 \times 10^6\ \mathrm{m}.$

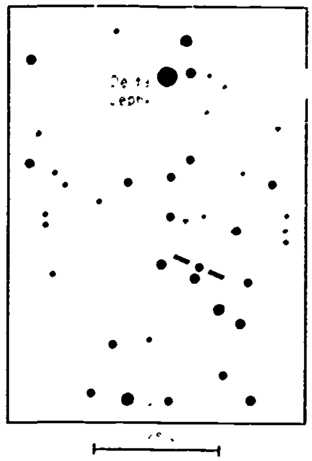
See Measuring G, The Von Jolly Experiment in the Article Section.

How to Find the Mass of a Double Star

Are, most stars mainly of double star systems? Sept. 1967 Physics Teacher Vol. 20 p. 45

To demonstrate the power of Newton's laws, let us study a double star. You can even derive its mass from your own observations.

An interesting double star of short period, which can be seen as a double star with a six-inch telescope, is Krüger 60. The find-chart shows its location less than one degree south of the variable star Delta Cephei in the northern sky.



A finding chart for Krüger 60, with north upward, east to the left.

The sequential photographs (Fig. 8.15), spaced in proportion to their dates, show the double star on the right. Another star, which just happens to be in the line of sight, shows on the left. The photographs show the revolution within the double-star system, which has a period of about 45 years. As you can see, the components were farthest apart, about 3.4 seconds of arc, in the mid-1940's. The chart of the relative positions of the two components (Fig. 8.16) shows that they will be closest together at 1.4 seconds of arc around 1971. The circles mark the center of mass of the two-star system. If you measure the direction and distance of one star relative to the other at five-year intervals, you can make a plot on graph paper which shows the motion of one star relative to the other. Would you expect this to be an ellipse? Should Kepler's Law of Areas apply? Does it? Have you assumed that the orbital plane is perpendicular to the line of sight?

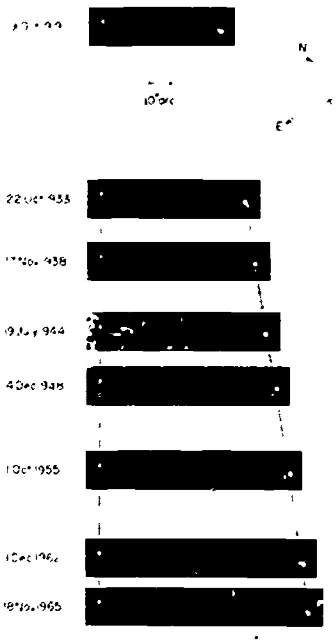


Fig. 8.15 The orbital and linear motions of the visual binary, Krüger 60, are both shown in this chart, made up of photographs taken at Leander McCormick Observatory (1919 and 1933) and at Sproul Observatory (1938 to 1965).

The sequential pictures show that the center of mass of Krüger 60 is drifting away from the star at the left. If you were to extend the lines back to earlier dates, you would find that in the 1860's Krüger 60 passed only 4 seconds of arc from that reference star.

The drift of Krüger 60 relative to the reference star shows that the stars do move relative to each other. For most stars, which are at great distances, this motion, called proper motion, is too small to be detected. Krüger 60 is, however, relatively nearby, only about 13 light-years away. (This is the distance light travels in 13 years at 3.0×10^8 meters per second.) The distance to Krüger 60 is then:

$$13 \text{ light-years} \times 3.0 \times 10^8 \text{ m/sec}$$

$$\times\ 3.2 \times 10^7 \text{ sec/yr},$$

or $$13 \times 10^{16} \text{ m}$$

or $$8.7 \times 10^5 \text{ A.U.}$$

(One year contains about 3.2×10^7 seconds. One A.U. is 1.5×10^{11} meters.)

From the sequential photographs and the scale given there we can derive the change in distance of Krüger 60 from the comparison star between 1919 and 1965. From the photographs our measurements give the distances as 55 seconds of arc in 1919 and 99 seconds of arc in 1965. (One second of arc is the angle subtended by a unit length, perpendicular to the line of sight, at a distance of 2.1×10^5 of the units.) Thus the proper motion was 44 seconds in 46 years, very nearly 1.0 second of arc per year. This angle is about $1/2.1 \times 10^5$ of the distance to the star. Then in one year the star moves 13×10^{16} m$/2.1 \times 10^5$, or 6.7×10^{11} m/yr. In one second the component of the star's velocity vector across the sky is 1.9×10^4 m, or its velocity perpendicular to the line of sight is 19 km/sec. Probably the star also has a component of motion along the line of sight, called the radial velocity, but this must be found from another type of observation.

Adapted from paper by James F. Wanner of the Sproul Observatory, Swarthmore College, in Sky and Telescope, January 1967.

The masses of the two stars of Krüger 60 can be found from the photographs shown in Fig. 8.15 and the application of Eq. (8.11). When we developed Eq. (8.11) we assumed that the mass of one body of each pair (sun-planet, or planet-satellite) was negligible. In the equation the mass is actually the sun or the two, so for the double star we must write $(m_1 + m_2)$. Then we have

$$\frac{(m_1 + m_2)\ \text{pair}}{m_{\text{sun}}} = \left[\frac{T_E}{T_{\text{pair}}}\right]^2 \left[\frac{R_{\text{pair}}}{R_E}\right]^3. \qquad (8.11)$$

The arithmetic is greatly simplified if we take the periods in years and the distances in Astronomical Units (A. U.), which are both unit for the earth. The period of Krüger 60 is about 45 years. The mean distance of the components can be found in seconds of arc from the diagram (Fig. 8.16). The mean separation is

$$\left(\frac{\text{max} + \text{min}}{2}\right) = \frac{3.4 \text{ seconds} + 1.4 \text{ seconds}}{2}$$

$$= \frac{4.8 \text{ seconds}}{2} = 2.4 \text{ seconds}.$$

Earlier we found that the distance from the sun to the pair is near 8.7×10^5 A.U. Then the mean angular separation of 2.4 seconds equals

$$\frac{2.4 \times 8.7 \times 10^5 \text{ A.U.}}{2.1 \times 10^5} = 10 \text{ A.U.},$$

or the stars are separated by about the same distance as Saturn is from the sun. (We might have expected a result of this size because we know that the period of Saturn around the sun is 30 years.)

Now, upon substituting the numbers into Eq. (8.11) we have:

$$\frac{(m_1 + m_2)\ \text{pair}}{m_{\text{sun}}} = \left(\frac{1}{45}\right)^2 \left(\frac{10}{1}\right)^3$$

$$= \frac{1000}{2020} = 0.50,$$

or, the two stars together have about half the mass of the sun.

We can even separate this mass into the two components. In the diagram of motions relative to the center of mass we see that one star has a smaller motion and we conclude that it must be more massive. For the positions of 1970 (or those observed a cycle earlier in 1925) the less massive star is 1.7 times farther than the other from the center of mass.

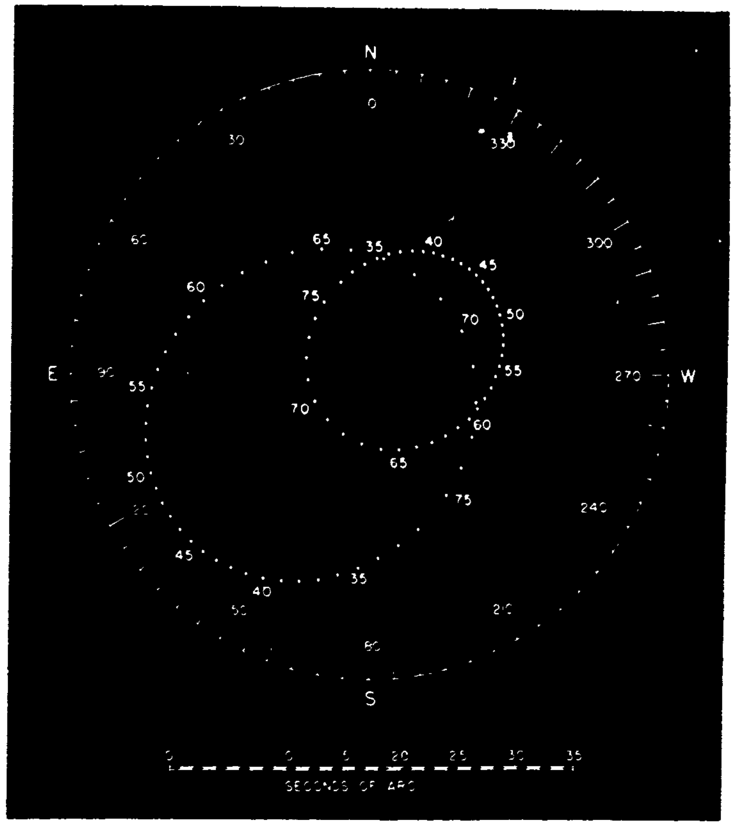


Fig. 8.16 Kruger 60's components trace elliptical orbits, indicated by dots, around their center of mass marked by circles. For the years 1932 to 1975, each dot is plotted on September 1st. The outer circle is calibrated in degrees, so the position angle of the companion may be read directly, through the next decade.

So the masses of the two stars are in the ratio 1.7 : 1. Of the total mass of the pair, the less massive star has

$$\frac{1}{1 + 1.7} \times 0.5 = 0.18 \text{ the mass of the sun,}$$

while the other star has 0.32 the mass of the sun. The more massive star is more than four times brighter than the smaller star. Both stars are red dwarfs, less massive and considerably cooler than the sun.

From many analyses of double stars astronomers have found that the mass of a star is related to its total light output, as shown in Fig. 8.17.

SG 8.15

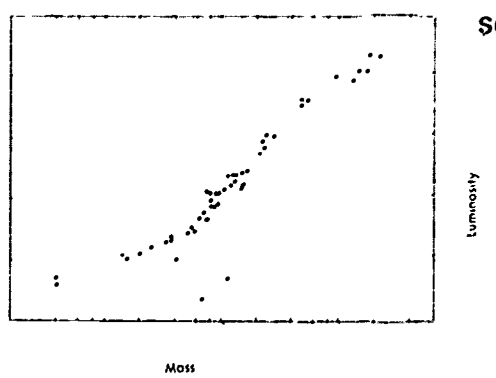


Fig. 8.17 The mass-luminosity relation. The three points at the bottom represent white dwarf stars, which do not conform to the relation.

Another method of measuring G is given in the Articles Section, page 64.

very large masses we compute for other large bodies. Perhaps for this reason Cavendish preferred to give his result not as the mass of the earth but as the mass divided by its volume (known quite well from geographical surveys). He concluded that the earth was 5.48 times as dense as water. It is a tribute to his experimental skill that his result is so close to the modern value of 5.52.

This large density for the earth raised questions of very great interest to geologists. The average density of rock found in the crust of the earth is only about 2.7, and even the densest ores found rarely have densities greater than the earth's overall average. Since much mass must be somewhere, and it is not at the surface, apparently the material making up the core of the earth must be of much higher density than the surface material. Some increased density of deep rocks should be expected from the pressure of the upper layers of rock. But squeezing cannot account for most of the difference. Therefore, the center of the earth must be composed of materials more dense than those at the surface and probably is composed mainly of one or more of the denser common elements. Iron and nickel are the most likely candidates, but geologists are also considering other alternatives.

SG 8.13

$$m_S = \frac{4\pi^2 R^3}{GT^2} \qquad (8.10)$$

With the value of G known by experiment we can also find the mass of the sun, or of any celestial object having some type of satellite. Once we know the value of G, we can substitute numbers into Eq. (8.10) and find m_S. In the case of the sun, the earth is the most convenient satellite to use for our computation. The earth's distance from the sun is 1.50×10^{11}m, and its period is one year, or 3.16×10^7 sec. We have already seen that $G = 6.67 \times 10^{-11}$ m^3/kg$\cdot$sec^2. Upon substituting these numbers in Eq. (8.10), we find the mass

$$m_S = \frac{4\pi^2(1.50 \times 10^{11}\,\text{m})^3}{(6.67 \times 10^{-11}\,\text{m}^3/\text{kg sec}^2)(3.16 \times 10^7\,\text{sec})^2} = \frac{4\pi^2(1.50 \times 10^{11})^3\,\text{m}^3}{(6.67 \times 10^{-11})(3.16 \times 10^7)^2\,\frac{\text{m}^3}{\text{kg sec}^2} \times \text{sec}^2}$$

$$m_S = 2.0 \times 10^{30}\,\text{kg}$$

of the sun to be 2.0×10^{30} kg. Earlier we found that the mass of the earth was about 6.0×10^{24} kg. Thus, the ratio of the mass of the sun to the mass of the earth is

$$\frac{2 \times 10^{30}\,\text{kg}}{6 \times 10^{24}\,\text{kg}},$$

which shows that the sun is 3.3×10^5 (about a third of a million) times more massive than the earth.

W.F. Magie's A Source Book of Physics, pp. 105-111, (Harvard University Press) contains a reprint of the original paper written by Cavendish about his experiment. This paper is fairly readable, with illustrations of the apparatus; some of your students would probably enjoy it.

This same procedure can be used to find the actual mass of any planet having a satellite. For planets not having satellites, their attraction on other planets can be used. But all methods rely at some point on the Law of Universal Gravitation. The masses of the sun, moon and planets relative to the earth are listed in Table 8.2. Notice that the planets taken together add up to not much more than 1/1000th part of the mass of the solar system.

Table 8.2

The mass of the sun, moon, and planets relative to the mass of the earth. The earth's mass is approximately 6.0×10^{24} kg.

Sun	333,000.00	Jupiter	318.
Moon	0.012	Saturn	95.
Mercury	0.056	Uranus	14.6
Venus	0.82	Neptune	17.3
Earth	- 1.00	Pluto	0.8?
Mars	0.108		

See "Life Story of a Galaxy" in Project Physics Reader 2.

See "Expansion of the Universe" in Project Physics Reader 2.

See "The Stars Within Twenty-two Light Years That Could Have Habitable Planets" in Project Physics Reader 2.

Q30 How was the value of G determined experimentally by Cavendish?

Q31 What geological problems result from the discovery that the mean density of the earth is 5.52?

8.16 Beyond the solar system. We have seen how Newton's laws have been applied to explaining much about the earth and the entire solar system. But now we turn to a new question. Do Newton's laws, which are so useful within the solar system, also apply at greater distances among the stars?

Over the years following publication of the Principia several sets of observations provided an answer to this important question. At the time of the American Revolutionary War, William Herschel, a musician turned amateur astronomer was, with the help of his sister Caroline, making a remarkable series of observations of the sky through his home-made high-quality telescopes. While planning how to measure the parallax due to the earth's motion around the sun, he noted that sometimes one star had another star quite close. He suspected that some of these pairs might actually be double stars held together by their mutual gravitational attractions rather than being just two stars in nearly the same line of sight. Continued observations of the directions and distances from one star to the other of the pair showed that in some cases one star moved during a few years in a small arc of a curved path around the other (see Fig. 8.18). When enough observations had been gathered, astronomers found that these double stars, far removed from the sun and planets, also moved around each other in accordance with

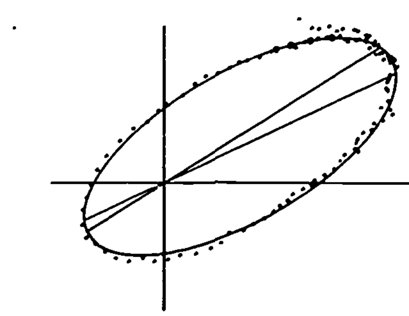


Fig. 8.18 The motion over many years of one component of a binary star system. Each circle indicates the average of observations made over an entire year. Open circles designate the years 1830-1873. Dark circles are used for the years after 1873.

Summary 8.16
By using Newton's law of universal gravitation, we can weigh (find the masses of) double stars.

See "Space, The Unconquerable" in Project Physics Reader 2.

Kepler's laws, and therefore in agreement with Newton's Law of Universal Gravitation. By the use of Eq. (8.11), p. 102, astronomers have found the masses of these stars range from about 0.1 to 50 times the sun's mass.

Q32 In what ways has the study of double stars led to the conclusion that Newton's Law of Universal Gravitation applies between stars?

Summary 8.17
Make clear that Newton did not stand alone, but, as he said, obtained his success by "standing upon the shoulders of giants." Refer back to the Development in Section 8.3 about the ability of men like Newton and Galileo to synthesize existing ideas, as well as create new ones.

His contacts with other scientists, like Halley and Flamsteed, who were members of the Royal Society (organized informally about 1660 and officially chartered by King Charles in 1662) were important and fruitful.

8.17 Some influences on Newton's work. The scientific output of Newton and his influence on the science of his time were perhaps unequaled in the history of science. Hence, we should look at Newton as a person and wonder what personal attributes led to his remarkable scientific insight. He was a complicated, quiet bachelor intensely involved with his studies, curiously close to the usual stereotype of a genius, which is so often completely wrong.

Newton was a man of his time and upbringing; some of his work dealt with what we today regard as pseudosciences. He had an early interest in astrology. He seems to have spent much time in his "laboratory," cooking potions that smelled more of alchemy than of chemistry. Yet in all his activities, he seems to have been guided and motivated by a search for simple underlying general principles and never for quick practical gains.

Throughout the discussion above we have been mixing the physics of terrestrial bodies with the motions of celestial bodies, just as Newton did. Since the relationships could be verified, we truly have a synthesis of terrestrial and celestial physics of great power and generality. A brief reconsideration of Eq. (8.4) can remind us of the inclusiveness of the results obtained. If we have

$$F_{grav} = \frac{Gm_1m_2}{R^2}, \qquad (8.4)$$

and if G is actually a universal constant, we are able to derive, and therefore to understand better, many particulars that previously seemed separate. For example, we can conclude:

1. that g should be constant at a particular place on earth;

2. that g might be different at places on earth at different distances from the earth's center, or at different latitudes which have different velocities of daily rotation as the earth turns;

3. that at the earth's surface the weight of a body must be related to its mass;

4. that Kepler's three Laws of Planetary Motion hold, and are interrelated;

5. that oceanic tides are the result of the net attraction of the sun and moon on the liquid waters.

Throughout Newton's work is his belief that celestial phenomena are explainable by quantitative terrestrial laws. He felt that his laws had a physical meaning and were not just mathematical tricks or conveniences covering unknowable true laws. Just the opposite; the natural physical laws governing the universe were accessible to man, and the simple mathematical forms of the laws were evidence of their reality.

Newton combined the skills and approaches of both the experimental and theoretical scientist. He made ingenious pieces of equipment, such as the first reflecting telescope, and performed skillful experiments, especially in optics. Yet he also applied his great mathematical and logical powers to the creation and analysis of theories to produce explicit, testable statements.

Summary Sec 8.18
1. Newton's success in physics led most scientists to view natural phenomena as mechanical or machinelike.

2. This mechanistic viewpoint made itself felt in all aspects of man's knowledge.

3. It became evident, by the beginning of the twentieth century, that certain phenomena could not be satisfactorily explained by a Newtonian analysis. One set of such phenomena concerns bodies moving at high speeds and/or in the vicinity of large masses. Another concerns the case of very small bodies as in the atomic and subatomic worlds.

Many of the concepts which Newton used came from his scientific predecessors and contemporaries. For example, Galileo and Descartes had contributed the first step to a proper idea of inertia, which became Newton's First Law of Motion. Kepler's planetary laws were central in Newton's consideration of planetary motions. Huygens, Hooke and others clarified the concepts of force and acceleration, ideas which had been evolving for centuries.

In addition to his own experiments, he selected and used data from a great variety of sources. Tycho Brahe was only one of several astronomers whose observations of the moon's motion he used. When he could not carry out his own measurements, he knew whom to ask.

Lastly, we must recall how fruitfully and exhaustively his own specific contributions were used and expanded throughout his work. For instance, in developing his Theory of Universal Gravitation, his Laws of Motion and his various mathematical inventions were used again and again. Yet Newton was modest about his achievements, and he once said that if he had seen further than others "it was by standing upon the shoulders of Giants."

This thorough interlocking of numerous theories, techniques, insights and deductions is one of the more satisfying aspects of all science.

Q33 What are some of the reasons that Newton is honored today?

8.18 Newton's place in modern science. The success of Newtonian mechanics greatly influenced scientific and philosophical thought in the early eighteenth century. The new mechanistic

L17: Perturbations

The impact of Newtonian mechanics on the civilized world is a fascinating area of study which can be used to link the physics course with other courses like history and literature.

approach suggested that all observations could be interpreted in terms of mechanical theories. In economics, philosophy, religion and the developing "science of man," the successful approach of Newton and the Newtonians encouraged the rising Age of Reason.

One consequence of the mechanistic attitude, lingering on to the present day, was a widespread belief that with Newton's laws (and later similar ones for electrodynamics) the future of the whole universe and each of its parts could be predicted. One need know only the several positions, velocities and accelerations of all particles at any one instant. As Kepler had suggested, the universe seemed to be a great clockwork. This was a veiled way of saying that everything worth knowing was understandable in terms of physics, and that all of physics had been discovered. As you will see later in this course, in the last hundred years scientists have been obliged to take a less certain position about their knowledge of the world.

The whole "age of reason" of the eighteenth century is tied to the Principia.

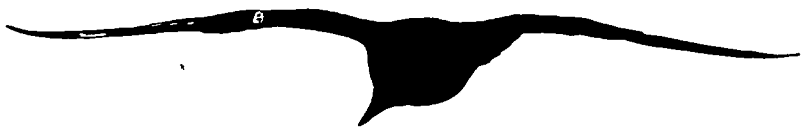
Today we honor Newtonian mechanics for less inclusive, but more valid reasons. The content of the Principia historically formed the basis for the development of much of our physics and technology. Also the success of Newton's approach to his problems provided a fruitful method which guided work in the physical sciences for the subsequent two centuries.

You might want to emphasize the tremendous impact that the Newtonian model had upon science, by using biology as an example. (See Development Section.)

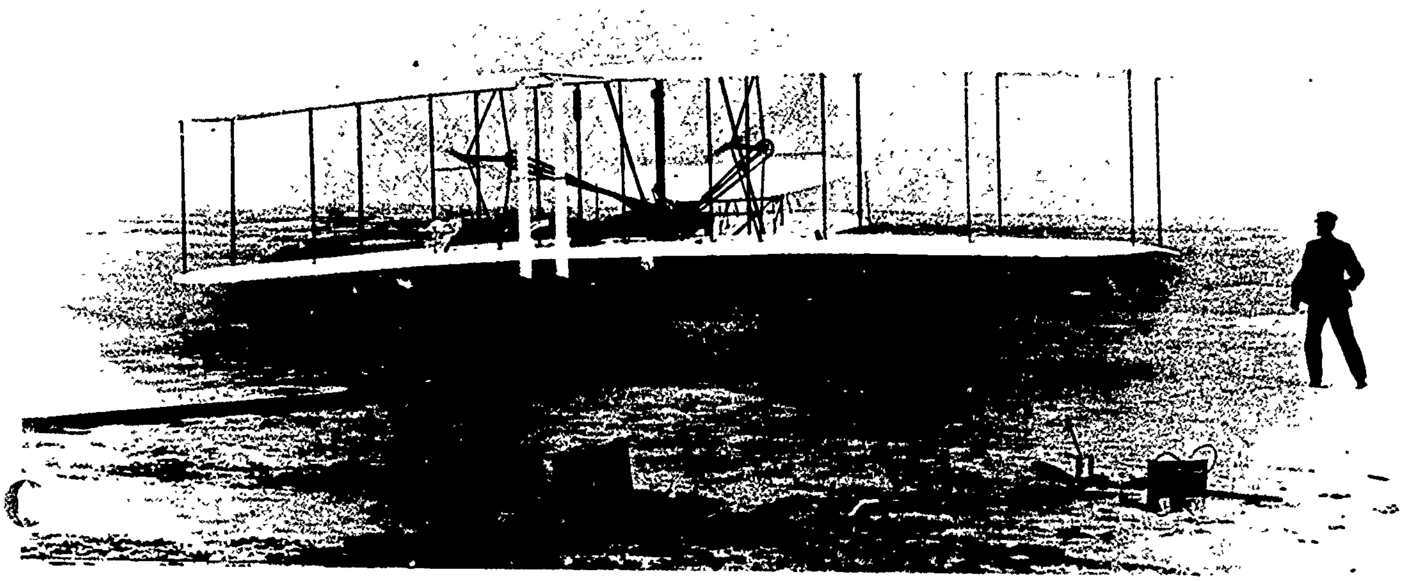
We recognize now that Newton's mechanics holds only within a well-defined region of our science. For example, although the forces within each galaxy appear to be Newtonian, one can speculate that non-Newtonian forces operate between galaxies. At the other end of the scale, among atoms and subatomic particles, an entirely non-Newtonian set of concepts had to be developed to account for the observations.

Even within the solar system, there are a few small residual discrepancies between predictions and observations. The most famous is the too great angular motion of the perihelion of Mercury's orbit: Newtonian calculations differ from the observations by some 43 seconds of arc per century.

Such difficulties cannot be traced to a small inaccuracy of the Law of Gravitation, which applies so well in thousands of other cases. Instead, as in the case of the failure of the Copernican system to account accurately for the details of planetary motion, we must reconsider our assumptions. Out of many studies has come the conclusion that Newtonian mechanics cannot be modified to explain certain observations. Newtonian science is joined at one end with relativity theory, which is important for bodies moving at high speeds. At the



A bird in flight and two interpretations of flight: Constantin Brancusi's "Bird in Space" (Museum of Modern Art, New York, 1919) and the Wright Brothers' "Kitty Hawk" flight (1903). Analogously, different theories may result from the different intentions of their originators.



Predictions from theories may require new observations. These often require improvements in the precision of apparatus, or the creation of new types of instruments. The annual parallax of the stars predicted by Aristarchus could not be observed until telescopes were invented and developed so that very small angles could be measured reliably.

Theories that have later been discarded may have been initially useful because they encouraged new observations. The idea that comets were some local phenomenon led Tycho to compare distant observations of the directions to a comet.

Theories that permit quantitative predictions are preferred to qualitative theories. Aristotle's theories of motion explained in a rough way how bodies moved, but Galileo's theories were much more precise. The qualitative planetary theory of Tycho and Descarte's vortex theory of motion were interesting, but did not rely upon measurements.

An "unwritten text" lies behind most terms in the seemingly simple statement of theories. For example, "force equals mass times acceleration" is a simple sentence. However, each word carries a specific meaning based on observations and definitions and other abstractions.

Communication between scientists is essential. Scientific societies and their journals, as well as numerous international meetings, allow scientists to know of the work of others. The meetings and journals also provide for public presentations of criticisms and discussions. Aristarchus' heliocentric theory had few supporters for centuries, although later it influenced Copernicus who read of it in the Almagest.

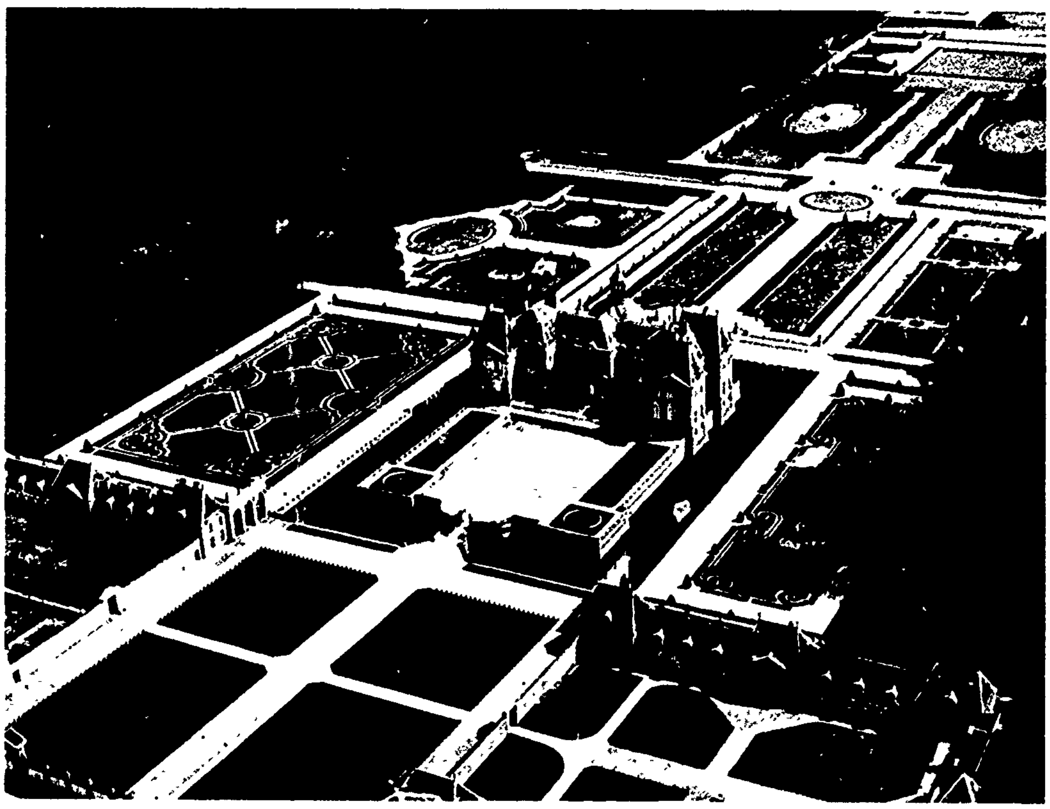
Some theories are so strange that they are accepted very slowly. The heliocentric theory was so different from our geocentric observations that many people were reluctant to accept the theory. Strange theories often involve novel assumptions which only a few men will at first be willing or able to consider seriously. Novelty or strangeness is, of course, no guarantee that a theory is important; many strange theories prove to be quite unuseful.

In the making of a theory, or in its later description to others, models are often used as analogies. Physical models are most easily understood. The chemists' ball-models of atoms are an example. Similarly, a planetarium projects images of heavenly bodies and their motions. Thus models may reproduce some of the phenomena or suggest behaviors predicted by the theory.

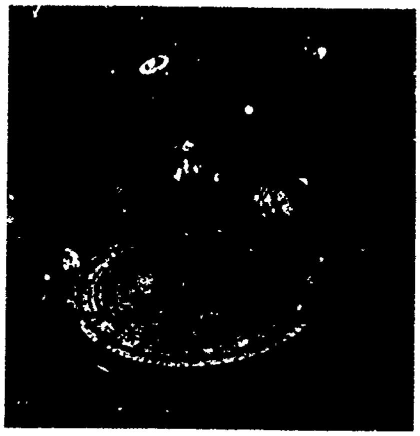
However, models are man-made and are not the real phenomena or the conceptions with which the theory deals in detail. Although models are often quite useful, they can also be misleading. They represent the theory only to the degree that the maker includes some aspects of what he is representing. A'so, the maker may add other aspects which do not relate to the original, e.g. chemists' atoms do not have definite sizes like the balls do that serve as visual models.

Other models may be statistical or mathematical. It is useful to consider all scientific theories as models which attempt to describe the suspected interworkings of some quantities abstracted from observations.

The power of theories comes from their generality. Theories are distillates from many observations, empirical laws and definitions. The more precise a theory is, the better it will agree with specific observations. But most important is the usefulness of a theory in describing a wide range of observations and predicting quite new observations. An aesthetic feeling of beauty and niceness, even of elegance is often stimulated by a concise yet broadly inclusive theory.



In areas other than science, conciseness and theory are used to achieve beauty and elegance. Even the gardens around this 17th-century French chateau reflect a preference for order.



Epilogue In this unit we have reached back to the beginnings of recorded history to follow the attempt of men to explain the regular cyclic motions observed in the heavens. Our purposes were double: to examine with some care the difficulties of changing from an earth-centered view of the heavens to one in which the earth came to be seen as just another planet moving around the sun. Also we wanted to put into perspective Newton's synthesis of earthly and heavenly motions. From time to time we have also suggested the impact of these new world views upon the general culture, at least of the educated people. We stressed that each contributor was a creature of his times, limited in the degree to which he could abandon the teachings on which he was raised, or could create or accept bold new ideas. Gradually through the successive work of many men over several generations, a new way of looking at heavenly motions arose. This in turn opened new possibilities for even further new ideas, and the end is not in sight.

See "An Appreciation of The Earth" in Project Physics Reader 2.

Prominent in our study have been references to scientists in Greece, Egypt, Poland, Denmark, Austria, Italy and England. Each, as Newton said, stood on the shoulders of others. For each major success there are many lesser advances or, indeed, failures. Thus we see science as a cumulative intellectual activity, not restricted by national boundaries or by time; nor is it inevitably and relentlessly successful, but it grows more as a forest grows, with unexpected changes in its different parts.

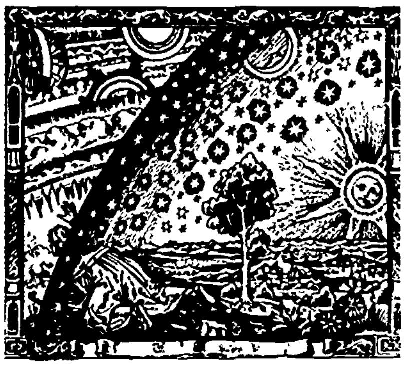
It must also be quite clear that the Newtonian synthesis did not end the effort. In many ways it only opened whole new lines of investigations, both theoretical and observational. In fact, much of our present science and also our technology had its effective beginning with the work of Newton. New models, new mathematical tools and new self-confidence—sometimes misplaced, as in the study of the nature of light—encouraged those who followed to attack the new problems. A never-ending series of questions, answers and more questions was well launched.

In the perspective of history it is intriguing to speculate why Newton turned to astronomy. Perhaps it was in part because the motions of the planets had been a major and persistent problem for centuries. But at least some of his interest—and reason for success—lay in the fact that the heavenly bodies do not move like those on the earth. In the heavens there is no friction, or air resistance. Thus the possibility that a few simple mathematical relationships between idealized factors could fit observations had its first

major application to conditions which were close to the idealized, simplified schemes—and it worked. Then scientists could return to earthly phenomena with renewed confidence that this line of attack could be profitable. We can wonder how mechanics would have developed if we lived on a cloud-bound planet from which the stars and planets were not visible.

Among the many problems remaining after Newton's work was the study of objects interacting not by gravitational forces but by friction and collisions. Experiments were soon to raise questions about what aspects of moving bodies were really important. This led, as the next unit shows, to the identification of momentum and of kinetic energy, and then to a much broader view of the nature and importance of energy. Eventually from this line of study emerged other statements as grand as Newton's Universal Gravitation: the conservation laws on which so much of modern physics—and technology—is based, especially the part having to do with many interacting bodies making up a system. That account will be introduced in Unit 3.

A : Haiku



Development of Equations in Chapter 8.

$F_{grav} \propto \frac{1}{R^2}$, initially an assumption

(8.1) $F_{grav} \propto m_P m_S$, a conclusion from the definition of force

(8.2) $F_{grav} \propto \frac{m_P m_S}{R^2}$, combines the two relations above

(8.3) $F_{grav} = G\frac{m_P m_S}{R^2}$, relation (8.2) restated as an equation by inclusion of constant G

(8.4) $F_{grav} = G\frac{m_1 m_2}{R^2}$, Law of Universal Gravitation for any two masses: m_1, m_2.

(8.5) $a_c = \frac{v^2}{R}$, centripetal acceleration, from Unit 1

but

$v = 2\pi R/T$,

and

$v^2 = 4\pi^2 R^2/T^2$,

so

(8.6) $a_c = 4\pi^2 R/T^2$.

However, because

$F = ma$,

(8.7) $F_c = ma_c$

$= m4\pi^2 R/T^2$, the centripetal force

SG 8.14

SG 8.17

Equate (8.3) and (8.7),

$F_{grav} = F_c$,

(8.8) $T_P{}^2 = \left[\frac{4\pi^2}{Gm_S}\right] R^3$

Rearrange (8.8)

(8.10) $m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{R^3}{T_P{}^2}\right)$

Compare two examples of (8.10) for planet and satellite

(8.11) $\frac{m_{sun}}{m_P} = \left[\frac{T^2_{sat}}{T_P{}^2}\right]\left[\frac{R_P{}^3}{R^3_{sat}}\right]$

$F_{grav} = \text{Weight}$

$G\frac{m_E m}{R^2} = mg$, which becomes

(8.9) $G = \left[\frac{R_E{}^2}{m_E}\right] g$,

or

(8.12) $m_E = \left[\frac{R_E{}^2}{G}\right] g$

8.1 If the velocity of a planet is greater at A than at B, where is the sun located? In what direction does the vector difference of the velocities at A and B point? *Sun is at lower focus. Toward the sun.*

8.2 a) Compare the force on the earth due to the sun's gravitational attraction with the force on the sun due to the earth's gravitational attraction. *Equal and opposite*

b) If the mass of the sun is $3.3 \times 10^5 \times$ the mass of the earth, compare the acceleration of the sun toward the earth with the acceleration of the earth toward the sun. $a_s = 3 \times 10^{-6} a_e$

8.3 Draw a rough graph of the weight of a 1-kilogram mass in terms of its distance from the center of the earth. Take the radius of the earth as R and go out to 5R. Points at 2R, 3R, 4R, and 5R would be sufficient. Will the weight be zero at 100R? At 1000R? *An activity*

8.4 Two bodies, A and B, are observed to be moving in circular orbits. The orbital radius and period of body A are both twice those of body B. Could both bodies be moving around the sun? Explain your conclusion. *No. Discussion.*

8.5 If the radius of planet A's orbit is twice that of planet B's, what is the ratio of (a) their periods, (b) their orbital speeds, (c) their accelerations toward the sun? $T_A = 2.8\,T_B$. $V_A = 0.71\,V_B$. $a_A = 0.25\,a_B$.

8.6 Two satellites revolve around different planets at the same distance, R. One satellite has a period three times that of the other.

a) Which planet has the larger mass? *shorter period satellite*
b) What is the ratio of the masses of the two planets? *9:1*

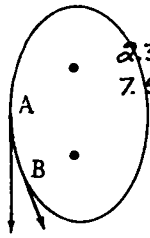
8.7 At what altitude above the earth's surface would the acceleration due to gravity be

a) 3/4 g? *0.15 earth radii (960 km)*
b) 1/2 g? *0.41 earth radii (2600 km)*
c) 1/4 g? *1.00 earth radii (6400 km)*

8.8 a) What is the acceleration due to gravity on the moon's surface? [Hint: Equate the weight of any mass m_1 on the moon with the force exerted on it according to the Universal Law of Gravitation.] The moon's radius is 1738 km, and its mass is 7.15×10^{22} kg. *1.58 m/sec²*

b) How much would a 72-kg astronaut weigh on the moon's surface? What would be his mass there? *114 N No change in mass.*

8.9 Mass of Jupiter: 1.90×10^{27} kg. Average distance from Jupiter to sun: 7.78×10^8 km.



a) At what point on an imaginary line connecting the sun's center with the center of Jupiter would a spacecraft have no net force from these two bodies? *2.33×10^7 Km* *7.55×10^8 Km*
b) How does the distance between this point and the center of Jupiter compare with the mean orbital radius of the orbit of Jupiter's outermost satellite (orbital radius = 2.37×10^7 km)? *About the same*
c) From your results in parts (a) and (b) what peculiarities, if any, would you expect to observe in the satellite's motion? *Unstable orbit. Discussion*
d) Would you expect to find a satellite even farther from Jupiter? *No. Discussion.*

8.10 If two planets in another solar system have the same average density, but the radius of one of them is twice that of the other:

a) which one would have the greater surface gravitational pull? *largest*
b) what is the ratio between the surface gravitational strengths of the two planets? *2:1*

8.11 Mars has two satellites, Phobos and Deimos—Fear and Panic. The inner one, Phobos, revolves at 5,800 miles from the center of Mars with a period of 7 hrs 39 min. *From Mars Phobos would rise in west.*

a) Since the rotation period of Mars is 24 hrs 37 min, what can you conclude about the apparent motion of Phobos as seen from Mars?
b) What is the mass of Mars? 6.7×10^{23} kg

8.12 The shortest earth-Mars distance is about 56×10^6 km; the shortest Mars-Jupiter distance is about 490×10^6 km. The masses of these planets are as follows:

Earth	5.96×10^{24} kg
Mars	6.58×10^{23} kg
Jupiter	1.91×10^{27} kg.

a) When (under what conditions) do these "shortest" distances occur? *Discussion*
b) What is the gravitational force between the earth and Mars when they are closest together? Between Jupiter and Mars? $F_{E,M} = 8.3 \times 10^{16}$ N $F_{J,M} = 3.5 \times 10^{17}$ N
c) Do you expect the motion of Mars to be influenced more by the attraction of the earth or of Jupiter? *Jupiter*

8.13 Callisto, the second largest satellite of Jupiter, is observed to have a period of revolution of 1.442×10^6 sec. Its mean orbital radius is 1.87×10^6 km. Using only these data and the value of

the Universal Gravitational Constant, calculate the mass of Jupiter. 1.86×10^{27} kg.

8.14 Making use of the expression $\frac{GM}{R^2} = \frac{v^2}{R}$, which follows directly from the Law of Gravitation, show that the time taken per revolution by a satellite at a distance R from the center of the earth is given by:

$$T = 2\pi\left(\frac{R^3}{GM}\right)^{\frac{1}{2}}.$$

Hint: make use of $T = \frac{2\pi R}{v}$. Derivation

8.15 Two balls, each having a mass m, are separated by a distance r. Find the point which lies along the line joining their centers where the gravitational attractions cancel one another.

a) What will happen to this point if both the balls are made twice their size? The same point

b) What will happen to this point if one of the balls is made twice as heavy as the other? $R_1 = 1.41 R^2$

8.16

Mass of the moon:	7.18×10^{22} kg
Mass of the sun:	1.99×10^{30} kg
Mass of the earth:	5.96×10^{24} kg
Average distance from sun to earth:	1.495×10^{8} km
Average distance from moon to earth:	3.84×10^{5} km.

a) Derive the gravitational pull of the moon, and of the sun on the earth. $F_M = 1.93 \times 10^{20}$N. $F_S = 3.52 \times 10^{22}$N.

b) Does the sun or the moon have the greatest effect on the earth's tides? Can you explain this quantitatively?

8.17 At the earth's surface, a 1-kg mass standard weighs 9.81 newtons. The Midas 3 satellite launched July 27, 1964 orbits in a nearly circular orbit 3,430 km above the earth's surface with a period of 161.5 minutes. What is the centripetal acceleration of the satellite? What is the condition for a circular orbit? (Remember, the radius of the satellite's orbit is the height above the ground plus the earth's radius.)

$\frac{r_E^2}{r_O^2}$. $a_c = \frac{4\pi^2 R}{T^2}$

Answers to End of Section Questions

Your self-checking answers need not be as elaborate as these.

Chapter 5

Q1 We conclude that the ancient peoples watched the skies from: cave paintings of star patterns, the orientation of the pyramids in Egypt, Stonehenge and similar structures in England, Scotland and France; the alignment of observing windows in buildings in Mexico and Peru. Written records were made in Babylon, Greece, Egypt, China since some centuries B.C.

Q2 Calendars were needed for establishing the proper times for agricultural activities and for religious rites.

Q3 During one year the sun shows three motions: daily rising and setting, seasonal drift eastward among the stars, and seasonal north-south variation.

Q4 The difference between the Gregorian calendar and the positions of the sun will add up to one day during an interval of 3,333 years.

Q5 During a month, which begins with new moon, as the moon passes the sun, the moon continually moves eastward but not at an exactly even rate. The moon also moves north and south so that it is always near the ecliptic.

Q6 Usually when the moon passes the sun, or the direction opposite to the sun, the moon is north or south of the ecliptic. Thus the moon's shadow misses the earth at new moon. Similarly, the moon moves above or below the earth's shadow at full moon. However, twice a year the moon is near the ecliptic at new and full moon. These are the times when eclipses of the sun and moon can occur.

Q7 Mercury and Venus are always observed near the sun. They will be low in the west after sunset, or low in the east before sunrise.

Q8 When in opposition a planet is opposite the sun. Therefore the planet would rise at sunset and be on the north-south line at midnight.

Q9 Mercury and Venus show retrograde, that is, westward motion among the stars, after they have been farthest east of the sun and visible in the evening sky. At this time they are brightest, and nearest the earth. They are then moving between the earth and sun.

Q10 Mars, Jupiter and Saturn show retrograde motion when they are near opposition.

Q11 The retrograde motion of Mars has the largest angular displacement, but the shortest period. Saturn has the longest period of the planets visible to the naked eye, but has the smallest angular displacement. The retrograde motion of Jupiter is intermediate in displacement and duration.

Q12 Plato assumed that the motions of the planets could be described by some combination of uniform motions along circles. He also assumed that the earth was at the center of the largest circle for each planet.

Q13 Our knowledge of Greek science, as well as that of every other ancient civilization, is incomplete because many of their written records have been destroyed by fire, weathering and decay. Yet each year new records are being unearthed and deciphered.

Q14 The Greeks around the time of Plato assumed that a theory should be based on self-evident propositions. Quantitative observations were rarely used as a basis for judging the usefulness of a theory.

Q15 A geocentric system is an earth-centered system. It is also an observer-centered system, because as observers we are on the earth.

Q16 The first solution to Plato's problem was made by Eudoxus. He described the planetary motions by a system of transparent crystalline spheres which turned at various rates around various axes.

Q17 Aristarchus assumed that the earth rotated daily—which accounted for all the daily motions observed in the sky. He also assumed that the earth revolved around the sun—which accounted for the many annual changes observed in the sky.

Q18 If the earth were moving around the sun, it would have a shorter period than would Mars, Jupiter and Saturn. When the earth moved between one of these planets and the sun (with the planet being observed in opposition), the earth would move faster than the planet. We would see the planet moving westward in the sky as retrograde motion.

Q19 The distance between the earth and sun was known to be some millions of miles. If the earth revolved around the sun during a year, the direction to the stars should show an annual shift—the annual parallax. This was not observed until 1836 A.D.

Q20 Aristarchus was considered to be impious because he suggested that the earth, the abode of human life, might not be at the center of the universe.

Q21 Ptolemy assumed: 1) that the heaven is spherical and rotates once each day around the earth, 2) that the earth is spherical, 3) that the earth is at the center of the heavens, 4) that the size of the earth is negligible in comparison to the distance to the stars, 5) that the earth has no motions, and 6) that uniform motion along circles is the proper behavior for celestial objects.

Q22 If the earth rotated, Ptolemy argued that birds would be left behind and that great winds would continually blow from the east.

Q23 The radii of the epicycles of Mars, Jupiter and Saturn must always be parallel to the line between the earth and sun. This also meant that each of these epicycles had a period of exactly one year.

Q24 The Ptolemaic system was purely a mathematical model and probably no one believed it was a physical description.

Q25 Ptolemy displaced the earth from the exact center of the universe with his equants and eccentrics.

Q26 The Ptolemaic system survived because it predicted the positions of sun, moon and planets; because it agreed with the philosophical and theological doctrines and because it made sense, and it was not challenged by a better, simpler model.

Chapter 6

Q1 Copernicus rejected the use of equants because a planet moving on an equant did not move at a uniform angular velocity around either the center of the equant or around the earth. This was essentially an aesthetic judgment.

Q2 Apparently Copernicus meant that the sun was not exactly at the center of motion for any planet (he used eccentrics). Yet in general the sun was in the center of all the various planetary motions.

Q3

	Assumed by	
	Ptolemy	Copernicus
a) The earth is spherical	Yes	Yes
b) The earth is only a point compared to the distances to the stars	Yes	Yes
c) The heavens rotate daily around the earth	Yes	No
d) The earth has one or more motions	No	Yes
e) Heavenly motions are circular	Yes	Yes
f) The observed retrograde motion of the planets results from the earth's motion around the sun	No	Yes

Q4 Copernicus derived distances to the planetary orbits in terms of the earth's distance from the sun. He also derived periods for the planetary motions around the sun. So far as we know, Aristarchus did not develop his heliocentric theory to the point that he reached similar results.

Q5 Copernicus argued that his heliocentric system was inherently simpler than the geocentric system of Ptolemy. Also, his results agreed at least as well as Ptolemy's with observations. Furthermore, Copernicus argued that his simple system reflected the mind of the Divine Architect.

Q6 The predictions made by both Ptolemy and Copernicus differ from observations by as much as 2°, four diameters of the moon.

Q7 Copernicus argued that the distance to the stars must be very great because they showed no annual shift (parallax). But the distance required to make the parallax unobservable was enormously greater than people wanted to accept. That the predicted shift was not observed was weak evidence. Negative evidence is not very convincing even when there seem to be only two possible alternative conclusions.

Q8 Simple is a difficult word to interpret. The actual computational scheme required by the Copernican system was not simple, even though the general idea of a moving earth and a fixed sun seemed simple.

Q9 As the quotation from Francis Bacon indicates, the evidence did not permit a clear choice between the two possible explanations. To many people the argument seemed to be a tempest in a tea cup.

Q10 The major difference between the Ptolemaic and the Copernican systems was the assumption about the mobility of the earth: daily rotation and annual revolution about the sun. This was a difference in the frame of reference.

Q11 Because the astronomical interpretations dealt with the structure of the whole universe, they overlapped with conclusions drawn from religious discussions. The synthesis by Thomas Aquinas of Aristotelean science and Christian theology increased the difficulty of discussing one separately from the other.

Q12 The Copernican system conflicted with the accepted frame of reference in which the earth was central and stationary. The distances which Copernicus derived for the separation of the planets from the sun implied that vast volumes of space might be empty. This conflicted with the old assumption that "nature abhors a vacuum."

Q13 Some conflicts between scientific theories and philosophical assumptions are:

a) the earth appears to be very old although strict interpretation of Biblical statements lead to ages of only a few thousand years, less than those attributed to ancient cities;
b) the geological assumption of uniformitarianism—slow changes throughout long times—conflicted with the assumption of abrupt changes in the earth's history from major floods, earthquakes, etc.;
c) the idea of slow genetic evolution in biology clashed with the idea of a unique and recent creation of mankind;
d) the astronomical conclusion that the earth was only one of several planets clashed with the assumption that the earth was created uniquely as the abode of human life.

Q14 Copernicus brought to general attention the possibility of a new explanation of the astronomical observations. This shift in assumptions permitted others—Kepler, Galileo and later Newton—to apply and expand the initial propositions of Copernicus.

Q15 The Copernican system proposed that the earth was just one of many planets and was in no way uniquely created as the abode of life. Therefore, there might be life on other planets.

Currently astronomers conclude that there might be some form of life on Mars or Venus. The other planets in our solar system seem to be too hot or too cold, or to have other conditions opposed to life-as-we-know-it. There is now general agreement that many other stars probably have planets and that life might well exist on some of them. A clear distinction must be made between some form of life and what we call intelligent life. Since the only means by which we could learn of any intelligent life on planets around other stars would be through radio signals, intelligent life means that the living organisms would transmit strong radio signals. On earth this has been a possibility for less than half a century. Thus we have, in these terms, been "intelligent life" for only a few years.

Q16 Tycho's observations of the new star and then of the comet of 1577 directed his attention to astronomical studies.

Q17 Tycho's conclusions about the comet of 1577 were important because the comet was shown to be an astronomical body—far beyond the moon. Also the comet moved erratically, unlike the planets, and seemed to go through the crystalline spheres of the Aristotelean explanation.

Q18 Tycho's observatory was like a modern research institute because he devised new instruments and had craftsmen able to make them, he had a long-term observing program, he included visitors from other places, and he worked up and published his results.

Q19 The Bayeaux tapestry shows the people cowering below tne image of Halley's comet. Other pictures may show similar scenes. Many allusions to the dire effects of comets appear in Shakespeare and other writers.

Q20 For several months during 1909-1910 Halley's comet moved westward—in retrograde motion. Because it stayed near the ecliptic, we might suspect that its

orbital plane makes only a small angle with the earth's orbital plane (the ecliptic plane). Whether the comet is moving in the direction of the planets or in the opposite direction is not clear. In Chapter 8 the motion of this comet is mentioned. Also there is an Optional Activity which leads to an explanation of the motion observed in 1909-1910.

Q21 Tycho made instruments larger and stronger. Also he introduced the use of finer scales so that fractions of degrees could be determined more accurately.

Q22 Tycho corrected his observations for the effects of atmospheric refraction. He established the amount of the corrections by long series of observations of objects at various angular distances above the horizon.

Q23 The Tychonic system had some features of both the Ptolemaic and the Copernican systems. Tycho held the earth stationary, but had the planets revolving around the sun, which in turn revolved around the earth.

Q24 Whether the Copernican system in its entirety could be interpreted as a "real" system of planetary paths in space is doubtful. If the minor cycles required to account for small observed variations were neglected, the major motions might be considered to represent "real" orbits. Copernicus did not discuss this aspect of his system.

Chapter 7

Q1 Tycho became interested in Kepler through Kepler's book, in which he tried to explain the spacing of the planetary orbits by the use of geometrical solids.

Q2 After some 70 attempts with circles, eccentrics and equants, Kepler still had a difference of 8' between his best prediction and Tycho's observed positions for Mars. Kepler finally decided that no combination of circular motions would yield a solution. (He might have been wrong.)

Q3 Kepler used the observations made by Tycho Brahe. These were the most accurate astronomical observations made up to that time.

Q4 First Kepler had to refine the orbit of the earth. Then he could use the earth's position to triangulate positions of Mars in its orbit.

Q5 Kepler discovered that the planets moved in planes which passed through the sun. This eliminated the necessity to consider the north-south motions of each planet separately from its eastward motion.

Q6 Kepler's Law of Areas: the line from the sun to the moving planet sweeps over areas that are proportional to the time intervals.

Q7 Kepler noted the direction of Mars when it was in opposition. He knew that after 687 days Mars was at the same point in its orbit, but the earth was at a different place. By reversing the directions observed from the earth to Mars and to the sun, he could establish positions on the earth's orbit.

Q8 The component of a planet's velocity perpendicular to the line from the sun to the planet changes inversely with the distance of the planet from the sun.

Q9 When a circle is not viewed from directly above its center, it has an elliptical shape.

Q10 Of the naked-eye planets which Tycho could observe, Mars has an orbit with the largest eccentricity. If the orbit of Mars was considerably less eccentric, Kepler would not have found that its orbit was an ellipse. He could not have found that any of the other orbits were ellipses.

Q11 Kepler knew that after 687 days Mars had returned to the same point in its orbit. Observations from the earth at intervals of 687 days provided sight-lines which crossed at the position of Mars on its orbit.

Q12 Kepler's Law of Periods: the squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. If the distance or period of a planet is known, the other value can be computed.

Q13 Kepler compared the "celestial machine" to a gigantic clockwork.

Q14 Kepler's reference to a "clockwork" is significant because it suggests a "world machine." This idea was developed as a result of Newton's studies.

others too. Perhaps you will come to agree that "there is nothing more practical than a good theory."

theory, as well as its great usefulness.

A theory is a general statement relating selected aspects of many observations. A theory:

1. should summarize and not conflict with a body of tested observations. Examples: Tycho's dissatisfaction with the inaccuracy of the Ptolemaic system, Kepler's unwillingness to explain away the difference of eight minutes of arc between his predictions and Tycho's observations.

2. should permit predictions of new observations which can be made naturally or arranged in the laboratory. Examples: Aristarchus' prediction of an annual parallax of the stars, Galileo's predictions of projectile motions and Halley's application of Newton's theory to the motions of comets.

3. should be consistent with other theories. Example: Newton's unification of the earlier work of Galileo and of Kepler. However, sometimes new theories are in conflict with others, as for example, the geocentric and heliocentric theories of the planetary system.

Every theory involves assumptions. The most basic are those of Newton's Rules of Reasoning, which express a faith in the stability of things and events we observe. Without such an assumption the world is just a series of happenings which have no common elements. In such a world the gods and goddesses of Greek mythology, or fate, or luck prevent us from finding any regularity and bases for predictions of what will happen in similar circumstances.

The major points developed about the nature of a theory are too central to the purpose of the entire course for us to consider them self evident to the pupils solely on the basis of the text. They must be discussed in class and illustrated by specific examples.

This section will be better understood as the students learn more about physical theory (and so should be referred back to). If possible, extend the discussion of theory-making into other fields: biology, chemistry, archaeology, even economics and sociology.

Q15 According to the Ptolemaic theory Venus was always between the earth and the sun. Therefore, it should always show a crescent shape. However, Galileo observed that it showed all phases, like the moon. Therefore, to show full-phase Venus had to pass behind the sun. Such positions for Venus were consistent with either the heliocentric or the Tychonic system.

Q16 With the telescope Galileo discovered the phases of Venus—which were contrary to the Ptolemaic system. He also discovered the system of satellites around Jupiter—a miniature Copernican system, but NOT around the earth or the sun. Thus the earth was not the only possible center of motion, as the Ptolemaeans had contended. Galileo's telescopic observations of the moon, the sun, Saturn and the stars were interesting, but not critically related to the heliocentric model.

Q17 Galileo's observation of the satellites of Jupiter showed that there could be motions around centers other than the earth. This contradicted basic assumptions in the physics of Aristotle and the astronomy of Ptolemy. Galileo was encouraged to continue and sharpened his attacks on those earlier theories.

Q18 Kepler and Galileo emphasized the importance of observations as the raw material which must be explained by theories.

Q19 Galileo was said to be impious. Also he was sharp in his criticisms of others and often used ridicule. Officially he was tried for breaking his agreement not to support the Copernican system as really correct.

Q3 Newton said that he was "contemplative" when he saw the apple fall. He wondered about what he saw and began to seek for explanations.

Q4 You write your own answer to this one.

Q5 The Principia was originally written in Latin. Even the translations are difficult to read because Newton adopted a very mathematical style with complicated geometrical proofs.

Q6 a) Orbits are ellipses + Newtonian inertia ⟶ there is a net force acting.

b) Step (a) + Kepler's Law of Areas around sun + Newton's Law of Areas around center of force ⟶ sun must be at the center of force.

c) For elliptical orbits (or any conic section) around sun at one focus, $F \propto 1/R^2$.

d) $F \propto 1/R^2$ ⟶ ? Law of Periods. Yes, only this force law will satisfy Kepler's Law of Periods.

Q7 Kepler's Law of Areas is satisfied by any central force when the areas are measured from the center of force.

Q8 Only Kepler's Law of Periods provides information about the behavior of planets which have different mean distances from the sun. Their motions allow us to study how the gravitational force from the sun changes with distance.

Q9 The derivation assumed that: a) Galileo's law of acceleration applied to planetary motions as well as to falling stones; b) that the strength of the

a partial list of operations in theory making should help you examine other theories which appear in later parts of this course. Do not attempt to memorize this list.

Observations are focused only upon selected aspects of the phenomena. Our interest centers upon the general question: "In what way do the initial conditions result in this reaction?" For example, Galileo asked, "How can I describe the motion of a falling body?"

A theory relates many selected observations. A pile of observations is not a theory. Tycho's observations were not a theory, but were the raw material from which one or more theories could be made.

Theories often involve abstract concepts derived from observations. Velocity is difficult to observe directly, but can be found by comparing observations of time and position. Similarly, acceleration is difficult to measure directly, but can be found from other observed quantities. Such abstractions, sometimes called "constructs," are created by scientists as useful ideas which simplify and unite many observations.

Empirical laws organize many observations and reveal how changes in one quantity vary with changes in another. Examples are Kepler's three laws and Galileo's description of the acceleration of falling bodies.

Theories never fit exactly with observations. The factors in a theory are simplifications or idealizations; conversely, a theory may neglect many known and perhaps other unknown variables. Galileo's theory of projectile motion neglected air resistance.

rise with it—that is, we see only the change in the stone's position; also 2) the mass of the stone is so minute compared to the mass of the earth that the motion of the earth would be undetectable.

Q14 The Constant of Universal Gravitation, G, is a number by which we can equate observed accelerations (and forces) with the masses and distances supposedly responsible for those accelerations. So far as we can tell, the value of G does not change with position in the universe or with the passage of time.

Q15 A universal law, like any other scientific law, can be tested only in a limited number of cases. The more varied the cases are, the more confident we can be about the usefulness of the law.

Q16 A mass point is a concept by which we consider all the mass of a spherical homogeneous body to be concentrated at the center of the body.

Q17 Even though Newton did not know the value of G, he could form ratios of quantities with the result that the value of G would cancel between the numerator and the denominator. Thus Newton could obtain relative masses of the planets and the sun.

Q18 Newton concluded that the moon was accelerated toward the earth just as an apple was. He compared the observed and computed accelerations of the moon and found that "they agreed tolerably well."

Q19 Newton, like other scientists, could think up many possible explanations for what is observed. Numerical results indicate whether or not the ideas fit rea-

earth. It also depends upon elevation, or distance from the earth's center. Local variations in the materials of the rocks, the presence of nearby canyons or peaks and other variations also affect the value of g at a particular point. But if g is constant at a point, then G is also constant.

Q23 We consider the solid earth to be a mass point responding all over to the moon's attraction at the earth's center. Although the solid earth bends a bit under the differences in the moon's attractions on the near and far sides, this change is much less than the effects of the moon's pull on the fluid waters, which move readily under the moon's changing attraction.

Q24 On the side of the earth away from the moon, the attraction of the moon is less than it is upon the center of the solid earth. As a result, the earth is pulled away from the fluid water on the side away from the moon and there is a net force away from the moon.

Q25 Newton had explained the motions of the planets, which moved in nearly circular orbits. Halley then showed that the same force laws would explain the motions of comets, which previously had been considered as unexplainable. Some of the comets, moving in very elongated ellipses, even had periods not much greater than Saturn (30 years).

Q26 By forming ratios of two expressions like Eq. (8.10), Newton could compare the masses of the sun and each planet which had a satellite of known period. These planets were, in Newton's time, the earth, Jupiter and Saturn.

$$= \left[4.62 \times 10^{-5}\right]^2 \left[2.20 \times 10^4\right]^3$$

$$= 21.3 \times 10^{-10} \times 10.65 \times 10^{12}$$

$$= 22.700.$$

Since the mass of the sun is 332,000 times the mass of the earth,

$$\frac{m_{Uranus}}{m_{earth}} = \frac{332,000}{22,700} = 14.6.$$

Q28 The moon's orbit around the earth is somewhat eccentric (e = 0.055). During its cycles of the earth, the moon's distance from the sun also changes. As a result the accelerations on the moon and therefore the orbit of the moon change continuously. This is an example of the "three-body" problem for which no general solution has yet been developed.

Q29 Following Newton's work careful observations were needed to determine the magnitude of many small variations which his theory predicted. Also the general problem of the interactions between the planets had to be worked out.

Q30 Cavendish used a torsion balance to measure the gravitational attraction between metal spheres.

Q31 Geologists and astronomers were obliged to explain how the mean density of the earth could be 5.52, which is much greater than the density of surface rocks. Apparently the earth has a central core of high density.

Q32 The motions of double stars can be explained quantitatively by Newton's Theory of Gravitation. Apparently the gravitational force between such stars is identical to that between the sun and planets.

Q33 Newton is honored today for his development of the Theory of Universal Gravitation, for his development of the mathematics called the infinitesimal calculus, for his work in optics, for his creation of the f t reflecting telescope, and for his ervices to the Royal Society and to his national government.

Q34 After Newton the idea of a great World Machine was widel accepted for several hundred years. More emphasis was put upon precise observations to determine the rate at which various predicted changes occurred. How to account for gravitation and other "actions at a distance" became an important problem. Scientists felt confident that their observations were revealing a "real world" which could be explained in terms of mechanical systems.

Brief Answers to Study Guide

Chapter 5

5.1 Discussion

5.2 Discussion

5.3 Discussion

5.4 Discussion

Chapter 6

6.1 (a) (b) Discussion
- (c) 87.5 days
- (d) Discussion
- (e) 224 days

Chapter 7

7.1 89.5 years

7.2 (a) 18 A.U.
- (b) 1.8 A.U.
- (c) 0.053

7.3 4%

7.4 249 years

7.5 0.594

7.6 Discussion

Chapter 8

8.1 Upper focus

8.2 3×10^{-6}

8.3 An activity

8.4 No

8.5 (a) 2.8
- (b) 0.71
- (c) 0.25

8.6 (a) Shorter period
- (b) 9:1

8.7 (a) 0.15
- (b) 0.41
- (c) 1.00

8.8 (a) 1.58 m/sec^2
- (b) 114 N

8.9 (a) 2.33×10^{7} km
  7.55×10^{8} km
- (b) About the same
- (c) Discussion
- (d) No

8.10 (a) Larger one
- (b) 2:1

8.11 (a) Discussion
- (b) 6.7×10^{23} kg

8.12 (a) Discussion
- (b) 8.3×10^{1} N
  3.5×10^{17} N
- (c) Discussion

8.13 1.86×10^{27} kg

8.14 Derivation

8.15 (a) The same point
- (b) 1.41

8.16 (a) 1.93×10^{20} N
  3.52×10^{22} N
- (b) Discussion

8.17 (a) 4.2 m/sec^2

Picture Credits

Cover photograph: the orrery on the cover was made in 1830 by Newton of Chancery Lane, London. The earth and moon are geared; the rest of the planets have to be set by hand. It is from the Collection of Historical Scientific Instruments at Harvard University. Photograph by Albert Gregory, Jr.

Prologue

P. 0 Aztec Calendar Stone in the Museo National, Mexico City. Photo courtesy of the American Museum of Natural History, New York.

P. 1 Collection of Historical Scientific Instruments, Harvard University.

P. 2 (top) Stephen Perrin; (bottom) courtesy of the Trustees of the British Museum, London.

P. 4 Frontispiece from Recueil de plusieurs traitez de Mathématique de l'Academie Royale des Sciences, 1676.

Chapter 5

Fig. 5.2 Emil Schulthess, Black Star Publishing Company, Inc.

Fig. 5.3 John Stofan.

Fig. 5.4 John Bufkin, Macon, Missouri, Feb. 1964.

Pp. 9, 10 Mount Wilson and Palomar Observatories.

Fig. 5.9 DeGolyer Collection, University of Oklahoma Libraries.

Chapter 6

P. 26 Deutsches Museum, Munich.

Fig. 6.1 Woodcut by Sabinus Kauffmann, 1617. Bartynowski Collection, Cracow.

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Chapter 7

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P. 49 Kepler, Johannes, Mysterium cosmographicum, Linz, 1596.

P. 54 Archives, Academy of Sciences, Leningrad, U.S.S.R. Photo courtesy of Dr. Owen Gingerich.

Fig. 7.13 Instituto e Museo di Storia della Scienza, Florence, Italy.

Fig. 7.14 DeGolyer Collection, University of Oklahoma Libraries.

Fig. 7.17 Lowell Observatory Photograph.

P. 68 (telescope) Collection of Historical Scientific Instruments, Harvard University.

P. 71 Alinari--Art Reference Bureau.

P. 73 Bill Bridges.

Chapter 8

P. 74 Yerkes Observatory.

P. 77 (drawing) from a manuscript by Newton in the University Library, Cambridge; (portrait) engraved by Bt. Reading from a painting by Sir Peter Lely. Trinity College Library, Cambridge.

Figs. 8.15, 8.16 Courtesy of Sproul Observatory, Swarthmore College.

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P. 117 Henrard - Air-Photo.

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PLANETS AND THEIR SATELLITES

The Planets

	Name	Symbol	Mean Distance from Sun: Astron. Units	Mean Distance from Sun: Million Miles	Period of Revolution: Sidereal	Period of Revolution: Synodic	Eccentricity of Orbit	Inclination to Ecliptic
					days	days		
Inner	Mercury	☿	0.3871	35.96	87.969	115.88	0.206	7° 0′
	Venus	♀	0.7233	67.20	224.701	583.92	0.007	3 24
	Earth	⊕	1.0000	92.90	365.256	.....	0.017	0 0
	Mars	♂	1.5237	141.6	686.980	779.94	0.093	1 51
					years			
	Ceres	①	2.7673	257.1	4.604	466.60	0.077	10 37
Outer	Jupiter	♃	5.2028	483.3	11.862	398.88	0.048	1 18
	Saturn	♄	9.5388	886.2	29.458	378.09	0.056	2 29
	Uranus	♅	19.1820	1783	84.015	369.66	0.047	0 46
	Neptune	♆	30.0577	2794	164.788	367.49	0.009	1 46
	Pluto	♇	39.5177	3670	247.697	366.74	0.249	17 9

Name	Mean Diameter in Miles	Mass ⊕ = 1	Density Water = 1	Period of Rotation	Inclination of Equator to Orbit	Oblateness	Stellar Magnitude at Greatest Brilliancy
Sun ☉	864,000	331,950	1.41	$24^d.65$	7° 10′	0	−26.8
Moon ☾	2,160	0.012	3.33	27 .32	6 41	0	−12.6
Mercury	2,900	0.05	6.1	88	7?	0	−1.9
Venus	7,600	0.81	5.06	30?	23?	0	−4.4
Earth	7,913	1.00	5.52	$23^h\ 56^m$	23 27	1/296	......
Mars	4,200	0.11	4.12	24 37	24	1/192	−2.8
Jupiter	86,800	318.4	1.35	9 50	3 7	1/15	−2.5
Saturn	71,500	95.3	0.71	10 02	26 45	1/9.5	−0.4
Uranus	29,400	14.5	1.56	10 45	98	1/14	+5.7
Neptune	28,000	17.2	2.29	15 48?	29	1/40	+7.6

The Satellites

Name	Discovery	Mean Distance in Miles	Period of Revolution	Diameter in Miles	Stellar Magnitude at Mean Opposition
Moon		238,857	$27^d\ 7^h\ 43^m$	2160	−12
Satellites of Mars					
Phobos	Hall, 1877	5,800	0 7 39	10?	+12
Deimos	Hall, 1877	14,600	1 6 18	5?	13
Satellites of Jupiter					
Fifth	Barnard, 1892	113,000	0 11 53	150?	13
I Io	Galileo, 1610	262,000	1 18 28	2000	5
II Europa	Galileo, 1610	417,000	3 13 14	1800	6
III Ganymede	Galileo, 1610	666,000	7 3 43	3100	5
IV Callisto	Galileo, 1610	1,170,000	16 16 32	2800	6
Sixth	Perrine, 1904	7,120,000	250 14	100?	14
Seventh	Perrine, 1905	7,290,000	259 14	35?	17
Tenth	Nicholson, 1938	7,300,000	260 12	15?	19
Twelfth	Nicholson, 1951	13,000,000	625	14?	19
Eleventh	Nicholson, 1938	14,000,000	700	19?	18
Eighth	Melotte, 1908	14,600,000	739	35?	17
Ninth	Nicholson, 1914	14,700,000	758	17?	19
Satellites of Saturn					
Mimas	Herschel, 1789	115,000	0 22 37	300?	12
Enceladus	Herschel, 1789	148,000	1 8 53	350	12
Tethys	Cassini, 1684	183,000	1 21 18	500	11
Dione	Cassini, 1684	234,000	2 17 41	500	11
Rhea	Cassini, 1672	327,000	4 12 25	1000	10
Titan	Huygens, 1655	759,000	15 22 41	2850	8
Hyperion	Bond, 1848	920,000	21 6 38	300?	13
Iapetus	Cassini, 1671	2,210,000	79 7 56	800	11
Phoebe	Pickering, 1898	8,034,000	550	200?	14
Satellites of Uranus					
Miranda	Kuiper, 1948	81,000	1 9 56	.....	17
Ariel	Lassell, 1851	119,000	2 12 29	600?	15
Umbriel	Lassell, 1851	166,000	4 3 28	400?	15
Titania	Herschel, 1787	272,000	8 16 56	1000?	14
Oberon	Herschel, 1787	364,000	13 11 7	900?	14
Satellites of Neptune					
Triton	Lassell, 1846	220,000	5 21 3	2350	13
Nereid	Kuiper, 1949	3,440,000	359 10	200?	19

I am fully aware that names are one thing and science another.
FARADAY, nineteenth century

Glossary

The following is a list of words that appear in the text, but which may not be familiar to the average reader.

action at a distance. The process in which one body exerts a force on another without any direct or indirect physical contact between the two.

angular altitude. The angle measured in degrees that a star, planet, the sun or the moon appears above the horizontal.

angular motion. The orbital motion of a planet, satellite or star, measured in degrees per unit time.

angular size. The angle subtended by an object. For instance, the sun, the moon and your thumbnail at arm's length have about the same angular size.

aphelion. The point of an orbit that is furthest from the sun.

arc length. The distance along an arc or orbit.

astrology. The study of the supposed influences of the stars and planets on human affairs (e.g., horoscopes).

Astronomical Unit (A.U.). The average distance between the earth and the sun.

atmospheric refraction. The bending of light rays that occurs when light enters the atmosphere at an acute angle.

cause. An event or relationship that is always followed by another particular event or relationship; the absence of the first implies the absence of the second, all other conditions being equal.

celestial. Of the sky or the heavens.

celestial equator. The great circle formed by the intersection of the plane of the earth's equator with the celestial sphere.

celestial sphere. The imaginary spherical shell containing the stars and having the earth as a center.

center of motion (or force). The point toward which an orbiting body is accelerated by a central force; the point toward which a central force is everywhere directed.

central force. A force that is always directed towards a particular point, called the center of force.

centripetal acceleration. Acceleration of an orbiting body toward the center of force.

comet. A luminous celestial body of irregular shape having an elongated orbit about the sun.

conic section. Any figure that is the total intersection of a plane and a cone. The four types are the circle, ellipse, parabola and hyperbola.

Constant of Universal Gravitation (G). The constant of proportionality between the gravitational force between two masses, and the product of the masses divided by the square of the distance of separation. (See also gravitation.)

constellation. One of eighty-eight regions of the sky; the star pattern in such a region.

cyclic variation. A change or process of change that occurs in cycles, i.e., that repeats itself after a fixed period of time, such as the phases of the moon.

deferent. The circle along which the center of an epicycle moves.

density. Mass per unit volume.

double star. A pair of stars that are relatively close together and in orbit about each other.

eccentric. A circular orbit with the earth displaced from the center, but on which the orbiting body moves with constant velocity.

eclipse. The shadowing of one celestial body by another: the moon by the earth, the sun by the moon.

ecliptic. The yearly path of the sun against the background of stars. It forms a great circle on the celestial sphere.

ellipse. A closed plane curve generated by a point moving in such a way that the sums of its distances from two fixed points is a constant. The two points are called the foci of the ellipse. If the two points are the same, then the figure is a circle, and the foci are its center.

empirical. Based on observation or sense experience.

empirical law. A statement that concisely summarizes a set of experimental data or observations. Example: Kepler's Law of Areas.

epicycle. Part of the geometrical construction formerly used to describe the motions of celestial bodies; it is the small circle whose center moves along the deferent.

equant. A circle on which the orbiting body moves with constant angular velocity about a point different from the center of the circle, and such that the center of the circle is midway between that point and the earth.

equatorial bulge. The bulging of a planet or star at its equator, due to the flattening effect of the body's rotation.

equinox. A time when the sun's path crosses the plane of the earth's equator. The vernal equinox occurs on approximately March 21, the autumnal equinox on approximately September 22.

essences. The four basic elements thought by the Greeks to compose all materials found on the earth. They are: earth, water, air and fire.

experimental philosophy. The term formerly used for experimental science; the systematic study of the external world based on observation and experiment.

focus. (See ellipse.)

frame of reference. The coordinate system, or pattern of objects, used to describe a position or motion.

geocentric. Having the earth as a center.

geologist. One who studies the material composition of the earth and its changes, both past and present.

gravitation. The attractive force that every pair of masses exerts on each other. Its magnitude is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

gravity. The gravitational force exerted by the earth on terrestrial bodies; gravitation in general.

Halley's comet. A very bright comet, whose orbital period (about 75 years) was first discovered by the astronomer Edmund Halley.

heliocentric. Having the sun (helios) as a center.

hypothesis. An idea that is tentatively proposed as a basis for a theory; a statement that is not yet accepted as true, because either it has not been proven for enough cases, or it fails to explain all the observed phenomena.

interaction. Action on each other, such as by a mutual attractive force, or a collision.

inverse-square force. A central force whose magnitude at any point is inversely proportional to the square of the distance of that point from the force center.

law. A prediction that under specified circumstances, a certain event will always occur, or a certain relationship will be true.

light year. The distance traveled by a ray of light in one year (about 5.88×10^{12} mi.)

mass-point. The center of gravitational force of an object with mass. (See center of force.)

model. A combination of physical or mathematical devices that represents and suggests an explanation for the behavior of some actual physical system. Synonym: mechanism.

natural motion. The Aristotelian concept that objects have a motion that is a property of the object itself and therefore the natural way in which the object moves. For example, the Aristotelians thought that the sun's natural motion was to revolve daily about the earth, that the natural motion of fire was to rise, of earth, to fall.

natural place. The concept in Aristotelian science that every object has a particular location where it belongs, and that if an object is removed from this proper location, it will tend to move back there. Thus gravity was explained by saying that objects fall to the surface of the earth because this is their natural place.

Newtonian synthesis. Newton's system of mechanics, which proposed that the same laws govern both terrestrial and celestial motions.

North Celestial Pole. An imaginary point on the celestial sphere about which the stars appear (to an observer in the northern hemisphere) to rotate. This apparent rotation is due to the rotation of the earth.

occult quality. A physical property or cause that is hidden from view, mysterious or undiscoverable, but whose existence is assumed in order to explain certain observed effects.

opposition. The moment when a body is opposite the sun in the sky, i.e., when the earth is between that body and the sun, and lies on the line joining them.

orb. A sphere; usually refers to the imaginary crystalline spheres formerly thought to move the planets. Synonym: spherical shell.

orbit. The path of a celestial body that is revolving about some other body.

pantheism. The belief that God is the same as nature or the physical universe.

parallactic shift. The quantity of apparent change in position of an object due to a change in position of the observer. (See parallax.)

parallax. The apparent displacement of an object due to a change in position of the observer.

perihelion. The point of an orbit that is nearest the sun.

period. The time taken by a celestial object to go once completely around its orbit.

perspective geometry. The study of how three-dimensional objects in space appear when projected onto a plane.

phases of the moon. The variations of the observed shape of the sunlit portion of the moon during one complete revolution of the moon about the earth.

phenomenon. An observed, or observable event.

physical cause. An action of one body on another by direct or indirect physical contact.

Principle of Parsimony. Newton's first "Rule of Reasoning" for framing hypotheses, which asserts that nature is essentially simple in its processes, and therefore that the hypothesis that explains the facts in the simplest manner is the "truest" hypothesis.

Principles of Unity. Newton's second and third "Rules of Reasoning," which assert that similar effects have similar causes, and that if every experiment confirms a certain hypothesis, then it is reasonable to assume that the hypothesis is true universally.

quadrant. A device for measuring the angular separation between stars, as viewed from the earth.

qualitative. Pertaining to descriptive qualities of an object, texture, color, etc.

quantitative. Involving numbers, or properties and relationships that can be measured or defined numerically. Weight is a quantitative property of an object, but texture is not.

quintessence. The fifth basic element or "essence" in the Greek theory of materials, supposed to be the material of which all celestial objects are composed.

regular geometrical solid. A three-dimensional closed figure whose surface is made up of identical regular plane polygons.

retrograde motion. The apparent backward (i.e., westward) motion of a planet against the background of stars that occurs periodically.

satellite. A small body that revolves around a planet (such as the moon around the earth).

Scholastics. Persons who studied and taught a very formal system of knowledge in the Middle Ages, which based truth on authority (the teachings of Aristotle and the Church fathers), rather than on observation.

solar year. The interval of time between two successive passages of the sun through the vernal equinox.

statistics. The study dealing with the classification and interpretation of numerical data.

sun-spots. Relatively dark spots that appear periodically on the surface of the sun. They are presumably caused by unusually turbulent gases that erupt from the sun's interior, and cool rapidly as they reach the surface, which causes them to appear darker than the surrounding solar matter.

systematic error. Error introduced into a measurement (e.g., by inaccuracies in the construction or use of the measuring equipment), whose effect is to make the measured values consistently higher or lower than the actual value.

terrestrial. Pertaining to the earth, or earthly.

theory. A system of ideas that relates, or suggests a causal connection between certain phenomena in the external world. (See cause.)

torsion balance. An instrument which measures very small forces by determining the amount of twisting they cause in a slender wire.

Index

Project Physics **Teacher's Guide**

An Introduction to Physics **2 Motion in the Heavens**

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Table of Contents

Experiments

Equipment Notes

Additional Background Articles

Bibliography

Film Loop Notes

Suggested Answers to Tests

*recommended experiments

Overview of Unit 2

Unit 2 is a brief story of the physics that developed as men attempted to account for the motions of heavenly bodies. It is not a short course in astronomy.

The climax of the unit is the work of Newton. For the first time in history, scientific generalizations to explain earthly events were found to apply to events in the heavens as well. This remarkable synthesis, summarized in Chapter 8, produced echoes in philosophy, poetry, economics, religion and even politics.

The early chapters are necessary to establish the nature and magnitude of the problem that Newton solved. They also show that observational data are necessary to the growth of a theory. Thus Chapters 5, 6 and 7 are a prelude to Chapter 8 and constitute a case history in the development of science.

The prologue to Unit 2 gives a brief overview of the unit.

Chapter 5 constructs a model of the universe based upon the kinds of observations made by the ancients, the Greeks and incidentally by modern man. It describes the motions of sun, moon, planets and stars as seen from a fixed-earth frame of reference. It relates Plato's model and Ptolemy's geocentric universe.

Chapter 6 describes the work of Copernicus and Tycho Brahe, and discusses the arguments that developed and historic consequences of this radical view of the universe. The diligent observations and recording of these kept by Tycho shows the importance of such works to science.

The work of Tycho's successor, Johannes Kepler, and that of Galileo are related in Chapter 7, again with world-shaking consequences that changed the course of religion and philosophy as well as that of science.

Chapter 8 gives the student a profile of Newton, the man, and an insight into the tremendous power of the synthesis of earthly and celestial mechanics. The cement of his synthesis, the law of universal gravitation, is developed and several tests of the law are discussed.

Experiments

*E1 Naked-eye astronomy

E13 The size of the earth

*E14 The height of Piton, a mountain on the moon

*E15 The shape of the earth's orbit

E16 Using lenses, making a telescope, using a telescope

*E17 The orbit of Mars

E18 The inclination of Mar's orbit

E19 Mercury's orbit

*E20 Step-wise approximation to an orbit

E21 Model of a comet orbit

Demonstrations

D28 Phases of the moon

D29 Alternate demonstration for model of geocentric motions

D30 Alternate demonstration for model of heliocentric motions

D31 Plane motions

D32 Conic sections from model

*recommended experiments

Teaching Aid Perspective — Unit 2

TRANSPARENCIES	FILMS	LOOPS
T13 Stellar motion	F6 Universe—NASA (Prologue)	L10 Retrograde motion —geocentric model
T14 Celestial sphere	F7 Mystery of Stonehenge —McGraw Hill (prologue)	L10A Retrograde motion of planets
T15 Retrograde motion	F8 Frames of reference —PSSC	L11 Retrograde motion —heliocentric model
T16 Eccentrics and equants	F9 Planets in orbit—EBF	L12 Jupiter satellite orbit
T17 Orbit parameters	F10 Elliptic orbits —PSSC	L13 Program orbit I
T18 Motion under a central force	F11 Measuring large distances —PSSC	L14 Program orbit II
	F12 Of stars and men (about Galileo) —Columbia University Press	L15 Central forces —iterated blows
	F13 Tides of Fundy	L16 Kepler's laws
	F14 Harlow Shapley-EBF	L17 Perturbations
	F15 Universal gravitation—PSSC	
	F16 Forces—PSSC	
	F17 The invisible planet	
	F18 Close-up of Mars	

Reader Articles — Unit 2 Perspective

CHAPTER 5	CHAPTER 6	CHAPTER 7	CHAPTER 8	MISCELLANY
R2 Roll Call (Asimov) (5.1)	R16 The Great Comet of 1965 (Gingerich) (6.6)	R7 Kepler (Holton) (7.1)	R9 Newton and The Principia (Gillispie) (8.1)	R23 Expansion of the Universe (Bondi)
R11 The Garden of Epicurus (France) (5.5)	R3 A Night at the Observatory (Cooper) (6.7)	R8 Kepler on Mars (Kepler) (7.1)	R10 The Laws of Motion and Proposition One (Newton) (8.2)	R18 Space The Unconquerable (Clarke)
		R6 Kepler's Celestial Music (Cohen) (7.1-7.5)	R1 The Black Cloud (Hoyle) (8.3)	R24 Negative Mass (Hoffmann)
		R5 The Starry Messenger (Galileo) (7.8)	R17 Gravity Experiments (Dicke et al) (8.6)	R21 U.F.O (Sagan,
			R20 The Stars Within Twenty-Two Light Years That Could Have Habitable Planets (Dole) (8.8)	R25 Three Poetic Fragments about Astronomy (Shakespeare, Butler and Ciardi)
			R14 A Search for Life on Earth at Kilometer Resolution (Kilston et al) (8.8)	R26 The Dyson Sphere (Shklovskii and Sagan)
			R12 Universal Gravitation (Feynman) (8.9-8.16)	R19 Is there Intelligent Life Beyond the Earth? (Shklovskii and Sagan)
			R15 The Boy Who Redeemed His Father's Name (Morris) (8.12)	R4 Preface to <u>De Revolutionibus</u> (Copernicus)
			R22 The Life-Story of a Galaxy (Burbidge) (8.16)	
			R13 An Appreciation of the Earth (Dole) (8.17)	

MULTI-MEDIA DAILY PLAN UNIT 2

1 Read The Black Cloud, Hoyle. Small-group discussion on article.	2 Exchange and plot data from E1, Naked-Eye Astronomy	3 TEACHER PRESENTATION Plato's and Aristotle's Views	4 LAB STATIONS Ptolemy and Copernicus
1a Text: Prologue to Unit 2; Prepare for debate	2a Text 5.1 through 5.4	3a Text: 5.5 and 5.6	4a Text. 5.7 Prepare for debate
5 CLASS DISCUSSION Ptolemy and Copernicus	6 STUDENT DEBATE	7 DEMONSTRATION Experiment E15, The Shape of the Earth's Orbit	8 SMALL-GROUP PROBLEM SOLVING
5a Reader: The Great Comet, Gingerich. Prepare for debate	6a Text: 6.1 through 6.3	7a Text: 6.4 and 6.5 Finish orbit plot	8a Text: 6.6 through 6.8
9 LAB STATIONS Brahe and Kepler	10 Experiment E17, Orbit of Mars	11 TEACHER PRESENTATION Brahe and Kepler	12 CLASS DISCUSSION Models and Frames of Reference
9a Reader: Starry Messenger, Galileo	10a Text: 7.1 through 7.4 Finish orbit plot	11a Text: 7.5 through 7.8	12a Text: 7.9 through 7.11
13 LAB STATIONS Galileo	14 FILM Harlow Shapley Small-group discussion on films	15 TEACHER PRESENTATION The Newtonian Synthesis	16 DEMONSTRATIONS Central Forces
13a Text: 8.1 through 8.3	14a Text: 8.4 through 8.9	15a Text: 8.10 through 8.15	16a Text: 8.16 through 8.18
17 LAB STATIONS Inverse-Square Law	18 DEMONSTRATION Experiments from day 17	19 Film PSSC #0309 Universal Gravitation Discuss Film	20 Student option
17a Finish reading Chapter 8	18a Reader: Life Story of a Galaxy, Burbridge	19a Prepare for day 20	20a
21 Option	22 Unit 2 Review	23 Unit 2 Exam	24 Discuss Unit 2 Exam Begin Unit 3
21a Text: Epilogue to Unit 2			

Details of the Multi-Media Schedule

Day 1

Read The Black Cloud, Hoyle

Some possible questions for small-group discussion:

Is it possible?
How big is it?
Where is it located?
What would Aristotle say?

Make assignments for student debates, day 6. You will need debaters, time keepers, judges, etc. This is a good chance to work with the English department.

Day 2

Small groups plot data from E1 and discuss questions from experiment. If data was not available, use data provided in the Teacher Guide.

An evening visit to a planetarium would be useful. An evening star-gazing session might be an alternate activity.

Day 3

Teacher presentation on Aristotle and Plato. The scientific and philosophical viewpoints of the ancients are presented and discussed.

Day 4

Lab Stations: Ptolemy and Copernicus

1. epicycle machine (see student activity)
2. film loop L10A
3. D28 phases of the moon
4. D29 geocentric-epicycle model
5. celestial-sphere model (student activity)
6. T14 celestial sphere

Day 5

Class discussion: Ptolemy and Copernicus

Answer questions that will arise with regard to geocentric celestial mechanics. T13 and T16 should be helpful.

Day 6

Student debate. The nature of the universe as described by Ptolemy and Copernicus. Students present both viewpoints in standard debate form.

Day 7

Students collectively do Experiment 15, The Shape of the Earth's Orbit. Several students read measurements of solar diameters from projected photographs. Every student makes an orbit plot. Large sheets of graph paper are very helpful for this experiment.

Day 8

Small-group problem solving. See p. 14, Student Handbook, for some possibilities. Alternate: devise "computer-based" problem.

Day 9

Lab Stations: Brahe and Kepler

1. drawing ellipses (see Student Handbook)
2. demonstrating satellite orbits (see Student Handbook)
3. T17 Orbit Parameters
4. film loop L11 Retrograde Motion, Heliocentric Model
5. conic sections model (see Student Handbook)
6. T18 Central Forces

Suggestion: students might enjoy taking star-trail photographs as activity.

Day 10

Experiment 17: The Orbit of Mars.

Students plot Mars' orbit on graph made on day 7.

Day 11

Teacher presentation: Brahe and Kepler

Points to make:

1. the role of instruments and measurements
2. looking for relationships in data
3. kinds of people in science (e.g., Brahe and Kepler)

See Kemble, Physical Science, Its Structure and Development, Chaps.1-5, 19. Also Koestler, The Watershed.

Day 12

Class discussion

Possible discussion topics:

1. models of the universe (Aristotle to Kepler)
2. the changing nature of physical laws
3. separation of celestial physics and terrestrial physics in history

Day 13

Lab Stations: Galileo

1. height of a mountain on the moon (E14)
2. making a telescope (Part of E16)
3. L12 Jupiter's Satellite Orbits

Since these stations require quantitative observations, more time should be given for each activity.

Optional: Check out telescopes for student use. Perhaps an evening star-gazing session on planets and the moon would be appropriate here.

Day 14

Film: (Harlow Shapley)

Encyclopedia Brittanica Film #1806 (30 min)

This film discusses major astronomical discoveries and how they have influenced philosophy, religion, and an orientation to the world. Follow with small-group discussion on film.

Day 15

Teacher presentation: The Newtonian Synthesis

Here Kepler's laws, Galileo's observations, and terrestrial physics are combined into one grand law. See Holton and Roller, Foundations of Modern Physical Science, Chaps. 11 and 12; Kemble, Physical Science, Its Structure and Development, Chap. 9; and Andrade, Sir Isaac Newton.

Day 16

Demonstrations: Central Forces

Begin with T18. E20 may be done as demonstration experiment. Loops 13 and 14 are other possibilities.

Day 17

Lab Stations: Inverse-Square Law

Students work at one station only, take data, plot, and compare.

1. Photometry. With a light meter measure intensity at various distances from a small source.
2. Radioactivity. Measure intensity of radiation at various distances.
3. Sound. Microphone and amplifier drive a "Vu" meter. Measure intensities at various distances.
4. Magnetism. See activity in Unit 4 Student Handbook, "Force law for bar magnets."
5. Electric Fields. See E34 or equivalent.

Day 18

Students demonstrate experiments (day 17) and show results to rest of class.

Day 19

Film PSSC #0309, "Universal Gravitation"

A tongue-in-cheek Hume and Ivey film; requires careful post-film discussion.

Day 20

Student option

In small groups or individually, students plan their own activity for this day. Possibilities include:

E13, Size of the Earth
E19, Mercury's Orbit
E21, Model of Comet Orbit
Activities from Handbook
Reader articles
Periodicals or other outside reading

Day 21

Option

Some possibilities:

1. class reading of play, "Lamp at Midnight" from the Hallmark Hall of Fame TV series, available in paperback
2. a field trip to planetarium
3. National Gallery of Art slide set and record, "Physics and Art"
4. write Haiku about Unit 2 topics

Day 22

Review Unit 2 in preparation for exam

Day 23

Unit 2 exam

Day 24

Discuss exam

Chapter 5 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period.

Read 5.0 – 5.3

Discuss naked-eye astronomy
Film strip on Mars "Retrograde"
Show film loop 10 A
Demonstrate lunar phases

Read Experiment 14

Discuss indirect measurement, exp. procedure,
Height of Piton exp.

Analyze Lab

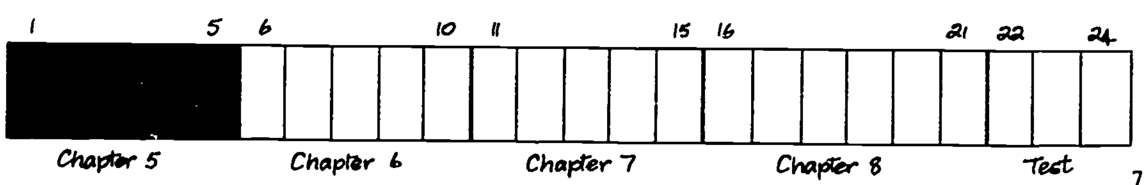
Discuss exp. procedure and results
Importance of indirect measurement
Problem Seminar

Read 5.4 – 5.7

Discuss details of Ptolemaic system
Eccentrics and equants transparency
Retrograde motion loop 10

Review Chapter 5

Chapter quiz



Chapter 5 Resource Chart

Text	Schedule	Problems	Experiments and Teacher Demonstration
5.1 Motions of the sun and stars		2	E1 Naked-eye astronomy (continu...) E13 The size of the earth
5.2 Motions of the moon			E14 The height of Piton, a mountain on the moon D28 Phases of the moon
5.3 The wandering stars		1	
5.4 Plato's problem		3	
5.5 The first earth-centered solution			D29 Alternate demonstration for model of geocentric motions
5.6 A sun-centered solution			D30 Alternate demonstration for model of heliocentric motions.
5.7 The geocentric system of Ptolemy		4	

Chapter 5 Resource Chart

T13 Stellar Motion R2 Roll Call T14 Celestial Sphere F6 Universe – NASA (Prologue) F7 Mystery of Stonehenge – McGraw-Hill (Prologue)	How long is a sidereal day? Global sundial
Film strip of Mars at three oppositions L10A Retrograde motions of planets	Making angular measurements Scale model of the solar system
T13 Stellar motion T14 Celestial sphere R12 The Garden of Epicurus T15 Retrograde motion	Celestial - sphere model Armillary sphere
T16 Eccentrics and equants L10 Retrograde motion	Epicycles and retrograde motion

Experiment 1*: Naked Eye Astronomy

In this experiment, students are asked to make quantitative sky observations over a period of several weeks. Data is then plotted on graphs, and graphs are interpreted. Sample data is provided in case of bad weather. This experiment should be started during the first few days of school year.

<u>Equipment</u>

Astrolabe

Constellation Chart

Experiment 13: The Size of the Earth

Two students, at least 200 miles apart, take simultaneous star sightings. Data is exchanged, and the size of the earth may be calculated. See list of consultants for possible cooperating teachers.

<u>Equipment</u>

Astrolabe

Experiment 14: The Height of Piton

A combination of measurements made from a moon photograph and assumptions made from a geometric model allow indirect measurement of the height of a mountain on the moon.

<u>Equipment</u>

Ruler

Student Handbook

10x magnifier

Chapter 6 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period.

Read Experiment E15

Discuss Exp. procedure using film strip, The shape of the earth's orbit E15

Read 6.1 – 6.5

Discuss the frames of reference, heliocentric universe and its simplicity Show retrograde motion L 11

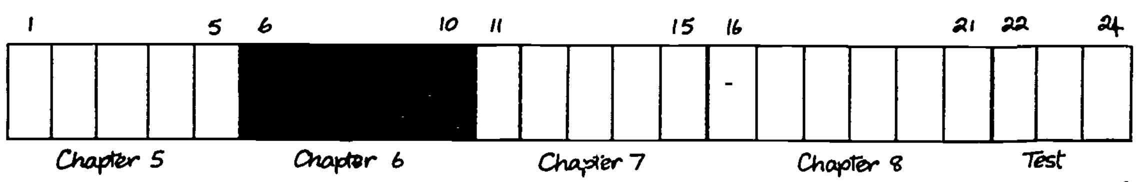
Problem Seminar

Read 6.6 – 6.8

Discuss importance of accurate measurement Frames of reference Brahe's system

Review Chapter 6

Chapter quiz



Chapter 6 Resource Chart

Text	Schedule	Problems	Experiments and Teacher Demonstration
6.1 The Copernican system			E15 The shape of the earth's orbit
6.2 New conclusions		1	
6.3 Arguments for the Copernican system			
6.4 Arguments against the Copernican system			
6.5 Historical consequences			
6.6 Tycho Brahe			
6.7 Tycho's observations			E16 Using lenses, making a telescope, using a telescope
6.8 Tycho's compromise system			

Chapter 6 Resource Chart

L 11	Retrograde motion-heliocentric model	Planetary positions relative to the sun Retrograde motion Proof that the earth rotates
F 8	Frames of reference — PSSC	
F9	Planets in orbit - EBF	
R17	The Great Comet of 1965	
R3	A Night at the Observatory	Telescope or binocular observations Observing sunspots

Experiment 15*: The Shape of the Earth's Orbit

The student uses a series of photographs of the sun to establish relative earth-sun distances on twelve dates throughout the year. Data is plotted as geocentric and heliocentric systems. Heliocentric graph is used as a starting point for E17, The Orbit of Mars.

Equipment

35 mm film strip projector

(screen)

Meterstick

Graph paper—20 × 20 size desirable or smaller sheets

Film strip of sun photos

Ruler

Compass

Protractor

Experiment 16: Using Lenses, Making A Telescope, Using a Telescope

By "playing" with lenses, students realize that convex lenses can be used to a) magnify, and b) form real images. By combining lenses in an appropriate way, a telescope can be formed.

This experiment is not intended to teach the details of geometrical optics.

Equipment

1 large lens

1 10x magnifier

1 small lens mounted in wooden cylinder — optics kit

2 cardboard tubes, telescoping

1 plastic cap for mounting large lens

Optional

Wooden saddle

Rubber bands

Camera tripod

Chapter 7 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period.

Read 7.1 – 7.5

Discuss and use Kepler's laws
Show Orbit parameters trans.

Read Experiment E17

Discuss triangulation

Orbit of Mars Experiment

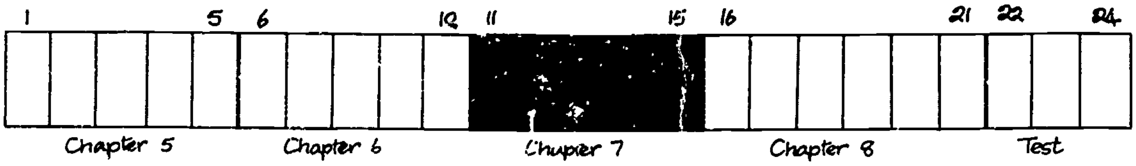
Discuss parameters of Mars' orbit
Problem Seminar

Read 7.6 – 7.11

Discuss Galileo's observations and their implications

Review Chapter 7

Chapter quiz



Chapter 7 Resource Chart

Text	Schedule	Problems	Experiments and Teacher Demonstration
7.1 The abandonment of uniform circular motion			
7.2 Kepler's second law			D31 Plane motions
7.3 Kepler's law of elliptical orbits			E17 The orbit of Mars E18 The inclination of Mars' orbit E19 Mercury's orbit D32 Conic sections from model
7.4 Using the first two laws		3 2 5	
7.5 Kepler's law of periods		1 4	
7.6 The new concept of physical law			
7.7 Galileo's viewpoint			
7.8 The telescopic evidence			
7.9 Galileo's arguments			
7.10 The opposition to Galileo			
7.11 Science and freedom			

Chapter 8 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period.

Read 8.0 – 8.5

Discuss motion under central force using trans. Show central forces loop as exp. preview

Read Experiment E20

Discuss exp. procedure Step-wise approx. to an orbit experiment E20

Analyze Lab

Show program orbit I & II loop Problem Seminar L13 and L14

Read 8.6 – 8.12

Discuss Kepler's laws and universal gravitation, theories in general

Read 8.13 – 8.18

Show and discuss Jupiter satellite orbit loop L12

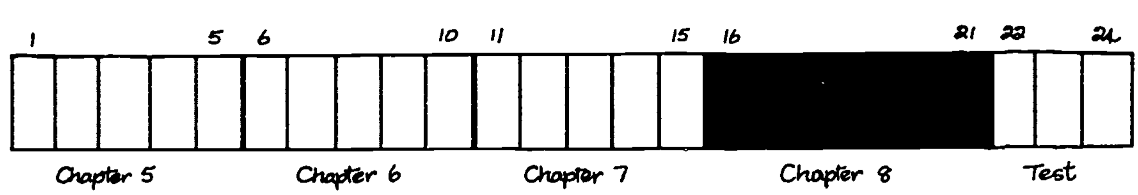
Read 8.19 and Review

Chapter quiz

Unit review

Unit test

Analysis



Chapter 8 Resource Chart

Text		Schedule	Problems			Experiments and Teacher Demonstration
8.1	A sketch of Newton's life					
8.2	Newton's Principia					
8.3	A preview of Newton's analysis					
8.4	Motion under a central force		1			E20 Stepwise approximation to an orbit
8.5	The inverse-square law of planetary force			4		
8.6	Law of universal gravitation					
8.7	The magnitude of planetary force			2	9 15	
8.8	Testing the general law		3	7 10		
8.9	The moon and universal gravitation			17	5	
8.10	Gravitation and planetary force		14	8		
8.11	The tides				16	
8.12	The comets					E21 Model of a comet orbit
8.13	Relative masses of the planets compared to the sun			13	6	
8.14	The scope of the principle of universal gravitation					
8.15	The actual masses of celestial bodies			12	11	
8.16	Beyond the solar system					
8.17	Some influences on Newton's work					
8.18	Newton's place in modern science					
8.19	What is a theory?					

Chapter 8 Resource Chart

R10	Newton and the Principia	
R11	The Laws of Motion and Proposition One	
R1	The Black Cloud	
T18 L14	Motion under central force L13 Program orbit I (Exp. 20) Program orbit II (Exp. 20) Film: Newton's Equal Areas	Three-dimensional model of two orbits
L15 L16	Central forces – iterated blows Kepler's laws	Demonstrating conic sections Graphical construction of conic sections A conic-sections model
R18	Gravity experiments	Demonstrating vortexes
R21	The Stars within 22 Light-Years That Could Have Habitable Planets	
R15	A Search for Life on Earth at Kilometer Resolution	
R13	Universal Gravitation	
F13	Tides of Fundy	
R16	The Boy Who Redeemed His Father's Name	Other comet orbits
L12 F14	Jupiter satellite orbit Harlow Shapley – EBF	
F15	Universal Gravitation – PSSC	
F16	Forces – PSSC	
R23	The Life Story of a Galaxy	
R14	An Appreciation of the Earth	
L17	Perturbations	
		Haiku – end

Experiment 20*: Stepwise Approximation to an Orbit

A geometrical technique is given that allows the student to plot the orbit of a short-period comet. The experience dramatizes the directional nature of a central force and the variation of acceleration due to an inverse-square force law. Other aspects of orbital motion can be investigated using Kepler's laws.

Equipment

20 × 20 graph paper

Straight edge

45° or 60° triangle

Dividers or compass

Experiment 21: Model of a Comet Orbit

From the six elements that define the orbit of Halley's comet, students can build a three-dimensional model which shows how the comet's orbit is related to the earth's. They can also develop a timetable (Ephemeris) for the comet's motion and so interpret the observations made of the comet in 1910.

Equipment

Cardboard or stiff paper, two sheets

Ruler

Protractor

Data for Halley's comet (Student Handbook)

Compass

Scissors

Solutions for Chapters 5 and 6 Study Guide

5.1

a) Observations	b) Reason for Importance
Apparent motion of the sun:	
1) daily westward motion	day-night determination
2) annual north-south motions	seasonal changes
3) annual eastward motion through the stars	length of the year, basis of the calendar
Apparent motions of the moon:	
1) phase changes	tie-in with other physical phenomena
2) continuous motion eastward among the stars	basis of the month, related to eclipses of sun and of moon
Apparent planetary motions:	
1) retrograde motions westward at opposition (Mars, Jupiter, Saturn)	a seemingly contrary motion that should be explained THE REAL BUGABOO!
2) periodic motions of Venus and Mercury near the sun	planets are different in some ways
Apparent fixed positions of the stars:	
1) continuous circumpolar rotation of the celestial sphere	the most uniform of all observed motions

5.2

Regardless of the earth's shape, the apparent motions of the stars would seem the same.

a) Saucer-shaped:
In the morning the sun would be seen earlier in western lands than in eastern lands. At sunset the sun would be seen later by observers in the eastern areas.

b) Sunrise would occur at the same moment to everyone.

c) Like a flat earth, only abrupt changes at the edges.

d) No variation of the sun's noon altitude with north-south position of the observer.

5.3

a) The time interval between recurring heavenly events, such as eclipses of the sun, is very often large. Intervals are such that periodic phenomena are not often seen as periodic unless records cover hundreds of years.

b) In terrestrial studies, lost experimental data can be recreated by repeating experiments. This is not possible in astronomy.

5.4

a) Each theory described a model through which certain phenomena were described adequately, e.g., retrograde motions.

b) The geocentric, earth-centered idea was consistent with theology, philosophy and common sense. The heliocentric put the sun, the light-giver, at the center of the universe.

c) Ptolemy used geometrical motions to improve the precision of predicting planetary positions.

6.1

a) When Mercury and Venus are moving from farthest east to farthest west relative to the sun, they are overtaking the earth and passing between the earth and sun. In the case of Venus only about one-quarter of its orbital period is required for the planet to move from farthest east to farthest west.

b) To find a period for Mercury's motion relative to the sun we have a choice of which intervals to measure. Probably the most significant would be the times required for Mercury to pass the sun. The three intervals for motion from west to east are: 110, 105 and 130 days. For motion from east to west the intervals are: 127 and 112 days. The average of the five intervals is 115 days. The variations result from the eccentric orbit of Mercury.

c) The mean cycle compared to the sun is 115 days, or 0.315 years. This is T in the equation of the chase problem. N is one. Then

$$\text{period of Mercury} = T/(T+N), \text{ or } = 0.315/1.315 = 0.240 \text{ y, or } 87.5 \text{ days.}$$

d) The major sources of uncertainty are:

1) Only three cycles for Mercury are shown. With more cycles a better average would result.

2) The orbit of Mercury is not circular, but is rather eccentric. Therefore the observed intervals depend upon the direction from which we on the moving earth see Mercury in its orbit.

e) Only a little more than half a cycle is shown for Venus. We assume that the motion is symmetrical. Since a half cycle takes 289 days, a full cycle would be 578 days. In one year we observe 365/578 of a cycle; this is N = 0.632. T is one year. Then

period of Venus = 1/(1+0.632)
= 0.613 y
= 224 days.

Solutions for Chapter 7 Study Guide

7.1

$T^2 = ka^3$ where $k = 1\ yr^2/A.U.^3$

$T^2 = 20^3 = 8000.$

$T = \sqrt{8000} = 89.5$ years.

7.2

a) $a^3 = 75^2 = 5625$

$a = \sqrt[3]{5625} = 17.8$ A.U.

b) $e = c/a$ where c = mean distance - perihelion distance.

$c = ae = 17.8 \times 0.90 = 16$ A.U.

least distance = $a - c = 17.8 - 16 = 1.8$ A.U.

c) From law of areas, velocities are inversely proportional to distances.

$v_{aphelion}/v_{perihelion}$

$= (a-c)/(a+c) = 1.8/(17.8 + 16)$

$= 1.8/33.8 = 0.053.$

7.3

1.02 - 0.98 = 0.04 or 4 percent change, since speeds depend upon distances.

7.4

$T = (39.6)^3 = 249$ years.

7.5

$c = ae = 0.254\ a$

ratio of speeds = inverse ratio of distances

$= (a-c)/(a+c) = (a-0.254a)(a+0.254a)$

$= 0.746a/1.254a = .549$

7.6

Discussion.

Solutions for Chapter 8 Study Guide

8.1

The upper focus is the sun because the nearer the sun the greater the speed. The vector difference $\vec{B} - \vec{A}$ over a relatively small time interval is proportional to and in the direction of the accelerating force. It is therefore directed at the sun.

8.2

(a) The forces are equal in magnitude and opposite in direction.

(b) $a = \frac{F}{m}$

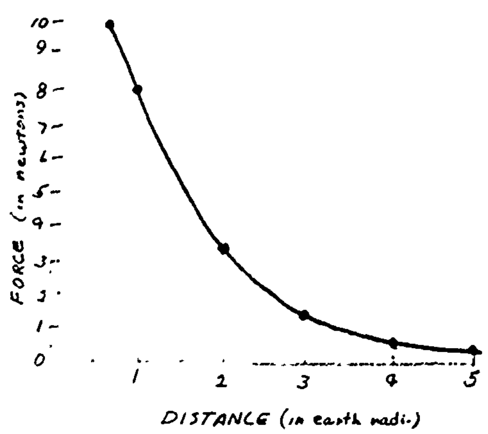
$$\frac{a_s}{a_e} = \frac{F_s/m_s}{F_e/m_e}$$

Since $F_s = F_e$ and $m_s = 3.3 \times 10^5\ m_e$

$$\frac{a_s}{a_e} = \frac{m_e}{3.3 \times 10^5\ m_e} \text{ or}$$

$$\frac{a_s}{a_e} = 3 \times 10^{-6}$$

8.3



The weight at 100R will be small but not zero. Likewise the weight at 1000R will be much smaller, but still not zero.

8.4

We assume that A and B are relatively small bodies revolving about the massive sun. The period of body A, whose orbital radius is $2R_B$ could be calculated:

$$\frac{T_A^{\,2}}{T_B^{\,2}} = \frac{R_A^{\,3}}{R_B^{\,3}}$$

$$T_A^{\,2} = T_B^{\,2}\frac{R_A^{\,3}}{R_B^{\,3}} = T_B^{\,2}\ (2)^3$$

$$T_A = T_B\ (8)^{\frac{1}{2}}$$

Then, $\underline{T_A = 2.82\ T_B}$.

This is considerably more than twice the period of body B. Therefore they are not revolving about the sun.

8.5

(a) $T^2 = kR^3$ or $\frac{T_A^{\,2}}{T_B^{\,2}} = \frac{R_A^{\,3}}{R_B^{\,3}}$

$$\frac{T_A}{T_B} = \left(\frac{R_A^{\,3}}{R_B^{\,3}}\right)^{\frac{1}{2}} = (8)^{\frac{1}{2}}$$

$$\underline{\frac{T_A}{T_B} = 2.8}$$

(b) If $R_A = 2R_B$, then the ratio of the circumferences is

$$\frac{C_A}{C_B} = \frac{2\pi(2R_B)}{2\pi(R_B)} = 2.$$

If it takes planet A 2.8 times as long to complete its orbit, the ratio of their speeds is

$$\frac{v_A}{v_B} = \frac{\frac{C_A}{2.8T_B}}{\frac{C_B}{T_B}} = \frac{C_A}{2.8C_B} = \frac{2}{2.8} = \underline{0.71}$$

(c) Their accelerations are proportional to the gravitational forces which are inversely proportional to R^2 $(F \propto \frac{1}{R^2})$.

$$\frac{a_A}{a_B} = \frac{F_A}{F_B} = \frac{R_B^{\,2}}{R_A^{\,2}} = \frac{R_B^{\,2}}{(2R_B)^2} = \tfrac{1}{4} = \underline{0.25}$$

It is interesting to compare this result with that obtained from using the kinematic value of centripetal acceleration, $A = v^2/R$.

$$\frac{a_A}{a_B} = \frac{v_A^{\,2}R_B}{R_A v_B^{\,2}} = \frac{v_A^{\,2}}{v_B^{\,2}} \times \frac{R_B}{R_A} = (0.71)^2(\tfrac{1}{2})$$

$$\underline{\frac{a_A}{a_B} = 0.25}$$

(a) If the masses of the two balls are doubled, the point in the center remains the point at which the gravitational forces balance.

(b) Let $M_1 = 2M_2$ and m be a mass at the point where the forces balance.

Then,
$$\frac{GM_1m}{R_1^2} = \frac{GM_2m}{R_2^2}$$

$$\frac{R_1^2}{R_2^2} = \frac{GM_1}{GM_2}$$

$$\frac{R_1}{R_2} = \left(\frac{M_1}{M_2}\right)^{\frac{1}{2}} = \left(\frac{2}{1}\right)^{\frac{1}{2}} = 1.41$$

The point is 1.41 times as far from M_1 as from M_2.

8.16

(a) $$F_m = \frac{GM_mM_e}{R_m^2}$$

$G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$

$M_m = 7.18 \times 10^{22} kg$

$M_e = 5.96 \times 10^{24} kg$

$R_m = 3.84 \times 10^8 m$

$$F_m = 6.67 \times 10^{-11} N \cdot m^2/kg^2 \times 7.18 \times \frac{10^{22} kg \times 5.96 \times 10^{24} kg}{(3.84 \times 10^8 m)^2}$$

$$F_m = 1.93 \times 10^{20} N$$

$$F_s = \frac{GM_sM_e}{R_s^2}$$

$M_s = 1.99 \times 10^{30} kg$

$R_s = 1.50 \times 10^{11} m$

$$F_s = 6.67 \times 10^{-11} N \cdot m^2/kg^2 \times 1.99 \times \frac{10^{30} kg \times 5.96 \times 10^{24} kg}{(1.50 \times 10^{11} m)^2}$$

$$F_s = 3.52 \times 10^{22} N$$

Thus the gravitational force of the sun on the earth is 182 times the force of the moon on the earth!

(b) The moon has the greatest effect on the tides. The difference in the force of the moon on the near and far sides of the earth is greater beacuse the moon is much nearer the earth than is the sun.

This difference can be determined in the following way. The tide-raising force of the sun or moon results from the difference between the forces exerted on the earth as a whole (mass-point) and on the fluid waters one earth-radius nearer or farther from the center of the earth.

By doing the analysis for such a difference in algebraic terms, we can avoid much arithmetic. The force, F, on the earth varies as $1/R^2$, where R is the distance between the center of the earth and the center of the other body (sun or moon). The force at a slightly different distance, $(R + \Delta R)$, is $(F - \Delta F)$ (if the distance increases, the force decreases), which is

$$(F - \Delta F) \sim \frac{1}{(R + \Delta R)^2}, \text{ or}$$

$$\sim \frac{1}{(R^2 + 2\Delta R + \Delta R^2)}$$

If ΔR is small compared to R, then ΔR^2 is negligible.

Then $$\Delta F \sim \frac{1}{(R^2 + 2R\Delta R)} + F$$

$$\sim \frac{1}{(R^2 + 2R\,\Delta R)} + \frac{1}{R^2}.$$

By cross multiplication this becomes

$$\sim \frac{R^2 + R^2 + 2R\,\Delta R}{R^2(R^2 + 2R\,\Delta R)}, \text{ or}$$

$$\sim \frac{2\Delta R}{R^2(R + 2\Delta R)}.$$

But F is $\sim \frac{1}{R^2}$, so we may write

$$\Delta F \sim \frac{2\Delta R}{(R + 2\Delta R)} F.$$

The square brackets contain the term which gives the change in force F for a small change in distance, ΔR. Since the force F has been determined in part a) above, the values of F can be found for points one earth radius nearer, or farther, from the attracting body, sun or moon.

For the moon, ΔR is $\frac{1}{60}$ of R, or 0.0167R.

For the sun, ΔR is $\frac{6,000 \text{ km}}{150,000,000 \text{ km}} =$

$4.0 \times 10^{-5} R$.

Then, for the moon,

$$F_M = \frac{2 \times 0.0167}{(1 + 0.0167)} \times 1.93 \times 10^{20} N =$$

$$6.35 \times 10^{18} N$$

while for the sun,

$$F_S = \frac{2 \times 4.0 \times 10^{-5}}{(1 + 0.00004)} \times 3.52 \times 10^{22} N =$$

$$2.82 \times 10^{18} N$$

We see then that the differential tide-raising force of the moon is more than twice that of the sun.

8.17

$$a_c = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2},$$

where R is distance of satellite from center of earth, namely radius of earth plus height of satellite—$(6.37 + 3.43) \times 10^6\text{m} = 9.80 \times 10^6\text{m}$.

$$T = 161.5 \times 60 = 9690 \text{ sec.}$$

$$a_c = \frac{4\pi^2(9.8 \times 10^6)}{(9690)^2} = 4.2 \text{ m/sec}^2$$

For a circular orbit this value should equal the acceleration due to the earth's attraction at the satellite's distance—namely, $\frac{GM_e}{R^2} =$

$$\frac{6.67 \times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2 \times 5.96 \times 10^{24}\text{kg}}{(9.8 \times 10^6)^2}$$

$$= 4.17 \text{ m/sec}^2.$$

The speed of a planet in an elliptical orbit

L12 Jupiter satellite orbit

R4 The starry messenger

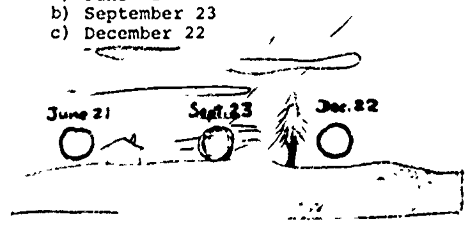
F12 Of stars and men (about Galileo) - Columbia University Press. Galileo

R5 Galileo

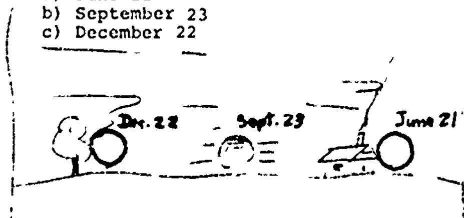
Study Guide
Review Problems

Answers to REVIEW PROBLEMS
from Student Handbook, p.14.

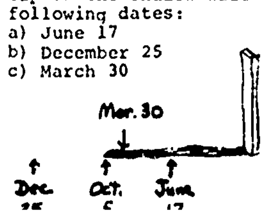
1. The sketch below illustrates sunrise on March 21.
Sketch, approximately, the positions of sunrise on each of the following dates:
a) June 21
b) September 23
c) December 22



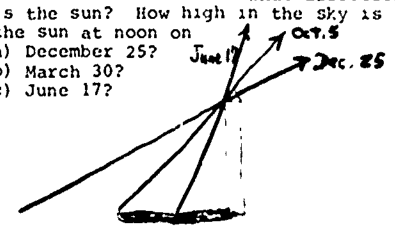
2. The sketch below illustrates sunset on March 21.
Sketch, approximately, the positions of sunset on each of the following dates:
a) June 21
b) September 23
c) December 22



3. At noon on October 5 a vertical stake casts a shadow as shown below.
Sketch, approximately, where the tip of the shadow will be on the following dates:
a) June 17
b) December 25
c) March 30



4. The shadow of the stake is shown for noon of October 5. In what direction is the sun? How high in the sky is the sun at noon on
a) December 25?
b) March 30?
c) June 17?



Orbit

This is a follow-up experiment to E17. Mars photographs are used to measure the inclination of the plane of Mars' orbit relative to that of the earth.

Equipment

Booklet of Mars photographs

Transparent overlays

Graph paper

Experiment 19: Mercury's Orbit

The orbit is obtained from observations of Mercury's maximum angular elongation from the sun. Results are plotted on earth orbit (E15) and Mars orbit (E17) graphs. Orbital eccentricity and Kepler's second law can both be studied.

Equipment

Graph of earth's orbit (E15)

Table of planet positions

Student Handbook

Protractor

Straight edge

Sec. 5.1: Motions of the Sun and Stars

When the change was made in England from Julian to Gregorian calendars in 1752, September 2 was followed by September 14 for a correction of 11 days. Many peasants are reported to have claimed they "wanted their eleven days back."

George Washington was actually born on February 11, 1732 according to the Julian calendar then used by the British. Scholars must be careful to distinguish Julian (Old Style) dates from Gregorian (New Style) dates on original documents from the latter half of the eighteenth century.

There are eighty-eight official constellations. By international agreement all the boundaries were defined along north-south or east-west lines, although older star maps show curved boundaries.

We have avoided referring in the text to both the zodiac and to sidereal time. Sidereal time is star time which gains on mean solar time by 3 minutes 56.6 seconds per day, due to the motion of the earth about the sun.

Students may wish to read "Stonehenge" by Jacquetta Hawkes (Scientific American, Vol. 188, No. 6, June 1953). See also: "Stonehenge Decoded" by Gerald Hawkins.

Sec. 5.2: Motions of the Moon

As recently as our Revolutionary War, navigators at sea depended strongly upon the position of the moon among the stars as the basis for determining their longitudes at sea. This was before chronometers were developed to keep accurate

Sec. 5.3: The Wandering Stars

Sections 5.1, 5.2 and 5.3 have reviewed the basic observations to be explained by a theory. If the students could fill in the following table correctly, they know the major motions to be explained.

Motion	Stars	Sun	Moon	Planets
Daily motion from eastern horizon to western horizon	x	x	x	x
Generally move eastward among the stars		x	x	x
Move N-S while moving eastward		x	x	x
Moves N-S in one year		x		
Moves N-S in one month			x	
Retrogrades				x

Sec. 5.4: Plato's Problem

In a recent book, Marshall Clagett (Archimedes in the Middle Ages, Vol. 1, Univ. of Wisconsin Press, 1964) has ferreted out the history of the manuscripts available on the works of Archimedes. Because Archimedes was one of the giants of Hellenistic Greece, we might expect that many manuscripts would be available. Not so. Modern knowledge of Archimedes is based mainly on three Byzantine Greek

Sec. 5.5: The First Earth – Centered Solution

To the ancients the earth was very large, immobile, and at the center of all motions. It seemed easy to explain the motions of the fixed stars with the earth at the center. Mention is made that Eudoxus, Plato's pupil, needed only twenty-seven spheres to explain the general observations. Aristotle added twenty-nine more mainly to tie together, like a great gear-train, the spheres of Eudoxus. Enough motions are needed to account for the various cycles observed. As more cycles were included for greater precision, more motions were needed.

From our point of view, one big drawback of the earth-centered scheme was its failure to predict precisely the positions of planets in the sky. But Greek science had different purposes than modern science; its theories were, at first, only intended to account for the general changes observed. The desire for greater precision came later.

Students should understand that it is impossible to describe the theory of the Greeks. There were many variations. Plato believed that the earth was spherical—from the shape of its shadow thrown on the moon at lunar eclipses. Heraclides of Pontus, who was, like Aristotle, a pupil of Plato, believed that the earth was at the center and rotated while the heaven was at rest.

Your students will probably be amazed to find that in the thirteenth century most of the astronomical explanations were still those of Greek antiquity. To elaborate is to trace western civilization; perhaps some students will want to present the class with a capsulated history. Or perhaps a student will want to explain to the class how Dante in the Divine Comedy (1300 A.D.) described the spherical earth in the center of the world, with the planets and stars moving in celestial spheres.

The Greek arrangement of the planets has come down to us in the names of the days of the week. Students might note that many of the names we use are from the Teutonic mythology, e.g., Thor's day, and match the characteristics of the various gods and goddesses in the Greek and Teutonic mythologies. Language students will note that the day names in French and Spanish and Italian are close to the original Greek names.

Sec. 5.6: A Sun-Centered Solution

Through the Almagest which circulated among scholars and students in the Middle Ages, the idea of a heliocentric system was known. Copernicus tried to defend a sun-centered system (Chapter 6) and to refute the argument in the Almagest against Aristarchus.

Figures 5.10a and 5.10b show the earth and a planet moving in circular orbits around the sun. The earth moves faster. Figure 5.10a shows that at point 1 the planet has a relative motion ahead (eastward) from the earth. At points 2 and 6 for both the earth and planet the components of motion perpendicular to the sight-line are equal, so the planet appears to be moving backward (westward or retrograde). At point 7, like point 1, the planet is seen to be moving eastward again.

See resource section, p. 140 for note on the sizes and distances to the sun and moon by Aristarchus.

Sec. 5.7: The Geometric System of Ptolemy

During the 500 years between Plato and Ptolemy the Greeks had made great achievements in geometry. Ptolemy applied some of these in his attempt to find ways to predict precisely the positions of the planets. He was willing to sacrifice Plato's assumption of uniform angular motion around the centers of circles for greater precision in his predictions. Emphasis was upon the longitudes, or position along the ecliptic, rather than upon the latitudes, or positions perpendicular to the ecliptic. The latitudes could be predicted, at least roughly, by tilting the planes of the epicycles a bit from the plane of the ecliptic.

Our intent is not to stress the details of the various geometrical devices used by Ptolemy, but rather to indicate his ability to introduce many different types of motions to satisfy the increasingly more precise observations. To satisfy a variety of cycles found in planetary motions, Ptolemy introduced a variety of geometrical models: the eccentric, epicycle and equant. His computational devices solved geometrically problems for which we would use equations in trigonometric terms of sines and cosines of various angles, with constants to give the amplitudes of terms. (An overhead projection on the eccentric should be available for class use.)

See resource section, p. 140, for notes on epicycles.

Sec. 6.1: The Copernican System

Since both the Copernican and the Ptolemaic systems had to account for the same observations, the two systems had about the same number of motions. Copernicus also used Ptolemy's numerical constants which described the magnitude of the motions. Consequently, the Copernican system was no more precise than the Ptolemaic system it proposed to replace.

But increased precision was not really Copernicus' primary intention. He wished to purify the model, to describe all the motions on the basis of combined uniform circular motions.

Copernicus had been requested by Pope Paul III to assist in the reform of the calendar, which resulted later in the Gregorian calendar in current use. But Copernicus declined, claiming that a better calendar should be based on an improved system for predicting celestial events. Some idea of the complexity of forming a calendar for civil and religious purposes is included in the article "Calendar" in the Encyclopaedia Britannica.

About 1530 Copernicus prepared and circulated to a few friends the *Commentariolus*, which was a sketch of his proposed system. Through it a number of people learned a bit about the ideas he was developing. Later in his *Revolutions* Copernicus made some changes in the argument and added other, small cyclic motions, perhaps as a result of criticisms from his friends.

Sec. 6.2: New Conclusions

The orbital distances for Mercury and Venus were found from the maximum angular elongations from the sun, as Fig. 6.4(a and b) indicates. An optional experiment, E19 uses such observations to yield an orbit for Mercury. The orbit of Venus is almost circular.

Orbital distances for Mars, Jupiter and Saturn were found by replacing their large annual epicycles by a single annual revolution of the earth. Then, as Fig. 6.5 indicates, the orbits (deferents) of planets would be moved inward or outward from their positions on the Ptolemaic system. Table 6.3 shows the arithmetic.

As the table below suggests, students can work out their own approximate values from measurements on Fig. 5.15 (which was not intended to be exact).

Derivation of Planetary Orbits from Fig. 5.15

Object	Diameter Epicycle, cm	Ratio Sun/ Planet	Radius* Deferent, cm	Radius, AU
Sun	2.00	1.00	1.00	1.0
Mars	3.25	0.61	2.62	1.6
Jupiter	2.15	0.93	5.40	5.0
Saturn	1.45	1.38	7.25	10.0

*The diameters of the deferents are not shown in the figure.

While the details of these calculations will interest some students, it is more important for all students to realize that the heliocentric model allowed such results to be obtained—for the first time in history.

At the end of this chapter we raise questions about the reality of these orbits. Certainly they seem much more "real" than the computing devices used by Ptolemy or the transparent crystalline spheres proposed earlier.

See resource section for note on derivation of periods.

Sec. 6.3: Arguments for the Copernican System

In some ways the Copernican system was relatively simple, but in its details this system was just as complex as the Ptolemaic. The simplicity is essentially aesthetic or philosophical, i.e., the basic idea as shown in Fig. 6.2 is simple. Yet the computations needed to make precise predictions were just as complex as ever. In this "messiness" lay the motivation of Tycho Brahe and of Kepler to attempt to find a simpler model.

Useful background reading would be Chapter 1 of Herbert Butterfield's *The Origins of Modern Science*, Collier paperback AS259V, revised edition.

Sec. 6.4: Arguments Against the Copernican System

Members of all the religious groups attacked Copernicus; the attacks and ridicule were not limited to any one group. As the religious leaders realized, if the sun were the center of the system, the stars must be very far away and very luminous, perhaps even themselves suns. If they were suns, they might have planets, and these might have intelligent life. This idea, that there might be other planets around other stars, was called the Plurality of Worlds. Then the earth and our religious experiences here might not be unique. The possibility of life existing on bodies other than the earth was voiced only slowly, in England by Thomas Digges and on the continent by Giordano Bruno—who was burned at the stake for heresy in 1600. The paper by Anatole France in the reader is relevant here.

Sec. 6.5: Historical Consequences

Follow text.

Sec. 6.6: Tycho

As you know, a star is new only in the sense of being observable or conspicuous. Current explanations describe the process as one in which the star's content of hydrogen has gradually been consumed to the point that the outward radiation pressure within the star no longer balances the gravitational attraction toward the center of the gaseous star. Then the star collapses. The process overshoots, very high central pressures are developed and the star flares up for a few years. Part of the outer envelope is blown off and may later be detected, for fairly nearby novae, as whispy filaments of outward moving gas. Apparently such stars eventually settle down as "white dwarfs," having surface temperatures around 10,000° C and internal specific gravities of 10^5 or 10^6, but still gaseous.

Other stars seem to have much greater outbursts and become supernovae. For a few years their luminosity—actual output of radiant energy—may be as great as 10^8 that of our sun. These stars collapse into strange bodies that are often the source of intense radio waves—detected only recently with the development of radio telescopes and masers. The novae observed in 1572 by Tycho, and in 1604 by Kepler and Galileo, were both supernovae. The supernova observed in Taurus in 1054 A.D. has resulted in the Crab Nebula. There is evidence that the Indians in western America saw this event and cut in a rock face the symbols of a star and the crescent moon. What else could have been so impressive that it produced this record? Probably a star like this supernova was visible during the day. From oriental records and recent computations, it has been found that the nova appeared in July 1054 near the crescent moon.

Tycho's Uraniborg might be likened to one of our present large research centers supported by the government; perhaps the Brookhaven Laboratory, the Lawrence Laboratory, the Argonne Laboratory, CERN in Switzerland, etc. An article in the February 1961 Scientific American, p. 118, discusses Tycho's observatory.

Tycho observed that the comet of 1577 (as mentioned on p. 43) showed no geocentric parallax when viewed simultaneously from places hundreds of miles apart. Because the moon's observed position is different over such a baseline, the comet must be farther away than is the moon. Perhaps some discussion of the many superstitions surrounding the unexpectedness of comets would be useful to suggest the variety of ways by which people interpret unexpected events. (The writings of Shakespeare contain many allusions to astronomical events as omens.) We still have many rudiments of such superstitions in us; but increasingly we may be conscious that we are reacting fearfully on the basis of unwarranted assumptions about the world. Refer students to the paper by Gingerich in the reader for Unit 2.

Figure 6.10 shows the apparent path of Halley's Comet in 1909-10 plotted on a star map showing some of the brighter stars. Note especially the changes in the motion of the comet. From November until April it moved westward, slowed down and remained almost stationary for nearly a month. Then, during only a few weeks in May 1910, the comet quickly moved across the sky nearly 180°. Notice also when the comet crossed the ecliptic from south to north (about January 25, 1910), and again from north to south (about May 19, 1910).

Interested students might read about comets and how they are studied. (See references.) The December 1965 issue of Sky and Telescope featured photographs of this bright comet of 1965.

Sec. 6.7: Tycho's Observations

As we saw earlier, accurate observations were not available to Copernicus who relied mainly upon the records of Ptolemy. Yet these old observations had been made by different men at different times. Scholars still discuss the extent to which the observations in the Almagest were made by Ptolemy or were in part adopted and corrected from earlier work by Hipparchus about 150 B.C. Tycho concluded that new and more precise observations made over a number of years were essential before any new description of planetary motions could be created or judged.

The sections on Tycho's equipment might stimulate some students interested in mechanics and equipment design. The inherent limitations of our eyes also could be investigated by reading, or by a project for those interested.

Atmospheric refraction is indicated in Fig. 6.13. See resource section for notes on refraction and the atmosphere, p. 143.

Sec. 6.8: Tycho's Compromise System

Tycho's observations were planned as the basis for the development of a new model of planetary motions. Although he died before much of the analysis could be completed, his general idea of a sys-

tem is that indicated in Fig. 6.16. The planets moved about the sun, but the sun moved about a fixed earth. In terms of describing the observed planetary motion, Tycho's system was equivalent to Copernicus'. However, it had one further advantage: no stellar parallax would be expected. A discussion would be profitable about why Tycho's system was not widely adopted.

At the end of the section we introduce the important question: are scientific models descriptions of reality, or only convenient computational devices? The Ptolemaic system permitted computations of the positions of the planets; it did not attempt to describe reality. The reality of the heavens was visualized by the Greeks and by medieval man in terms of crystalline spheres; this is the vision described by Dante in the Divine Comedy (1300). But Copernicus and Tycho raised the question of the real motions of the planets. Here as well as elsewhere in the text, we raise the question about the reality of conclusions based on scientific theories. The point should not be omitted in class discussion.

Sec. 7.1: The Abandonment of Uniform Circular Motion

Kepler was a strange blend of mystic and scientist with a deep Pythagorean feeling for the numerical perfection in the world and the music of the spheres. His early paper on the spacing of planets and his later work on the third law suggest that sometimes scientists begin with aesthetic or artistic premises. The recent stress upon "symmetry" in particle physics is another example.

Due largely to the fact that Kepler inherited all of Tycho's data on Mars and had access to the writings of preceding astronomers, the time was ripe for new ideas not prejudiced by the assumption of uniform circular motion. You might remind students that in many instances in life one may be forced to reexamine early assumptions and perhaps to replace them.

Tycho and Kepler, seen in historical perspective, made an ideal pair. Tycho stressed the importance of improved observations and devoted his life to obtaining such observations. Without them, Kepler would have been in the same difficulties as had been his predecessors.

After more than 70 unsuccessful trials Kepler found that he could not fit the observations with any combination of circular motions. Perhaps you would wish to dramatize the situations in which Kepler found himself. He felt that some satisfactory solution could be found. Since Mars continued to move across the sky, oblivious to Kepler's efforts, the troubles must lie with the theory makers. Therefore, he was obliged to look at the problem in a new way. This is always difficult for us, but Kepler did it.

His work, both unsuccessful and eventually successful, involved a staggering amount of labor because the mathematical techniques of his time were cumbersome. This labor is suggested by the reproduction on p. 54 of a page from Kepler's notebooks.

Kepler was caught in the middle of the religious conflicts of the Thirty Years War and the struggles between the Catholics and Protestants. At best he had a difficult time earning a living, despite the promises of the king. The witchcraft trial of his mother might be paralleled with the similar occurrences a bit later in the American colonies. This is an indication of the cultural and social context within which Kepler, and also Galileo, were working. (The popular book on Kepler by Max Caspar might interest some students; see references.)

Sec. 7.3: Kepler's Law of Elliptical Orbits

On page 57 we quoted Kepler's comment that, "Mars alone enables us to penetrate the secrets of astronomy which otherwise would remain forever hidden from us." This almost surely refers to the sizable eccentricity of the orbit of Mars ($e = 0.09$). Of the outer planets, only Mars is near enough to be studied accurately by Kepler's triangulation method. Although Mercury has a more eccentric orbit ($e = 0.21$), studies of it were practically impossible; Mercury is only seen in the twilight when few stars can be observed to determine accurate positions. Today telescopic observations of position can be made of Mercury and even of bright stars in the daytime. Of course, Uranus, Neptune and Pluto were unknown at the time of Kepler.

An excellent background on the mathematics of conic sections appears in the SMSG publication: Intermediate Mathematics, Teacher's Commentary (Unit 19), Yale University Press, New Haven, 1961.

Sec. 7.4: Using the First Two Laws

Each of the factors in Kepler's statements are themselves concepts several steps removed from the actual astronomical observations. Careful definitions are necessary for such concepts to match explicitly with observations.

Sec. 7.5: Kepler's Law of Periods

Follow text.

Sec. 7.6: The New Concept of Physical Law

Kepler's work reflects the change from a mystical interpretation of how the world ought to be to a reliance upon observations as the final basis for decisions. He had a growing feeling that some mechanism was essential to move the planets. We know that he often wrote to Galileo, but that after a few letters from Galileo the correspondence was one way. Why Galileo did not accept the elliptical orbits of Kepler is difficult to understand; except, as one authority noted, Kepler wrote in a flowery style and was often most difficult to understand. Unfortunately, his major contributions are buried in masses of words. (Is there a moral in this for your students? Have they examined some technical writing in science in professional journals?)

In the historical study of science, it is often difficult to establish who actually had an idea first. Ideas often grow as various men consider them and their consequences. The idea of the universe operating like a clockwork, or a giant machine was also implicit in the sequence of invisible celestial spheres proposed by Eudoxus, Chapter 5. However, Kepler's analogy is important because, as Chapter 8 shows and other more recent quotations indicate, this idea became firmly entrenched. Perhaps the ultimate form of the idea was the statement of one later scientist to the effect that—if he knew the initial positions and velocities of all the bodies in the universe, he could accurately predict the future of the universe. The great success of Newtonian mechanics, discussed in Chapter 8, supported such a mechanistic view. Only within the current century have physicists been obliged by new types of observations to abandon such sweeping general assertions; see Units 4, 5 and 6.

Sec. 7.7: Galileo's Viewpoint

A number of lines of evidence, including Galileo's work in mechanics and the astronomical models of Copernicus and Kepler, were undermining the premises on which the Aristotelians based their arguments. Even such men as the poet and writer John Milton in England were aware of what was happening. Milton made a journey to southern Europe and visited Galileo during the summer of 1638. In the quoted section of Paradise Lost the poet raises the question which had been rejected by the Ptolemaics.

Galileo was incensed that his contemporaries would not even use the telescope and try to refute his observations. They remained entrenched in their own ideas and wouldn't consider either challenging them by looking for themselves or accepting his reports. We all experience great difficulty making a major shift in concepts. Certainly, the shift from an earth-centered universe to a sun-centered system was gigantic in its implications. Can students suggest other comparable shifts in interpretation that have caused us to reorient our interpretation of the world and man's place in it? Do not restrict the list to those shifts which seemed to appear abruptly. Even the sun-centered system required nearly 1800 years to be considered seriously. Possibilities: determination of the age of the earth, the Darwinian theory of evolution, relativity, Freud's psychological theories, etc.

Sec. 7.8: The Telescopic Evidence

Figure 7.13 shows two of Galileo's telescopes preserved in a museum. Galileo saw and interpreted many new objects. His conclusions are even more important than his drawings of what he saw. Possibly others could have viewed the moon, but not found the mountains he recognized. The difference between raw data and interpretation might be developed. Although new instruments permit new observations, instruments only provide data which must be interpreted.

Sec. 7.9: Galileo's Arguments

Follow text.

Sec. 7.10: The Opposition to Galileo

Follow text.

Sec. 7.11: Science and Freedom

Students may want to report on the history of the Catholic Church in the seventeenth century and to compare it, perhaps, with the Church in the twentieth. Others may want to discuss the rise of the Protestant groups. Would it be likely that a community hospitable to the followers of Martin Luther or John Calvin would be hospitable to new ideas in science?

Do not attempt to create a "hero and villain" image of Galileo and the Church, for this is only divisive. Rather try to have students examine the known facts objectively and conclude that there probably was error and provocation on both sides. The book by de Santillana (see bibliography) presents Galileo as a martyr. Is it a fair position?

Chapter 8

Perhaps the persistent question that might be raised is whether this great theory (universal gravitation) is ever "proved." Hopefully, students will conclude that a rigorous proof is not possible. Yet in spite of this, the theory seems to work well; it explains so much that is known and predicts so many other phenomena and quantities.

The arguments follow rather closely to Newton's, although some have been modified, or created in the spirit of Newton for this discussion.

The emphasis here is upon the growing acceptance in northern Europe of the "new philosophy" of empirical, experimental science. In addition to the Royal Philosophical Society of London, in France there was the Academie des Sciences, in Italy, the Accademia de Lincei (Lynxes) at Rome and the Accademia de Cimento, at Florence. The scientific societies, first in Italy and then in England and France, were important because they allowed scientists to work and argue together and to publish journals that could be sent to their colleagues in other countries. Stress how the work of many people is related, as illustrated by the quote on page 77 from Lord Rutherford and demonstrated by the achievement of Newton.

Sec. 8.1: A Sketch of Newton's Life

Follow text.

Sec. 8.2: Newton's Principia

The first edition of the Principia was published in 1686. The second edition in 1713 included many corrections to the first printing, some new arguments, and considerably more data on comets based mainly on Halley's work.

Various authors have repeatedly pointed out that Newton did not attempt to explain gravitation. He postulated an inverse square force of attraction between bodies—and it worked. He did not know how it worked or why it seemed to be associated with masses. In his famous General Scholium at the end of the Principia he observed that "he framed no hypotheses"—on the nature of gravity. He was concerned, but had no conclusions that he wished to present.

At this point you may wish to ask students about the usefulness of an undefined concept such as gravity. We can measure its effects, predict the outcome of certain experiments, and in general make some use of gravity—yet we do not know what it is. Einstein was working on a unification of several aspects of gravity at the time of his death. People now involved in this research have still not explained gravity.

Sec. 8.3: A Preview of Newton's Analysis

One of the exciting understandings that can come out of this section is that the great scientific geniuses like Newton and Galileo were great synthesizers of ideas. Why didn't his contemporaries come to the same conclusions? How may one analytical mind tend to operate differently from another on the same set of data and laws?

Point out the shift in Newton's assumptions; first, from the Greek notion of circular motion as perfect to the inertial circular motion of Galileo discussed in Chapter 4; then, to Newton's definition of inertial motion in an optically straight line. Added to this is Newton's idea that circular motion is caused by a force in action, an idea which he extended to include the elliptical motion of Kepler's laws.

The origins of the great generalizations of science can be traced in preceding decades. This preview of Newton's analysis should emphasize the importance of the historical background leading to Newton's great synthesis of his laws of motion and of universal gravitation. The barrier between celestial and terrestrial motions set up by Aristotle was gradually being broken down. Tycho Brahe located comets beyond the moon. Kepler replaced "perfect" circular motions by those in elliptical orbits. Jeremiah Horrocks, born in the year Kepler's third law was published, entered Emmanuel College of the University of Cambridge at the age of thirteen. When nineteen and curate of Hoole in Lancashire, he applied Kepler's first law to the motion of the moon around the earth and even showed that the eccentricity of its orbit changed periodically and the major axis of the ellipse slowly rotated. This was 25 years before the youthful Newton conceived his ideas on universal gravitation, which he did not publish for another 20 years.

Sec. 8.4: Motion Under a Central Force

One of the major difficulties your students may encounter in the geometric development of Newton's argument is the inability to see how the way of measuring the triangular areas changes. To minimize this difficulty, make use of Fig. 8.3 which demonstrates how each side of a triangle may be used as a base, and how a perpendicular may be dropped from each vertex. This figure should be associated especially with the second paragraph of point 2 of the text: "These have equal bases, PQ and QR;..."

It might also be a good idea to have the students refer back to the contrast listing of Newton's and Kepler's laws on p. 82 while the geometrical argument is being developed. Emphasize the unexpectedness of the conclusion that the law of areas holds even when no central force is acting.

It might be useful to see Newton's original development of this argument on pp. 40-55 of Book I of the Principia (in Vol. I of the edition mentioned in the bibliography). While they will not be able to follow the text, at least the wording of the propositions and scholiums, and the illustrations will serve to demonstrate how neatly Newton tied his argumentative package.

The universal law of gravitation was a very bold proposal. Take advantage of this to dramatize the audacity of Newton to propose the universality of physical laws whose action could generally only be observed on the earth. The people of Newton's time were still bound by the concepts of separate worlds and other Aristotelian doctrines.

Sec. 8.5: The Inverse-Square Law of Planetary Force

Follow text

Sec. 8.6: Law of Universal Gravitation

The discussion raises the question of action at a distance. Note the quotation from Newton on the middle of page 91. Direct the student's attention to the fact that from the observations of Tycho and the empirical relations of Kepler and of Galileo, Newton had been able to fashion an exceedingly general and abstract description of heavenly motions. But in the process, he had been obliged to postulate the gravitational force which he could not explain. In much of science, as in mathematics, there are some postulates and axioms which cannot be anlyzed within the problems to be considered. Occasionally somebody comes along who can interpret one or more of these axioms by a more basic proposition.

Descartes' theory of vortices was first published in 1644 and received a wide acceptance on the continent. An English edition was finally published in London in 1682—before the Principia was published.

This theory was a popular non-mathematical statement, read by large numbers and readily accepted as a better explanation than none. It sounded good and was not too radically different from the Aristotelian attitudes on which the people had been raised.

Descartes' theory was widely taught, even at Cambridge, long after the publication of the Principia! It might be interesting to point out that Voltaire's famous essay, "Elements of Newtonian Philosphy," was banned in France, because the man in charge of permissions to publish was a Cartesian.

Sec. 8.7: The Magnitude of Planetary Force

Note that the discussion of geometric points in the case of Kepler's law of areas changes to discussion about the masses of stones and planets. The idea of "mass" has already been introduced by Newton's second law of motion. Note particularly the argument on pp. 93-94 that it is the mass of a body that is tied in with the notion of gravitational force. This argument really marks Newton's great contribution—a big leap in understanding—from a consideration of the direction of the force to that of the amount of the force.

The gravitational constant g serves the same function as any constant which changes a proportion into an algebraic equation. In the case of equations involving physical quantities, the constant also serves as a "balancer" of units. It might be worthwhile to remind the students at this point that symbols which stand for physical quantities are not sacred cows, and that they only mean what you want them to mean. It could also be pointed out, for example, that this same kind of operation involving a constant turns up in the algebraic form of Kepler's third law, where $T^2 \sim a^3$ becomes $T^2 = ka^3$.

Since $R_1 = R_2$

$$\frac{M_1}{M_2} = \frac{T_2^{\,2}}{T_1^{\,2}} = \frac{(3)^2}{(1)^2} = \frac{9}{1}.$$

.celeration of gravity is proportional to ·. Let R_E be the radius of the earth and g_E the acceleration of gravity at the earth's surface. Then the acceleration g at a distance R from the center is

$$\frac{g}{g_E} = \frac{R_E^{\,2}}{R^2}$$

so that

$$R = R_E \times \sqrt{\frac{g_E}{g}}$$

(a) For $g = \frac{3}{4} g_E$,

$$R = \sqrt{\frac{4}{3}}\, R_E \approx 1.15\ R_E$$

This is about $0.15\ R_E$ = 960 km above the earth's surface.

(b) For $g = \frac{1}{2} g_E$

$$R = \sqrt{2}\, R_E \approx 1.41\ R_E$$

This is about $0.41\ R_E$ = 2,600 km above the earth's surface.

(c) For $g = \frac{1}{4} g_E$

$$R = \sqrt{4}\, R_E = 2\ R_E$$

This is one earth radius (about 6,400 km) above the earth's surface.

$g_m = 1.58$ m/sec2

The value on the earth, 9.80 m/sec2, is 6.20 times greater.

(b) On the moon, the 72-kg astronaut weighs approximately:

$$W = mg = 72 \times 1.58 = 114N$$

His mass doesn not change.

8.9

(a) For there to be no net force, the forces exerted on the satellite must be equal in magnitude (and opposite in direction).

$$\text{Sun - Satellite force} = F_s = \frac{GM_s m}{R_s^{\,2}}$$

$$\text{Jupiter - Satellite force} = F_J = \frac{GM_J m}{R_J^{\,2}}$$

But these two forces are equal.

$$\frac{GM_s m}{R_s^{\,2}} = \frac{GM_J m}{R_J^{\,2}}$$

$$M_J = 1.90 \times 10^{27}\text{kg}$$

$$M_s = 1.99 \times 10^{30}\text{kg}$$

$$\left(\frac{R_s}{R_J}\right)^2 = \frac{M_s}{M_J} \qquad \frac{R_s}{R_J} = \left(\frac{M_s}{M_J}\right)^{\frac{1}{2}}$$

$$\frac{R_s}{R_J} = \left(\frac{1.99 \times 10^{30}\text{kg}}{1.90 \times 10^{27}\text{kg}}\right)^{\frac{1}{2}} = (1.047 \times 10^3)^{\frac{1}{2}}$$

$$\frac{R_s}{R_J} = 32.4$$

Therefore R_s is 32.4 times greater than R_J.

Sec. 8.8: Testing a General Law

How can certain quantities be measured? How exact must measurements be in order to insure the validity of a physical law? What different kinds of errors lie in waiting to trap the physicist? How can he deal with such errors? You might point out that when Newton first solved the earth-moon problem (which is discussed in Sec. 8.9), he got a wrong answer (off about 10%) because he used an incorrect value for the earth's radius; in disgust, he put his work away, thinking that his conceptual approach was wrong. Some time later, a new, more exact measurement showed his theory to be valid.

In infinitesimal calculus we "chop" a quantity into a large number of very tiny pieces, so small that each is virtually like the next. In a sense, this is the imaginative process of "averaging out" on a microscopic scale. It might be helpful for the class to look back at the concept of instantaneous velocity in Unit 1.

They should be able to see that the breaking-up of the time measurement into tiny Δt's is exactly parallel to what Newton did with spherical masses in order to arrive at the concept of the mass-point.

Sec. 8.9: The Moon and Universal Gravitation

The student has already learned in Experiment 14 that special measurement techniques must be used in astronomy, where the bodies being observed are far away. Later in the course, for example, it will be shown that the sun's temperature can be measured without using a thermometer. You might even ask the

The experiments that man can carry out in order to determine that G is a universal constant are limited in number. The extension to universality must be carried out in terms of a kind of well-sustained faith. Such a conclusion may come as a shock to those students who feel that science is a cut-and-dried rational process that has little or no room for imagination or statements based upon "revelation"; you have a ready-made "situation" for an exciting discussion.

Sec. 8.11: The Tides

Some of your students may have difficulty in understanding the concept of differential forces in the case of the sun's pull versus that of the moon on the earth. They ought to see that this difference is really a function of distance. Even though the gravitational pull of the sun on the earth is much greater than that of the moon, the sun does not "distinguish" between the near and far side of the earth. Where the sun does not exert significantly different pulls on the near and far sides of the earth, the moon does.

If one simply had all the data for high and low tides from different parts of the earth, would that information be enough for the formation of a general predictive theory for the tides? In what way does the principle of universal gravitation become a "breakthrough" here?

Sec. 8.12: Comets

Students will be able to find allusions to comets as omens in Shakespeare, Chaucer, Julius Caesar, and in all kinds of

Jupiter's gravitational field. The orbit, however, will change and not be a very stable one.

(d) A satellite even farther from Jupiter would have an even more unstable orbit, and would probably eventually escape from being a satellite of Jupiter. The several outer satellites of Jupiter may be asteroids which have been captured temporarily as satellites of Jupiter.

8.10

Since the surface gravity depends on the mass and radius of the planet, we need to compare these values for the two planets.

(a) Let planet A be the smaller one. Therefore $R_B = 2R_A$.

The density $\rho = \frac{\text{mass}}{\text{volume}}$ for the two planets can be equated:

$$\frac{M_A}{\frac{4}{3}\pi R_A{}^3} = \frac{M_B}{\frac{4}{3}\pi (2R_A)^3}$$

$$\frac{M_A}{M_B} = \frac{R_A{}^3}{8R_A{}^3} = \frac{1}{8}$$

Since $F = mg = \frac{GMm}{R^2}$

we can make the ratio

$$\frac{mg_A}{mg_B} = \frac{GM_A m}{R_A{}^2}\frac{R_B{}^2}{GM_B m}\text{, or}$$

$$\frac{g_A}{g_B} = \frac{(R_A)^2}{(R_A)^2}\frac{M_A}{M_B}.$$

Substituting,

$$\frac{g_A}{g_B} = \frac{(2R_A)^2}{R_A{}^2} \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

(b) From Unit 1 we know that F = ma and that the acceleration in circular motion is

$$a = \frac{4\pi^2 R}{T^2} \text{ (Eq. 8.6).}$$

Then $F = ma = \frac{m4\pi^2 R}{T^2}$

(m is the mass of the satellite).

This force is the same given by the law of gravitation:

$$\frac{m4\pi^2 R}{T^2} = \frac{GMm}{R^2}$$

The mass of Mars is then

$$M = \frac{4\pi^2 R^3}{GT^2} \text{ (Eq. 8.10).}$$

Since the units of G are in the MKS system, we must convert R and T:

$$T = 7\text{hrs } 39\text{min} = 7\text{hrs} \times 3.6 \times 10^3 \frac{\text{sec}}{\text{hr}} + 39\text{min} \times 60\frac{\text{sec}}{\text{min}}$$

$$T = 2.77 \times 10^4 \text{sec}$$

$$R = 5{,}800 \text{ miles} = 5{,}800 \text{ miles} \times 1.61 \times 10^3 \frac{\text{m}}{\text{mile}} = 9.34 \times 10^6 \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{N}\cdot\text{m}^2/\text{kg}^2$$

$$M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 (9.34 \times 10^6 \text{m})^3}{(6.67 \times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2 (2.77 \times 10^4 \text{sec})^2}$$

$$M = 6.7 \times 10^{23} \frac{\text{m}^3}{\frac{\text{kg}\cdot\text{m}}{\text{sec}^2}\cdot\frac{\text{m}^2}{\text{kg}^2}\cdot\text{sec}^2}$$

$$\underline{M = 6.7 \times 10^{23} \text{kg}^2}$$

Should any students wish to make a model of the comet's orbit, the elements (based on early observations) are:

T October 21.18, 1965
q 0.00777 AU
i 141.8°
Ω 345.9°
ω 068.7°

Similar sun-grazing comets were observed in 1880, 1882, and 1887. They, like 1965f, showed partial disintegration into several nuclei at about the time of perihelion passage. Because they all have periods which are essentially infinite, these could not possibly be reappearances of the same comet.

It is interesting to note that already computations are being made for the modified orbit of Halley's comet when it returns in 1986. The first part of the computation is based upon 2,800 observations made during the two-year appearance of the comet during the period, 1909-1911. Even during this time, the comet's orbit was changing its shape, size, and orientation (why?). We can only observe Halley's comet during 2 years of its 75-year trip, while it is inside the orbit of Jupiter. The other 73 years are spent at greater distances from the sun.

Sec. 8.13: Relative Masses of the Planets

The reason that the masses of the sun and Jupiter can be compared is that Jupiter acts like a miniature solar system (this was Galileo's immediate thought upon identifying the satellites of Jupiter). The only difference between the sun system and the Jupiter system, insofar as Kepler's law is concerned, is that each k involves a different central mass. As long as we can measure the R and T for a revolving body in each system, the two central masses can easily

they could not exist without a gravitational field. In describing them, we should write $W = m_g a_g$ and

$$F_g = \frac{GM_g m_g}{R^2}$$

Sec. 8.15 The actual masses of celestial bodies

As long as Newton had to depend upon the use of ratios, without G entering into the quantitative results he obtained his statement about gravitation was really a <u>hypothesis</u>. Once G was measured, the hypothesis could really be called a <u>law</u>, since all quantities in the statement were now measurable.

Sec. 8.16: Beyond the Solar System

Follow text.

Sec. 8.17: Some Influences on Newton's Work

Flamsteed was the first Astronomer Royal, appointed after the establishment of the Royal Astronomical Observatory in 1675. Note that Surveyor I landed on the moon in the crater named for Flamsteed. Halley succeeded Flamsteed upon his death in 1720. The French had already founded a national observatory in Paris in 1671.

Sec. 8.18: Newton's Place in Modern Science

Some students may want to look up references related to the Encyclopedist movement in France, begun by Denis Diderot; also, the influence of Newton's work upon the great Voltaire is worth some research. A good encyclopedia will certainly have much to say about this

$G = 6.67 \times 10^{-11} N{\cdot}m^2/kg^2$

$M_E = 5.96 \times 10^{24} kg$

$M_M = 6.58 \times 10^{23} kg$

$R = 5.6 \times 10^{10} m$

$$F_1 = 6.67 \times 10^{-11} N{\cdot}m^2/kg^2 \times 5.96 \times \frac{10^{24} kg \times 6.58 \times 10^{23} kg}{(5.6 \times 10^{10} m)^2}$$

$$\underline{F = 8.3 \times 10^{16} N},$$

the force of the earth on Mars.

$M_J = 1.91 \times 10^{27} kg$

$R_2 = 4.90 \times 10^{11} m$

$$F = 6.67 \times 10^{-11} N{\cdot}m^2/kg^2 \times 6.58 \times \frac{10^{23} kg \times 1.91 \times 10^{27} kg}{(4.90 \times 10^{11} m)^2}$$

$$\underline{F = 3.5 \times 10^{17} N},$$

the force of Jupiter on Mars.

(c) The force on Mars due to Jupiter is greater than the force due to the earth. Therefore Jupiter has on the average the greater effect on the motion of Mars.

8.13

$F = ma = \frac{GMm}{R^2}$ where a is the centripetal acceleration

$$a = \frac{GM}{R^2}$$

$$(6.67 \times 10^{-11} N{\cdot}m^2/kg^2)(1.44 \times 10^6 sec)^2$$

$$\underline{M = 1.86 \times 10^{27} kg}$$

8.14

The first expression came from:

$$F_{(grav)} = GMm/R^2$$

and

$$F_{(cent)} = mv^2/R.$$

These must be equal for a satellite in circular orbit, i :

$$F_{(grav)} = F_{(cent)}.$$

$$\frac{GMm}{R^2} = \frac{mv^2}{R},\ \frac{GM}{R^2} = \frac{v^2}{R},\ \text{or } v^2 = \frac{GM}{R}$$

Now, if $T = \frac{2\pi R}{v}$, then solve for v:

$$v = \frac{2\pi R}{T}$$

$$v = \frac{4\pi^2 R^2}{T^2}$$

$$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$$

Solving for T^2,

$$T = \frac{4\pi^2 R^3}{GM}$$

$$\underline{T = 2\pi \frac{R^3}{GM}}$$

8.15

Since $F = \frac{GMm}{R^2}$ and the masses of the balls are the same, at the point equidistant from the balls along their line of centers the gravitational attractions are equal and opposite.

Transparencies

SUMMARY SHEET—OVERHEAD PROJECTION TRANSPARENCIES

T13 Stellar Motion
(relevant to Secs. 5.1, 5.5)

Displays a two-sphere universe explanation of apparent stellar motion as observed at mid-northern latitudes, the equator and north pole.

T14 The Celestial Sphere
(relevant to Secs. 5.1, 5.5, Experiments 15, 17)

Illustrates scheme of the celestial sphere, indicates the meaning of equinoxes and solstices, shows sun's path in relation to zodiac and gives meaning of declination, right ascension and celestial longitude and latitude.

T15 Retrograde Motion
(relevant to Sec. 5.6)

Explains apparent retrograde (westward) motion of an outer planet by means of heliocentric model.

T16 Eccentrics and Equants
(relevant to Sec. 5.7)

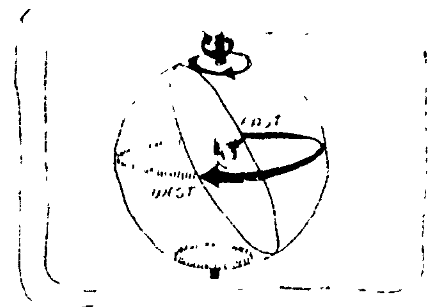
Displays features of geocentric schemes of Ptolemy for accounting for observed planetary motion.

T17 Orbit Parameters
(relevant to Secs. 7.3, 7.4, Experiments 18, 21)

Illustrates the six elements that define any orbit.

Transparency Notes

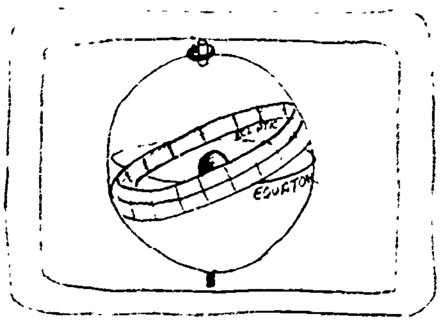
T13 STELLAR MOTION



After students have observed stellar motion directly, and have been made aware of the motions observed elsewhere on earth by means of photographs (Sec. 5.1), you may present the ancient conceptual scheme of the two-sphere universe as a model which explains these observations. Use each overlay independently to describe motions observed at different locations on earth. An observer will see all portions of the celestial sphere which lie above his horizon plane. Of course, the diagrams here are not drawn to scale in order to avoid miniscule dimensions for the earth. As a result, the horizon plane is drawn through the center of the celestial sphere rather than tangent to the place of observation.

Overlay 1 (right) displays three horizontal circles which represent the paths of selected stars attached to the sphere as it rotates daily. These circles indicate the diurnal motion of the stars as seen by an observer in the mid-northern latitudes. Some stars will appear to

T14 THE CELESTIAL SPHERE



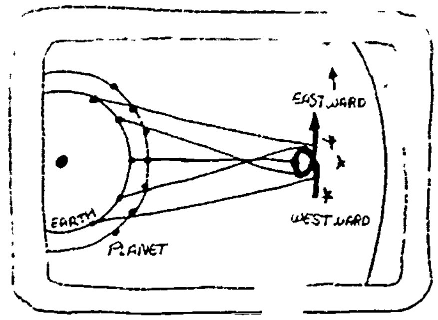
Use this transparency to aid in visualizing some of the features of the two-sphere universe. Overlays are relevant to Secs. 5.1 and 5.5, as well as to Experiments 15 and 17.

Overlay 1 (right) and the base display the celestial sphere with its poles and equator corresponding to the same positions on the sphere of the earth. The sun's apparent path across the skies as seen from earth is known as the ecliptic. The point of intersection of the ecliptic and celestial equator as the sun travels from south to north along the ecliptic is known as the vernal equinox. The crossing occurs approximately on March 21. The identifications SS, AE and WS refer respectively to the summer solstice (June 21), autumnal equinox (September 22) and winter solstice (December 21).

Overlay 2 (top) used with overlay 1 depicts the zodiac, a belt of twelve constellations which circle the sky close to the ecliptic. Remove overlay 2, retain overlay 1 and introduce overlay 3 (left). Use overlay 3 to assist your explanation of the coordinate system used in experiments 15 and 17. Stars and planets in the zodiac are easily identified by measuring celestial longitudes in degrees, 0° to 360°, beginning at the vernal equinox and proceeding along the ecliptic. Celestial latitudes are measured in degrees, 0° to 90°, beginning at the ecliptic. Remove overlay 3 and introduce overlay 4.

This overlay presents another scheme for identifying star locations. The hour circle of the vernal equinox serves as the origin of the coordinate called the right ascension. The right ascension of a star is measured eastward along the celestial equator in hours and minutes from the vernal equinox to the intersection of the star's hour circle with the equator. The declination of a star is measured in degrees from the celestial equator along the hour circle of the star.

T15 RETROGRADE MOTION



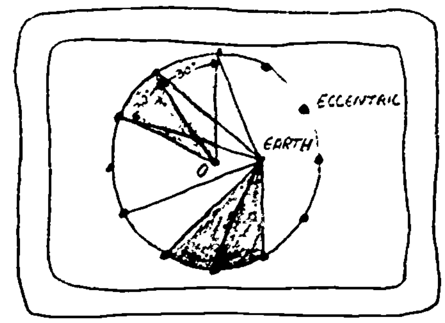
This transparency is useful in discussing the heliocentric explanation of an outer planet's apparent retrograde motion. It is relevant to Sec. 5.6.

The base transparency is a "view" of the sun-centered universe with the speed of the earth and an outer planet indicated by a "stroboscopic" representation along circular orbits.

The overlay hinged at the right is a blank piece of acetate film. Use it to draw sight lines on the transparency at positions 1-1, 2-2,...etc. This will help to position the planet in the sky relative to an observer on earth as both planets travel about the sun.

When the students have seen the step-by-step development, introduce overlay 1 (left) to show a neat finished drawing of the development. Remove your drawing and discuss the retrograde path.

T16 ECCENTRICS AND EQUANTS



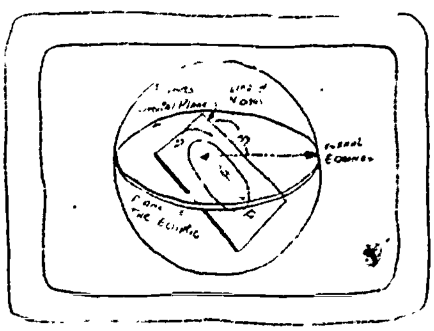
Use this transparency to present a rapid treatment of the geometrical devices of the Ptolemaic geocentric model of the universe (Sec. 5.7). Do not belabor the details but simply reveal the pragmatic value of the devices, emphasizing their function of accounting for variations observed in planetary motion while at the same time preserving the Platonic scheme of uniform angular rate at a constant distance from a center.

Overlay 1 (right) represents a planet moving with uniform angular speed at a constant distance from the center O. Its path is a perfect circle and an observer at O would measure 30° increments in successive equal time intervals.

Introduce overlay 2 (right) to relocate the earth for the eccentric scheme. It is immediately evident that the planet does not exhibit uniform speed relative to an observer on earth. Remove overlays 1 and 2.

Overlay 3 (left) introduces the geometrical dodge known as the equant. The planet proceeds with uniform angular motion about off-center point C but traces out a circular path of radius R about point O. Bring overlay 4 (left) into view to complete the scheme of the equant. Angular displacements measured from the off-centered earth will yield results different from those obtained in the eccentric device. This was precisely the need for the equant, viz., to explain variations in planetary motion not accounted for by the eccentric scheme.

T17 ORBIT PARAMETERS



Use this transparency to enhance your discussion of Kepler's first two laws (Secs. 7.3, 7.4) and Experiments 18 and 21.

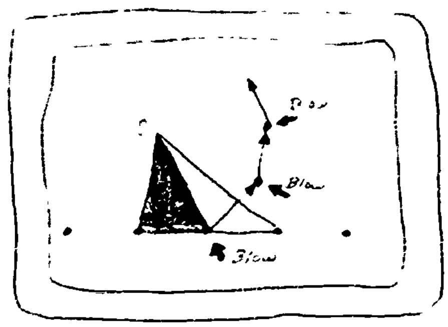
Overlay 1 (right) shows the orbital plane of a planet with the elements of an elliptical path indicated:

c = one-half distance between foci
a = semi-major axis
A = aphelion
P = perihelion
(e = eccentricity [e = c/a])

Overlay 2 (left) displays the plane of the earth's orbit known as the plane of the ecliptic. The remaining elements for determining an orbit are shown:

ω = argument of the perihelion
Ω = longitude of the ascending node
i = inclination

T18 MOTION UNDER CENTRAL FORCE



A step-by-step geometric presentation which parallels the text treatment of motion under a central force (Sec. 8.4) is presented in this transparency.

The base transparency displays a stroboscopic representation of an object traveling to the right with uniform velocity.

Use overlay 1 (right) to illustrate Kepler's law of areas for this uniform rectilinear motion example. An observer at O will see equal areas swept out by the moving object as shown by the two blue triangles. The areas can be shown to be equal since the bases are equal and the (dashed) altitudes of both triangles are identical.

Introduce overlay 2 (right). If the object is subjected to an (impulsive) force at B directed toward O, the resultant motion is toward C'. A stationary object at B would have moved to B' when given the blow. If it had not received the blow it would have proceeded to C in the same time interval.

Overlay 3 (right) is now introduced to illustrate that the area of the light blue triangle is equal to that of the red one. With the aid of the construction lines you can show that the altitudes of the two triangles are equal. The bases of both triangles, of course, are coincident.

Overlay 4 (left) continues the process of applying small periodic central forces. You may illustrate the law of areas by writing on the overlay with a wax pencil. You may also indicate the eventual path which will result if the time interval is made vanishingly small and the force is applied continuously.

Film Loops

(E) = evaluation print.
(L) = lab-loop; quantitative measurements can be made, but these loops can also be used qualitatively.
(R) = release print.

L10 Retrograde Motion—Geocentric Model

A machine was constructed in which the planet is represented by a lamp bulb on an epicyclic arm revolving around a deferent; the camera is at the position of the stationary earth, pointing in a fixed direction in space. (R)

L10A Retrograde Motion of Planets

By animation the retrograde motions of Mercury, Mars and Saturn during 1963 are shown against the stars. This loop helps define retrograde motion. (R)(L)

L11 Retrograde Motion—Heliocentric Model

The epicycle machine is used with the camera on an arm revolving around the sun; the camera points in a fixed direction in space. (R)

L12 Jupiter Satellite Orbit

Time-lapse photography, at one-minute intervals, of the motion of Jupiter's satellite Io. The period of revolution can be measured, the scale is given, and hence Jupiter's mass found. (E)(L)

L13 Program Orbit I

A computer is programmed to calculate the same orbit that a student calculates in the laboratory; the result is displayed on an X-Y plotter. Because of the step-wise approximation used, the orbit fails to close up exactly. (E)

L14 Program Orbit II

The computer calculates an orbit using many more points than in the preceding loop; this time the orbit closes up. The display on the X-Y plotter is repeated on the face of a cathode-ray tube (CRT). All other computer loops in this series use CRT display. (E)

L15 Central Forces—Iterated Blows

The computer is programmed to give sharp blows to a mass at equal time intervals. The blows are directed (at random) toward and away from a center of force, and the magnitude of the blows is also random. The law of areas can be verified. (E) (L)

L16 Kepler's Laws

Two planetary orbits in an inverse-square force field are programmed for display on the CRT; the positions of the planets are shown at successive equally-spaced time intervals. All three of Kepler's laws can be verified. (E)(L)

L17 Perturbations

The computer is programmed to display two motions which take place in central fields which are not exact inverse-square fields. One perturbation gives an advance of perihelion, as for Mercury's orbit; the other perturbation gives a catastrophic orbit in which the planet spirals into the sun. (E)

16mm Films

F6 Universe

B & W, 26 min. 10 sec. A triumph of film art, creating on the screen a vast, awe-inspiring picture of the universe as it would appear to the voyager through space. Realistic animation takes one out beyond our solar system, into far regions of space perceived by the modern astronomer. Beyond the reach of the strongest telescope, past moon, sun, Milky Way, into galaxies yet unfathomed, one travels on into the staggering depths of the night, astonished, spellbound at the sheer immensity of the universe. Starting point for this journey is the David Dunlap Observatory, Toronto. Seventeen film awards, including International Film Festival, Cannes, France; International Film Festival, Edinburgh, Scotland; British Film Academy, London, England. National Film Board of Canada, available from NASA Films.

F7 Mystery of Stonehenge

This film could be shown to awaken interest in the explanation of such structures built long ago. The film, B & W, is available from McGraw-Hill Films at a rental of $25. The film comes in two parts each running for 29 minutes. It was filmed by the Columbia Broadcasting System and shown on television in the United States and in Britain. The vigorous conflict of interpretations between Professor Hawkins and others is notable.

F8 Frames of Reference

PSSC, Modern Learning Aids, 28 minutes, B & W, sound. If you haven't shown it previously, Sec. 6.4 might be a good place. Although it presents much more than is necessary, it is an excellent film. It does give students the idea that the appearance of events may depend upon the frame of reference.

F9 Planets in Orbit

EBF, 10 minutes, B & W, sound. In this film are shown animated representations of some of the differences between the Ptolemaic and Copernican system.

F10 Elliptic Orbits

PSSC, Dr. A. V. Baez, Cat. #0310, might be used o make clear to students what area is being discussed in the law of areas. Modern Learning Aids.

F11 Measuring Large Distance

PSSC, Dr. F. G. Watson, Cat. #0103, might be shown here. With a series of models the film stresses the use of triangulation as the primary means for determining large distances. Toward the end of the film other techniques based on photometry are illustrated as means of extending the distance scale when triangulation is no longer possible. Modern Learning Aids.

F12 Of Stars and Men (About Galileo)

It is available from Center for Mass Communication, Columbia University Press, New York City, 10025.

F13 Tides of Fundy

Color, 14 minutes 38 seconds. A fascinating study of the phenomenal tides in the Bay of Fundy on Canada's Atlantic coast and how they affect the life of the region.

An ocean freighter turning about in what seems a hay field, a waterfall reversing its direction—these are only two scenes that will startle and amaze in this film of natural wonders. Animated pictures explain the forces of moon and ocean and earth's rotation that together create in the Bay of Fundy the highest tides in the world.

Other scenes, equally interesting, show the complete engulfing by the sea of the falls of St. John River, the steady advance of a tidal bore.

Filmed with an eye for the dramatic, this film brings to the screen scenes that are truly amazing. It shows, in this tiny pocket of the sea, a sequence of cause and effect that involves the very forces of the universe. It is a film that will appeal to every audience. National Film Board of Canada, available from NASA Films.

F14 Harlow Shapley

Thirty minute film, Encyclopedia Britannica Films, #1806. This film discusses major astronomical discoveries and how they have influenced philosophy, religion and our orientation to the world.

F15 Universal Gravitation

PSSC, 31 minutes, available from Modern Learning Aids, #0309. In this film the law of universal gravitation is derived for an imaginary solar system of one star and one planet.

F16 Forces

This PSSC film (23 minutes) is relevant here. This film introduces mechanics in general and shows a qualitative Cavendish experiment, in which the gravitational force between two small masses is demonstrated. Modern Learning Aids.

F17 The Invisible Planet

As this film opens, students meet Peter Van de Kamp, director of the Sproul Observatory at Swarthmore College, and learn of his interest in Barnard's star, a small star near us in the solar system. With Dr. Van de Kamp and Mr. Herbert as guides, the student learns about the operation of the large refractor telescope, the use of photographic plates, the recording and analysis of data and the results of data carefully recorded for over 25 years of time. From this data, Dr. Van de Kamp and his colleagues were able to determine the apparent presence of a small planet near Barnard's star that causes a small perturbation or wobbling. The precision, time and care in astronomical observations are portrayed with impact in this film. Recommended for use in physics or in earth science courses. NET Film Service.

F18 Close-up of Mars

This is the story of the development of the camera system aboard the spacecraft, Mariner IV, that took the historic photographs of the surface of the planet Mars in mid-July, 1965. The audience follows Robert Leighton, professor of physics at the California Institute of Technology, as he and the scientist-engineers working with him tackle the problem of designing, building and using a camera system that can weight no more than 11 pounds and use only 10 watts of electricity. In viewing this film, students can sense the difficulties surrounding the assignment and the excitement of success as the first films are relayed back to earth from 325 million miles out in space. Particularly recommended for students of physics or electronics. NET Film Service.

F19 Of Stars and Men

53 min, color

Produced and adapted by John and Faith Hubley from the book by Harlow Shapley. In the film, man (king of the animals) helps the audience to locate man's place in the universe of atoms, protoplasm, stars and galaxies. His relationship to space, time, energy, and matter are searched out. Available from Brandon Films Inc.

F20 NEWTON'S EQUAL AREAS

8 minutes, color

Bruce and Katherine Cornwell, Alfred Bork

This animated film is based on Isaac Newton's simple geometrical proof of the law of areas for any central force. It first establishes the laws of motion, in the form needed by Newton, goes through Newton's proof for several different cases, including the limit considerations, and then shows several examples (first simple then complex) of equal areas being traced out with a central force.

Although not produced within the Project, some copies are available for trial teachers. Write to Harvard Project Physics.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION FILMS

Requests for the free loan of NASA films should be addressed to the library assigned responsibility for your area, as indicated by the area map on the opposite page.

Note: there is no service area #3.

IF YOU LIVE IN	SERVICE AREA	ADDRESS YOUR REQUEST TO
Washington, Oregon, Idaho, Montana, Wyoming, No. California (north of the Los Angeles Metropolitan area), Alaska	#1	NASA Ames Research Center Public Affairs Office Moffett Field, California 94035
Arizona, southern California, (San Luis Obispo, Kings, Tulare and Inyo Counties), Hawaii, Nevada and Utah	#2	NASA Western Operations Office Public Affairs Office 150 Pico Boulevard Santa Monica, California 90406 Also NASA Jet Propulsion Laboratory Photographic Services 4800 Oak Grove Drive Pasadena, California 91103
Arkansas, Missouri, Texas, Oklahoma, Kansas, Nebraska, New Mexico, Colorado	#4	NASA Manned Spacecraft Center Public Affairs Office, AP2 Houston, Texas 77058
Alabama, Mississippi, Tennessee, Louisiana, Kentucky	#5	NASA Marshall Space Flight Center Public Affairs Office Community Services Huntsville, Alabama 35812
Ohio, Indiana, Illinois, Wisconsin, Michigan, North Dakota, South Dakota, Minnesota, Iowa	#6	IDEAL PICTURES, INC. 2110 Payne Avenue Cleveland, Ohio 44114 Phone: MA 1-9173
Southern Virginia (Richmond-South), West Virginia, North Carolina, South Carolina	#7	NASA Langley Research Center Public Affairs Office Mail Stop 154 Langley Station Hampton, Virginia 23365
Florida, Bermuda, Georgia	#8	Photographic Operations Section NASA John F. Kennedy Space Center Code SOP 323 Kennedy Space Center, Florida 32809
Maryland, Delaware	#9	NASA Goddard Space Flight Center Photographic Branch, Code 253 Greenbelt, Maryland 20771
North Virginia (north of Richmond), D. C., Pennsylvania, New Jersey, New York, Canada, Latin Americas and Overseas	#10	NASA Headquarters Code FAD-2 Washington, D. C. 20546

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION FILMS (continued)

IF YOU LIVE IN	SERVICE AREA	ADDRESS YOUR REQUEST TO
Maine, Vermont, New Hampshire, Connecticut, Massachusetts, Rhode Island	#11	NASA Electronics Research Center Educational Programs Office 575 Technology Square Cambridge, Massachusetts 02139

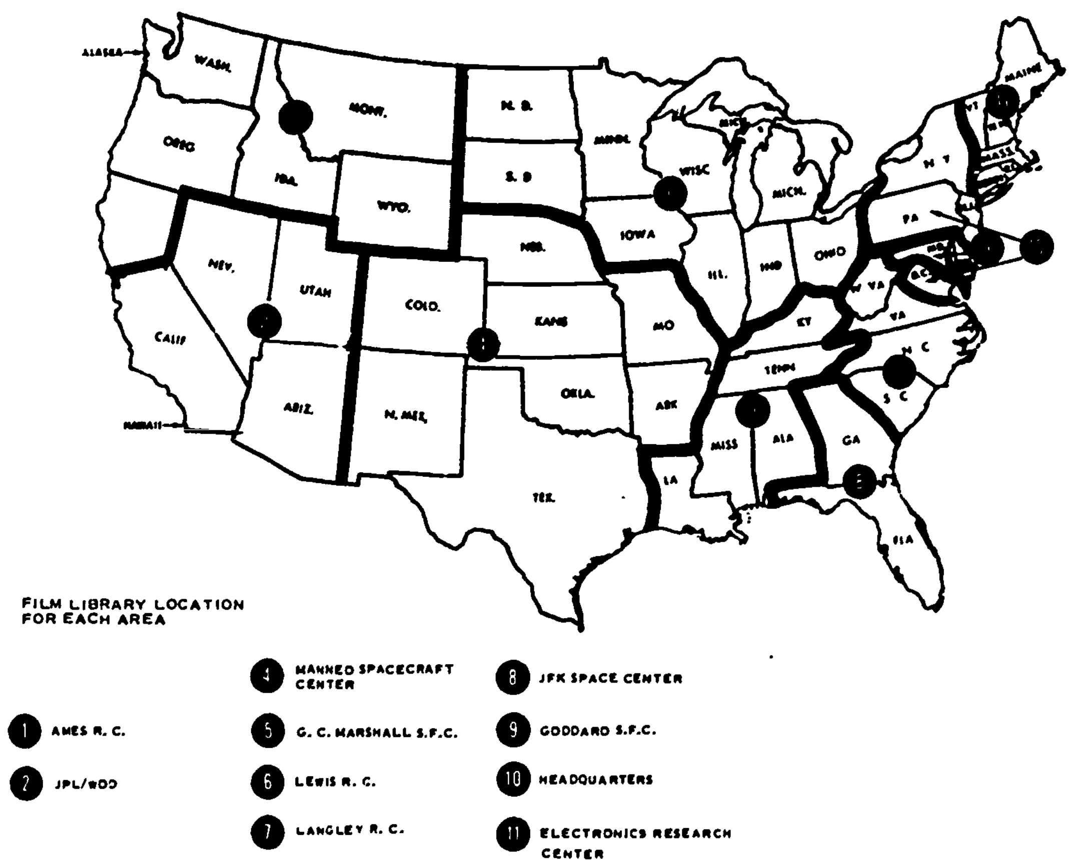
WHO MAY BORROW FILMS FROM NASA

Residents of the United States and Canada, who are bona fide representatives of educational, civic, industrial, professional, youth activity and government organizations are invited to borrow films from the NASA Film Library which services their area. There is no film rental charge, but the requestor must pay return postage and insurance costs. In view of the wear and tear that results from repeated projection, films are loaned for group showings and not for screening before individuals or in homes. Because custody of the films involves both legal and financial responsibility, films cannot be loaned to minors.

To expedite shipment of film, requestor should give name and address of person and organization specifying showing date and alternate date. It is also advisable to indicate a substitute film.

Television stations may order films unless otherwise noted, for unsponsored public service or sustaining telecasts.

NASA MOTION PICTURE FILM SERVICE AREAS



PSSC FILM SOURCES

PSSC films are available from

Modern Learning Aids
1212 Avenue of the Americas
New York, New York 10036

To order prints or for additional information, contact your MLA representative.

District Offices

714 Spring Street N.W.
Atlanta, Georgia 30308

1168 Commonwealth Avenue
Boston, Massachusetts 02134

160 E. Grand Avenue
Chicago, Illinois 60611

1411 Slocum Street
Dallas, Texas 75207

16 Spear Street
San Francisco, California 94105

2100 North 45th Street
Seattle, Washington 98103

1834 K Street N.W.
Washington, D.C. 20006

1875 Leslie Street
Don Mills, Ontario, Canada

ADDITIONAL FILM SOURCES FROM WHICH PHYSICS FILMS ARE AVAILABLE

Physics films may be obtained from the distributors listed below. However, these films have not as yet been reviewed.

AEC: Atomic Energy Commission Film Libraries. Lists available from Mr. Frank T. Richardson, Director, Public Information Service, U. S. Atomic Energy Commission, New York Operations Office, 376 Hudson Street, New York, New York 10014.

American Telephone and Telegraph Company, 208 West Washington Street, Chicago, Illinois. (Contact local office.)

Bausch & Lomb, Inc., Rochester, New York.

Brandon Films, inc., 200 West 57th Street, New York, N.Y. 10019

British Information Services, 45 Rockefeller Plaza, New York, New York. (Films distributed by Contemporary Films Inc., 267 25th Street, New York, New York.)

Carousel Films, Inc., 1501 Broadway, Suite 1503, New York, New York.

Center for Mass Communication, Columbia University Press, New York, New York 10025.

Contemporary Films, Inc., 267 West 25th Street, New York, New York.

Convair Motion Picture Section, Department 98-80, P. O. Box 1950, San Diego, California.

Corning Glass Works, Advertising Department, Technical Products Division, Chicago, Illinois.

Coronet, 42 Midland Road, Roslyn Heights, New York.

Edited Picture System, Inc., 165 West 46th Street, New York, New York.

EBF: Encyclopaedia Britannica Films, 38 West 32nd Street, New York, New York 10001.

First U. S. Army Central Film and Equipment Exchange, Fort Jay, Governors Island, New York, New York; Regional Film and Equipment Exchange, Schenectady General Depot, U. S. Army, Schenectady, New York.

General Electric Company, Film Distribution Section, Building 6, Room 210, 1 River Road, Schenectady, New York.

ADDITIONAL FILM SOURCES FROM WHICH PHYSICS FILMS ARE AVAILABLE (continued)

IFB: International Film Bureau, Inc., 332 South Michigan Avenue, Chicago, Illinois 60604.

Imperial Chemical Industries, Ltd., 488 Madison Avenue, New York, New York.

Leeds and Northrup Company, 4901 Stenton Avenue, Philadelphia, Pennsylvania.

MIT: Massachusetts Institute of Technology (Office of John J. Rowlands), 69 Massachusetts Avenue, Cambridge, Massachusetts.

McGraw-Hill Book Company, Text Film Department, 330 West 42nd Street, New York, New York.

Moody Institute of Science, 11428 Santa Monica Boulevard, P. O. Box 25575, Los Angeles, California 90025.

National Film Board of Canada, Suite 819, 680 Fifth Avenue, New York, New York.

NSTA: National Science Teachers Association, 1201 16th Street, N. W., Washington, D. C.

NET Film Service, Audio-Visual Center, Indiana University, Bloomington, Indiana 47401.

RCA Victor Division, Camden, New Jersey.

Rocketdyne Division of North American Aviation, Inc., Public Relations Department, 6633 Canoga Avenue, Canoga Park, California.

Scientific Film Company, 6804 Windsor Avenue, Berwyn, Illinois.

Shell Oil Company, 50 West 50th Street, New York, New York.

Sperry Gyroscope Company, Central Film Service, Great Neck, New York.

Teaching Film Custodians, Inc., 25 West 43rd Street, New York, New York.

United World Films, 1445 Park Avenue, New York, New York.

University Film Producers Association, Charles N. Hockman, President, Motion Picture Production, Extension Division, University of Oklahoma, Norman, Oklahoma.

U. S. Navy, Motion Picture Section, Office of Public Information, Executive Office of the Secretary, Washington, D. C.

Western Electric Company, Inc., Coordinator of College Relations, 195 Broadway, New York, New York.

Westinghouse Electric Corporation, Film Division, 3 Gateway Center, Pittsburgh, Pennsylvania.

Yale University Audio-Visual Center, Attention: Mr. David G. Anderson, Sterling Chemistry Laboratory, New Haven, Connecticut.

Reader

1. "The Black Cloud"

by Fred Hoyle

1957

In this introductory chapter to his science fiction novel, the noted astronomer Fred Hoyle gives a realistic picture of what goes on within an astronomy laboratory. The emphasis is on experimental astronomy.

2. "Roll Call"

by Isaac Asimov

1963

This pleasant introduction to the planets and the solar system is by a writer well known as a scientist, a popularizer of science and a writer of science fiction. Asimov approaches the solar system historically, briefly considering the discovery of some of the planets.

3. "A Night at the Observatory"

by Henry S. F. Cooper, Jr.

1967

What is it like to work at a major observatory? A reporter spends a night on Mt. Palomar talking about astronomy with Dr. Jesse L. Greenstein as he photographs star spectra with the 200-inch telescope.

4. "Preface to De Revolutionibus"

by Nicolaus Copernicus

1543

Copernicus addresses this preface of his revolutionary book on the solar system to Pope Paul III.

5. "The Starry Messenger"

by Galileo Galilei

1610

The introduction to Galileo's Starry Messenger not only summarizes his discoveries, but also conveys Galileo's excitement about the new use of the telescope for astronomical purposes.

6. "Kepler's Celestial Music"

by I. Bernard Cohen

1960

The end of this summary of Kepler's work in mechanics shows how seriously Kepler took the idea of the harmony of the spheres.

7. "Kepler"

by Gerald Holton

1960

This brief sketch of Johannes Kepler's life and work was initially written as a review of Max Caspar's definitive biography of Kepler.

8. "Kepler on Mars"

by Johannes Kepler (translated by Owen Gingerich)

1609

Kepler's description of how he came to take up the study of Mars, from his greatest book, The New Astronomy. Kepler records in a personal way everything as it occurred to him, not merely the final results.

9. "Newton and the Principia"

by C. C. Gillispie

1960

This article describes briefly the events which transpired immediately before the writing of the Principia.

10. "The Laws of Motion and Proposition One"

by Isaac Newton

1687

The Latin original of Newton's statement of the three laws of motion and the proof of proposition one is followed here by the English translation by Andrew Motte and Florian Cajori.

11. "The Garden of Epicurus"

by Anatole France

1920

Anatole France is best known as the writer of novels such as Penguin Island. This brief passage shows that he, along with many writers, is interested in science.

12. "Universal Gravitation"

by Richard P. Feynman, Robert B. Leighton and Matthew Sands

1964

A physical concept, such as gravitation, can be a powerful tool, illuminating many areas outside of that in which it was initially developed. As the authors show, physicists can be deeply involved when writing about their field.

13. "An Appreciation of the Earth"

by Stephen H. Dole

1964

The earth, with all its faults, is a rather pleasant habitation for man. If things were only slightly different, our planet might not suit man nearly as well as it now does.

14. "A Search for Life on Earth at Kilometer Resolution"

by Stephen D. Kilston, Robert R. Drummond and Carl Sagan

1965

15. "The Boy who Redeemed his Father's Name"

by Terry Morris

1966

A dramatized account of the boyhood of the Japanese astronomer who discovered a recent comet. This same comet, Ikeya-Seki, is described also in the article by Owen Gingerich.

16. "The Great Comet of 1965"

by Owen Gingerich

1966

The director of the Central Bureau for Astronomical Telegrams describes the excitement generated by a recent comet, and reviews current knowledge of comets.

17. "Gravity Experiments"

by R. H. Dicke, P. G. Roll and J. Weber

1966

The delicate modern version of the Eötvös experiment described here shows that the values of inertial mass and gravitational mass of an object are equal to within one ten-billionth of a percent. Such precision is seldom attainable in any area of science.

18. "Space the Unconquerable"

by Arthur C. Clarke

1962

Arthur Clarke began to think seriously about space travel before almost anyone else. His conclusions, as seen in the article s very first sentence, are somewhat more pessimistic than are now fashionable.

19. "Is there Intelligent Life Beyond the Earth?"

by I. S. Shklovskii and Carl Sagan

1966

Many scientists have argued recently that intelligent life may be quite common in the universe. This work was originally by Shklovskii, in Russian, and the "Annotations, Additions, and Discussions" which Sagan has added are bracketed by the symbols ∇ and Δ.

20. "The Stars within Twenty-two Light Years that Could Have Habitable Planets"

by Stephen H. Dole

1964

This table lists only those stars within twenty-two light years of the earth that have probabilities for the existence of planets which could support human life. The reader with astronomical interests should scan books on astronomy for a detailed explanation of most of the terminology used in this table.

21. "U.F.O."

by Carl Sagan

1967

The existence or non-existence of unidentified flying objects has been a subject for debate in the United States and elsewhere for many years. Here an astronomer reviews our current knowledge in an impartial way.

22. "The Life-story of a Galaxy"

by Margaret Burbidge

1962

A noted woman astronomer discusses current knowledge, and lack of knowledge, concerning the evolution of galaxies. Dr. Burbidge concludes, "It is difficult to understand in detail how one sort of galaxy can evolve into another, yet in a general way we know that it must happen."

23. "Expansion of the Universe"

by Hermann Bondi

1960

Bondi, a noted theoretical physicist and astronomer, presents the evidence for the overall expansion of the universe, evidence which depends greatly on the observed red shift of light from distant galaxies. The number mentioned at the end of the paper, ten billion years, is sometimes picturesquely called the "age of the universe."

24. "Negative Mass"

by Banesh Hoffmann

1965

Does mass, like electric charge, exist in both positive and negative forms? If so, negative mass must have the most extraordinary properties—but they could explain the immense energies of the star-like objects known as quasars.

25. "Three Poetric Fragments about Astronomy"

    From Troilus and Cressida
    by William Shakespeare

    From Hudibras
    by Samuel Butler

    My Father's Watch
    by John Ciardi

26. "The Dyson Sphere"

    by I. S. Shklovskii and Carl Sagan

    1966

The imagination of scientists often exceeds that of the science fiction writer. The question asked is how an advanced technological civilization could capture most of the sun's energy.

D28 Phases of the Moon

The following model will help to clarify the phases of the moon. Attach a ping-pong or tennis ball to a thread. Then in a darkened room have students watch the phases of this moon as you swing it:

a) around a single lamp bulb (not too bright), or

b) around their heads.

The latter matches best with our observations of the moon's phases.

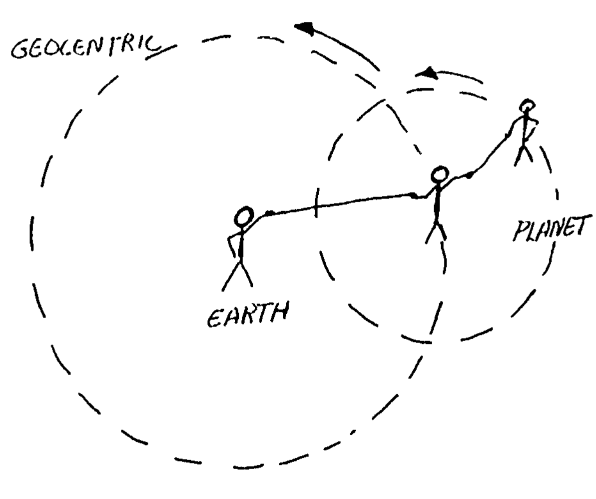
D29 Geocentric-Epicycle Model (an alternative to Demonstrations 5.5A, 5.6A)

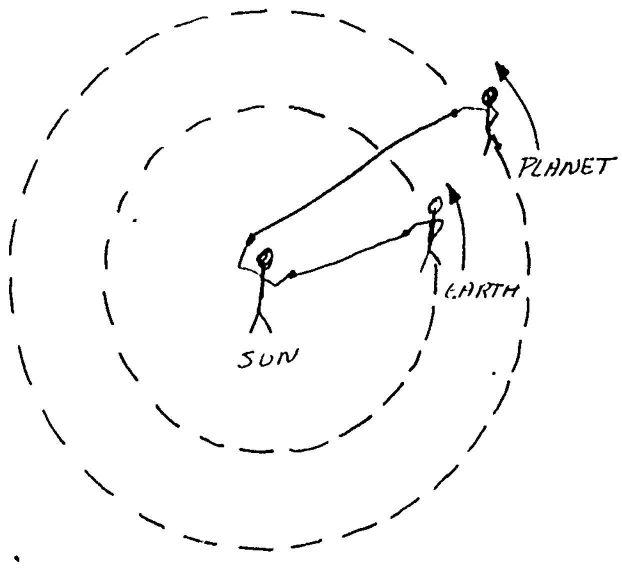
In this demonstration students themselves play the part of planets. As well as showing that the geocentric-epicycle model of the solar system gives retrograde motion, it demonstrates the effect that an observer's motion can have on his view of another object's motion.

Student A, representing the earth, stands still while two others, B and C, move around him—B in a circle, and C, representing a planet, in an epicycle. A length of string (about 5 yards) between A and B, and a shorter one (about 2½ yards) between B and C keep the radii of the circles constant. Student C will have to move fairly fast to make one or more revolutions about B while B, walking at a steady rate, makes one revolution about A. Once they have established appropriate speeds, they must try to maintain them as constant as possible. In this demonstration A is the earth, B is merely a point in space, C is a planet. "Earth" observes the motion of the "planet" with reference to a distant [illegible] ound—trees, goalposts, school [illegible] ngs, etc.—"the fixed stars." [illegible]es the planet always appear to be moving in the same direction? When does it retrograde? How long does the retrograde motion last (see diagram)?

D30 Heliocentric Model

In this model the stationary student A represents the sun. The earth, now displaced from its position at the hub of the universe, moves round in a circle (radius about 4 yards). The third student—representing Mars—moves around the sun in the same sense in a larger circle (6 yards). If earth and Mars walk in step, but with Mars taking a shorter step, earth's period will be considerably less than Mars and their relative motions will approximate fairly well to actual movement of the two planets. Again earth is asked to describe the relative motion of Mars as it appears to him against the distant background of fixed stars. Does the motion appear uniform? Is there retrograde motion? When does it occur? Retrograde motion of an inner planet may be more difficult to spot. Try these parameters: earth's orbit—10-yard radius; Mercury's orbit—4-yard radius; Mercury takes two paces for every one taken by earth (see diagram).





D31 Plane Motions

The importance of Kepler's use of motion in a plane cannot be overstressed. With Unit 1 in mind, have students make a fist with their left hand to represent the sun. Have them hold a pen or pencil in their right hand to represent a point in space and a velocity vector. You can anticipate Chapter 8 and ask students what forces are acting on the body. (The only force is the central pull of the sun). What initial motion does the body have? (The initial velocity vector is represented by the pen or pencil.) But one point and a line define a plane. What would you infer if the body did NOT move in a plane? (Some other force acting from a place not in the orbital plane.) This planar assumption is applied in Experiment 18 when the orbital inclination of Mars is derived from the observations of the positions of Mars north or south of the ecliptic.

D32 Conic Section Model

If the mathematics department has a model of a cone, use it to let the student see the natural occurrence of ellipses and other conic sections.

Introduction to Unit 2 Experiments

The following pages on experiments include both the directions for the student (centered on facing pages) and the notes for the teacher on the outside areas of these pages. There is sufficient blank space for the teacher to add his own notes and he is encouraged to do so. Essential experiments carry a star *.

EXPERIMENT 1 Naked-Eye Astronomy (continued from Unit 1 Student Handbook)

At the very beginning of this course, it was suggested that you might want to begin making some basic astronomical observations in order to become familiar with the various objects in the heavens and with the ways in which these objects move. Now the time has come to analyze your data more carefully and to continue your observations. From observations much like your own, scientists in the past have developed a remarkable sequence of theories. The more aware you are of the motions in the sky, the more easily you can follow the development of these theories.

If you have been careful and thorough in your data-taking, (and if the weather has been mostly favorable), you have your own data for analysis. If, however, you do not have your own data, similar results are provided in the following sections.

See Teacher Guide page 65.

a) One Day of Sun Observations

One student made the following observations of the sun's position during September 23.

Eastern Daylight Time	Sun's Altitude	Sun's Azimuth
7:00 A.M.	---	---
8:00	08°	097°
9:00	19	107
10:00	29	119
11:00	38	133
12:00	45	150
1:00 P.M.	49	172
2:00	48	197
3:00	42	217
4:00	35	232
5:00	25	246
6:00	14	257
7:00	03	267

If you plot altitude vs. azimuth and mark the hours for each point, you will be able to answer these questions.

1. What was the sun's greatest altitude during the day?

2. What was the latitude of the observer?

3. At what time (EDT) was the sun highest?

4. When during the day was the sun's direction (azimuth) changing fastest?

5. When during the day was the sun's altitude changing fastest?

6. Remember that daylight time is an hour ahead of standard time. On September 23 the apparent sun, the one you see, gets to the meridian 8 minutes before the mean sun. Can you determine the longitude of the observer? Near what city was he?

b) A Year of Sun Observations

See Teacher Guide page 66.

One student made the following monthly observations of the sun through a full year. (He had remarkably good weather!)

Dates	Sun's Noon Altitude	Sunset Azimuth	Interval to Sunset After Noon
Jan 1	20°	238°	4^h25^m*
Feb 1	26	245	4 50
Mar 1	35	259	5 27
Apr 1	47	276	6 15
May 1	58	291	6 55
Jun 1	65	300	7 30
Jul 1	66	303	7 40
Aug 1	61	295	7 13
Sep 1	52	282	6 35
Oct 1	40	267	5 50
Nov 1	31	250	5 00
Dec 1	21	239	4 30

*h = hour, m = minutes.

In terms of the dates make three plots (different colors or marks on the same sheet of graph paper) of the sun's noon altitude, direction at sunset and time of sunset after noon.

1. What was the sun's noon altitude at the equinoxes (March 21 and September 23)?

2. What was the observer's latitude?

3. If the observer's longitude was 79°W, near what city was he?

4. Through what range (in degrees) did his sunset point change during the year?

5. By how much did the observer's time of sunset change during the year?

6. If the interval between sunrise and noon equalled the interval between noon and sunset, how long was the sun above the horizon on the shortest day? On the longest day?

See Teacher Guide page 67.

c) Moon Observations

During October 1966 a student in Las Vegas, Nevada made the following observations of the moon at sunset when the sun had an azimuth of about 255°.

Date	Angle from Sun to Moon	Moon Altitude	Moon Azimuth
Oct.			
16	032°	17°	230°
18	057°	25	205
20	081°	28	180
22	104°	30	157
24	126°	25	130
26	147°	16	106
28	169°	05	083

1. Plot these positions of the moon on the chart.

2. From the data and your plot, estimate the dates of new moon, first quarter moon, full moon.

3. For each of the points you plotted, sketch the shape of the lighted area of the moon.

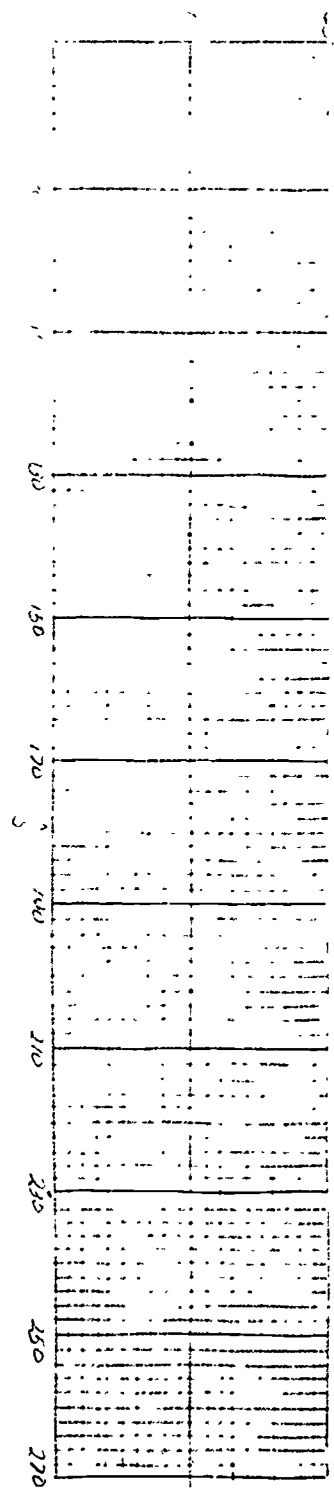


CHART FOR PLOTTING MOON OBSERVATIONS

See Teacher Guide page 67.

d) Locating the Planets

Table I in the appendix tells you where to look in the sky to see each of the planets whose wanderings puzzled the ancients. One set of positions is given, accurate to the nearest degree, for every ten-day interval; by interpolation you can get the planets' positions on any given day.

The column headed "J.D." shows the corresponding Julian Day calendar date for each entry. This calendar is simply a consecutive numbering of days that have passed since an arbitrary "Julian Day 1" in 4713 B.C.: September 26, 1967, for example, is the same as J.D. 2,439,760.

Look up the sun's present longitude in the table. Locate the sun on your SC-1 Constellation Chart: its path, the ecliptic, is the curved line marked off in 360 degrees; these are the degrees of longitude.

A planet that is just to the west of the sun's position (to the right on the chart) is "ahead of the sun," that is, it rises just before the sun does. One that is 180° from the sun rises near sundown and is in the sky all night.

When you have decided which planets may be visible, locate them along the ecliptic. Unlike the sun, they are not exactly on the ecliptic, but they are never more than eight degrees from it. The Constellation Chart shows where to look among the fixed stars.

See Teacher Guide page 67.

e) Graphing the Position of the Planets

Here is a useful graphical way to display the information in the planetary longitude table in the appendix.

On ordinary graph paper, plot the sun's longitude versus time. Use Julian Day numbers along the horizontal axis, beginning as close as possible to the present date. The plotted points should fall on a nearly straight line, sloping up toward the right until they reach 360° and then starting again at zero. (See Fig. 1.)

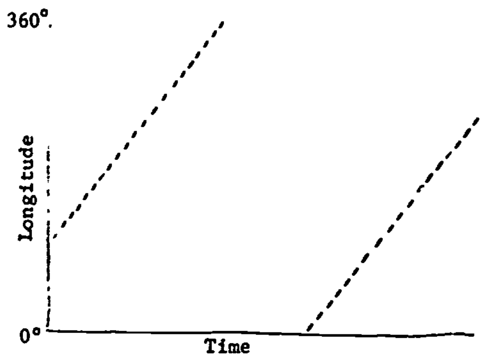


Fig. 1 The sun's longitude

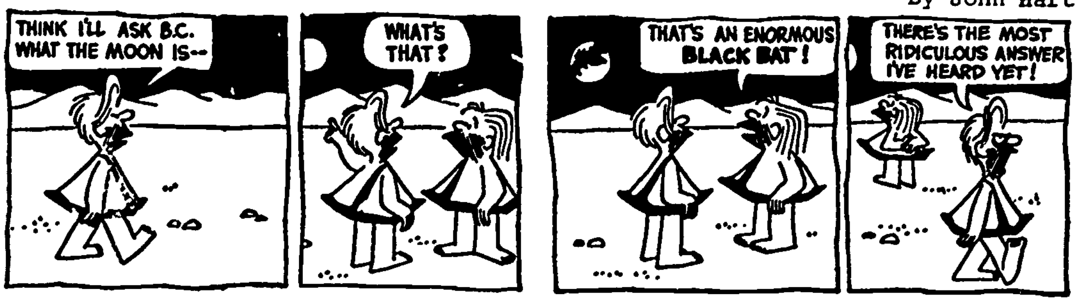
How long will it be before the sun again has the same longitude as it has today? Would the answer to that question be the same if it were asked three months from now? What is the sun's average angular speed over a whole year? By how much does its speed vary? When is it fastest?

Plot Mercury's longitudes on the same graph (use a different color or shape for the points). According to your plot, how far (in longitude) does Mercury get from the sun? (This is Mercury's maximum elongation.) At what interval does Mercury pass between the earth and the sun (forget for the time being about latitudes)? (This interval is Mercury's synodic period: the period of phases.)

Plot the positions of the other planets (use a different color for each one). The resulting chart is much like the data that puzzled the ancients. In fact, the table of longitudes is just an updated version of the tables that Ptolemy, Copernicus and Tycho made.

The graph contains a good deal of useful information. For example, when will Mercury and Venus next be close enough to each other so that you can use bright Venus to help you find Mercury? Can you see from your graphs how the ancient astronomers decided on the relative sizes of the planetary spheres? (Hint: look at the relative angular speeds of the planets.) Where are the planets, relative to the sun, when they go through their retrograde motions?

B.C. By John Hart



ANSWERS

EXPERIMENT 1: NAKED-EYE ASTRONOMY (continued from Unit 1 Student Handbook)

I. ONE DAY OF SUN OBSERVATIONS

1. About 49°.

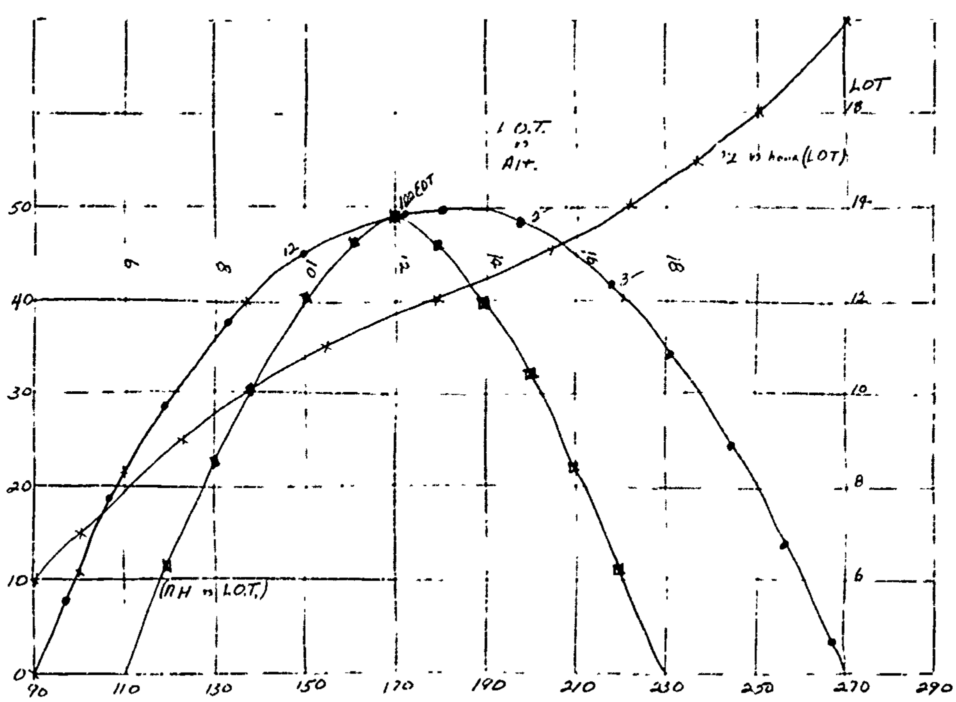
2. Since the sun was on the equator September 23, the latitude of the observer is 90° - sun's noon altitude, or 90° - 49° = 41° N.

3. The sun was highest at about 1:00 EDT.

4. The sun's azimuth changes most rapidly around noon.

5. The sun's altitude changes most rapidly when the sun is near the horizon—after sunrise and before sunset.

6. Local noon (highest sun) occurred about 1:00 EDT or at 12:00 EST. But the mean sun, on which our time is kept, would cross the meridian 8^m later, or at 12:08 EST The observer was therefore 8^m west of the mid-line for the EST zone. Since 4^m = 1° of longitude, the 8^m = 2° west. The mid-longitude for EST is 75° W. Then the observer must have been at longitude 75° W + 2 = 77° W. and at latitude 41° N. This is near Akron, Ohio.



ONE DAY OF SUN OBSERVATIONS—TEACHER GUIDE

II. A YEAR OF SUN OBSERVATIONS

1. About 46°.

2. Latitude = 90° - sun's noon altitude on March 21 or September 23

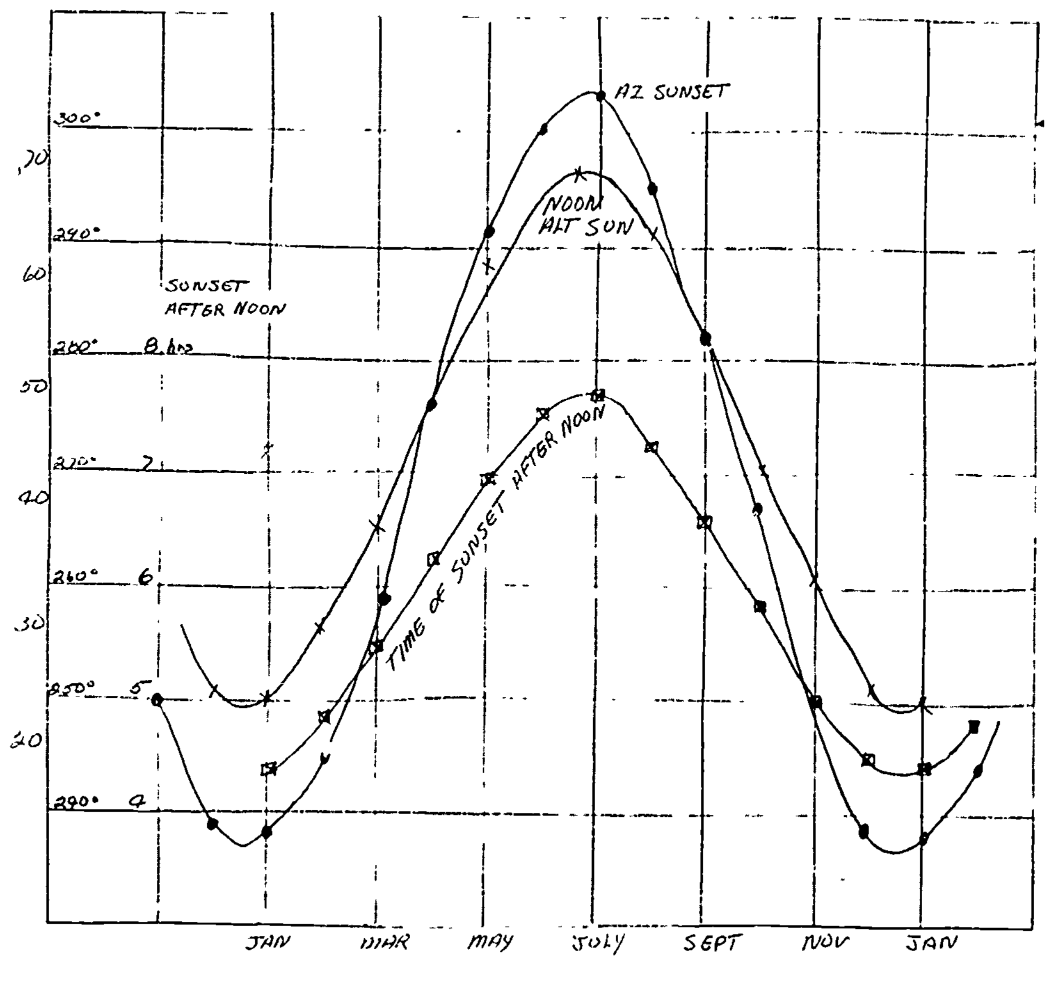
= 90° - 46° = 44°

3. He was near Toronto or Buffalo, New York.

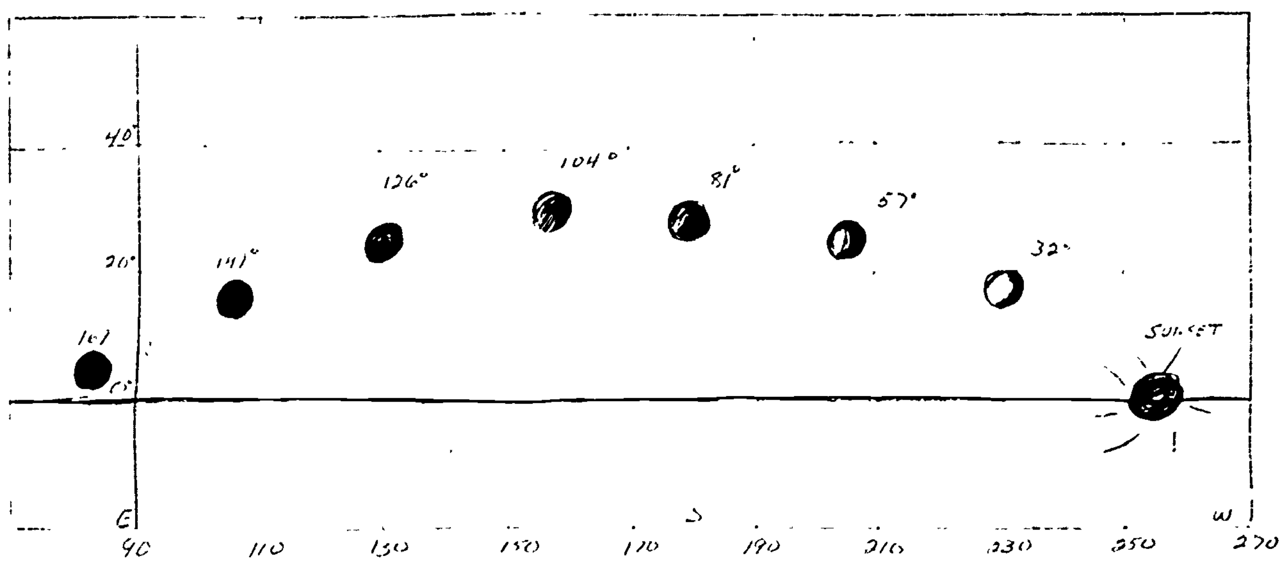
4. The sunset point changed in azimuth from about 237° in December to about 303° in June, or a change of 303° - 237° = 66°.

5. The interval between noon and sunset changed from about 4^h25^m in December to about 7^h40^m in June, or a change of 3^h15^m during the year.

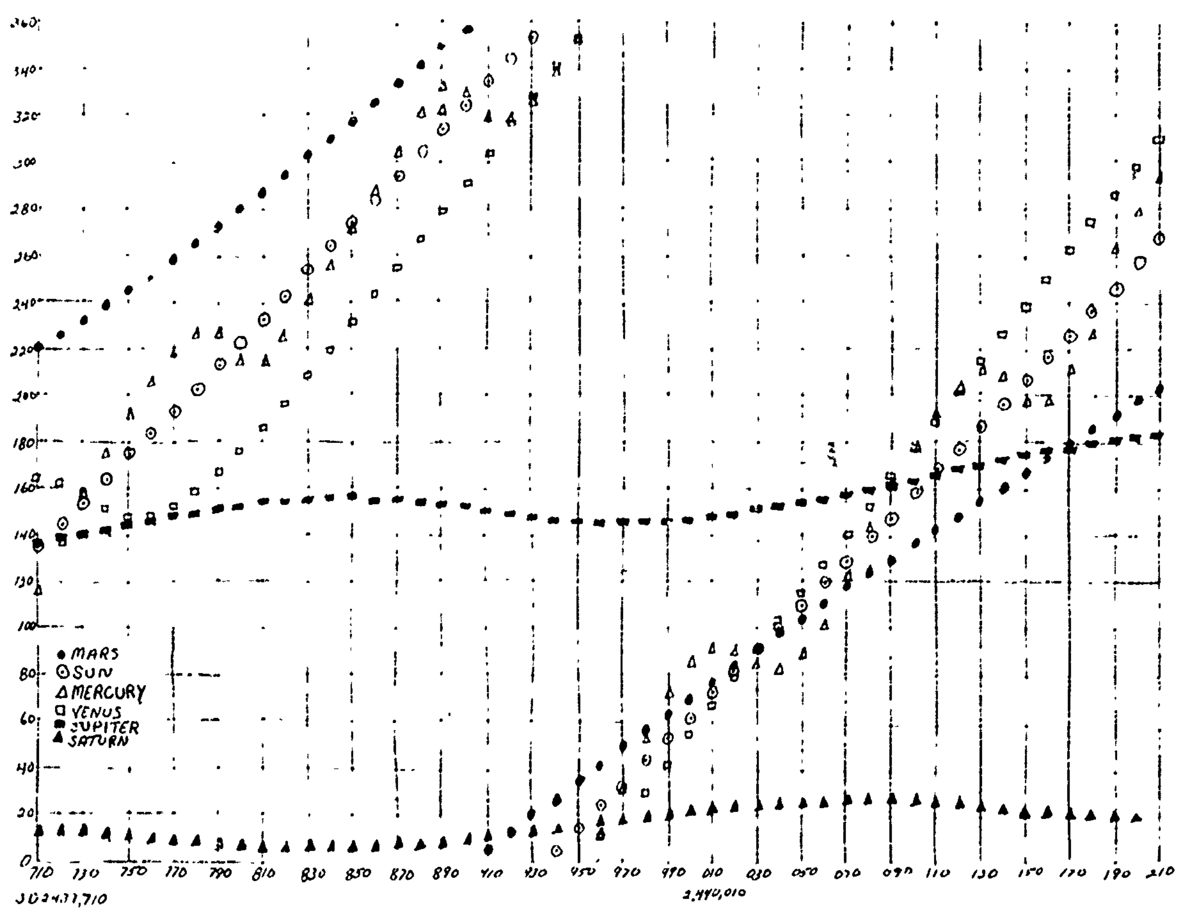
6. On the shortest day the sun was up for 2 × 4^h25^m or about 8^h50^m. On the longest day the sun was up for 2 × 7^h40^m or about 15^h20^m.



A YEAR OF SUN OBSERVATIONS—TEACHER GUIDE



d) & e) PLOTTING PLANET POSITIONS



Eratosthenes used two points in Egypt, Alexandria, on the Mediterranean Sea, and Syene, which was near the present location of the Aswan Dam. He measured the distance between these points in "stadia." Unfortunately we do not know the proper conversion from his stadia to our miles. According to one interpretation, Erathosthanes' value for the circumference of the earth was about 20% larger than the modern value. However, according to another interpretation, his value agrees with the modern value to about 1%.

EXPERIMENT 13 The Size of the Earth

The first recorded approximate measurement of the size of the earth was made by Eratosthenes in the third century B.C. His method was to compare the lengths of shadows cast by the sun at two different points rather far apart but nearly on a north-south line on the earth's surface. The experiment described here uses an equivalent method. Instead of measureing the length of a shadow you will measure the angle between the vertical and the sight line to a star or the sun.

You will need a colleague at least 200 miles away due north or south of your position to take simultaneous measurements. You will need to agree in advance on the star, the date and the time for your observations.

Theory

The experiment is based on the assumptions that

1) the earth is a perfect sphere.

2) a plumb line points towards the center of the earth.

3) the distance from stars and sun to the earth is great compared with the earth's diameter.

The two observers must be located at points nearly north and south of each other (i.e., they are nearly on the same meridian). They are separated by a distance s along that meridian. You (the observer at A) and the observer at B both sight on the same star at the prearranged time, preferably when the star is on or near the meridian.

Each of you measures the angle between the vertical of his plumb line and the sight line to the star.

Light rays from the star reaching locations A and B are parallel (this is implied by assumption 3).

You can therefore relate the angle at A, θ_A, to the angle at B, θ_B, and the angle between the two radii, ϕ, as shown in Fig. 1.

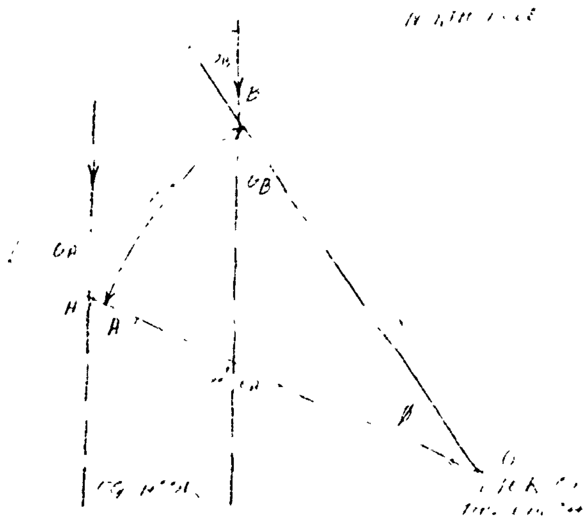


Fig. 1

In the triangle A'BO

$$\phi = (\theta_A - \theta_B) \quad (1)$$

If C is the circumference of the earth, and s is an arc of the meridian, then

$$\frac{s}{C} = \frac{\phi}{360} . \quad (2)$$

Combining Eqs. (1) and (2),

$$C = \left(\frac{360}{\theta_A - \theta_B} s \right)$$

where θ_A and θ_B are measured in degrees.

Procedure

For best results, the two locations A and B should be directly north and south of each other. The observations are made just as the star crosses the local meridian, that is, when it reaches its highest point in the sky.

You will need some kind of instrument to measure the angle θ. One such instrument is an astrolabe. One can be made fairly easily from the design in Fig. 2.

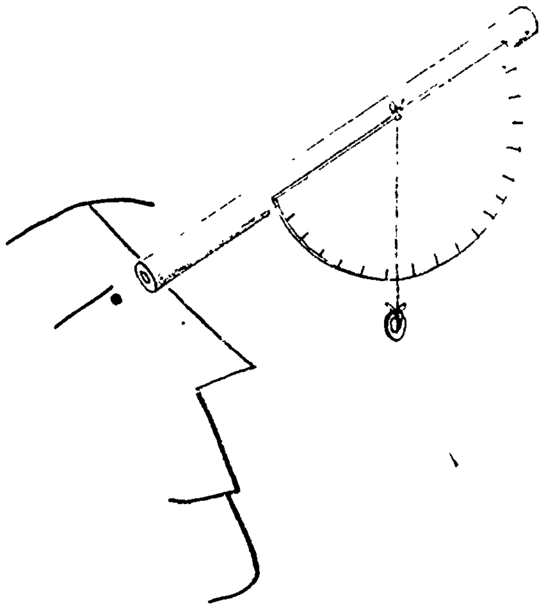


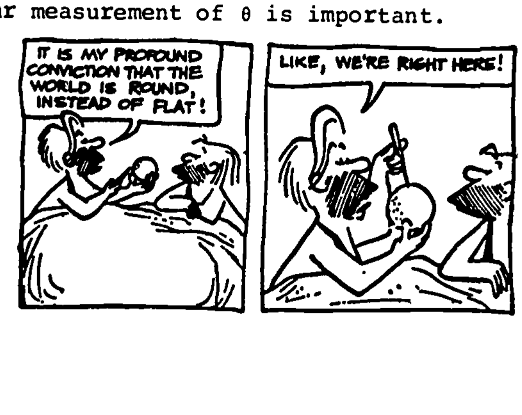
Fig. 2

Align your astrolabe along the meridian (north-south line) and measure the angle from the vertical to the star as it crosses the meridian.

If the astrolabe is not aligned along the meridian, the altitude of the star will be observed before or after it is highest in the sky. An error of a few minutes from the time of transit will make little difference.

An alternative method would be to measure the angle to the sun at (local) noon. (Remember that this means the time when the sun is highest in the sky, and not necessarily 12 o'clock.) You could use the shadow theodolite described in Experiment 1. Remember that the sun, seen from the earth, is itself ½° wide. Do not try sighting directly at the sun. You may damage your eyes.

An estimate of the uncertainty in your measurement of θ is important.



Take several measurements on the same star (at the same time) and take the average value of θ. Use the spread in values of θ to estimate the uncertainty of your observations and of your result.

If it can be arranged, you should exchange instruments with your colleague at the other observing position and repeat the measurements on the same star on another night. (Remember that it will cross the meridian 4 minutes earlier every night, so your time of observation will be different.) By taking the average of the two values of θ given by the different instruments you can eliminate errors due to differences in construction between the two.

The accuracy of your value for the earth's circumference also depends on knowing the over-the-earth distance between the two points of observation. See: "The Shape of the Earth," Scientific American, October, 1967, page 67.

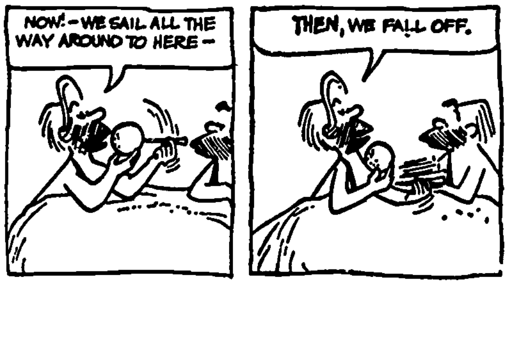
Q1 How does the uncertainty of the over-the-earth distance compare with the uncertainty in your value for θ?

Q2 What is your value for the circumference of the earth and what is the uncertainty of your value?

Q3 Astronomers have found that the average circumference of the earth is about 24,900 miles. What is the percentage error of your result?

Q4 Is this acceptable, in terms of the uncertainty of your measurement?

B.C. By John Hart



For Discussion

It is often said that Columbus' voyage to the New World was a daring feat because it was not known at the time (1492) that the earth was spherical. However, by 250 B.C., the Greeks concluded that the earth was a sphere. What observations may have caused them to make that conclusion? How might you explain the popular concept of Columbus' time that the earth was flat and that one could fall off the edge?

Since about 1673 scientists have known that the earth is slightly oblate, i.e., the polar diameter is slightly less than the equatorial diameter. At the pole there are about 69.4 miles per degree of latitude along the earth's surface; while there are only about 68.7 miles per degree at the equator.

This is an example of indirect measurement. Our ability to make measurements by direct methods (i.e., direct application of our senses) is severely limited. We can't see very small things. A mountain is just too large to measure by direct means. Some objects, such as a cloud, are just too inaccessible to measure. In each of these cases and in countless others, we use indirect methods to "estimate" dimensions. In this exercise students combine observations with a geometrical model to measure a very large, very inaccessible object.

The photograph on page 96 of Unit 1 shows the large crater Copernicus which is near the equator in the moon's Eastern hemisphere (3rd quarter).

EXPERIMENT 14 The Height of Piton, A Mountain on the Moon

You have probably seen photographs of the moon's surface radioed back to earth from an orbiting space ship or from a vehicle that has made a "soft landing" on the moon. The picture on page 95 of the Unit 1 text shows an area about 180 miles across and was taken by Lunar Orbiter II from a height of 28.4 miles.

But even before the space age, astronomers knew quite a lot about the moon's surface. Galileo's own description of what he saw when he first turned his telescope to the moon is reprinted in Sec. 7.8 of Unit 2.

From Galileo's time on, astronomers have been able, as their instruments and techniques improved, to learn more and more about the moon, without ever leaving the earth.

In this experiment you will use a photograph taken with a 36-inch telescope in California to estimate the height of a mountain on the moon. Although you will use a method similar in principle to Galileo's, you should be able to get a much more accurate value than he could working with a small telescope (and without photographs!).

Materials

You are supplied with a photograph of the moon taken at the Lick Observatory near the time of the third quarter. The North Pole is at the bottom of the photograph. This is because an astronomical telescope, whether you look through it or use it to take a photograph, gives an inverted image.

Why Choose Piton?

Piton, a mountain in the northern hemisphere, is fairly easy to measure because it is a slab-like pinnacle in an otherwise fairly flat area. It is

quite close to the terminator (the line separating the light portion from the dark portion of the moon) at third quarter phase.

You will find Piton on the moon photograph above (to the south) and left of the large crater Plato at the moon latitude of about 40° N. Plato and Piton are both labeled on the moon map.

The important features of the photograph are shown in Fig. 1. The moon is a sphere of radius R. Piton (P) is a distance s from the terminator and casts a shadow of apparent length ℓ.

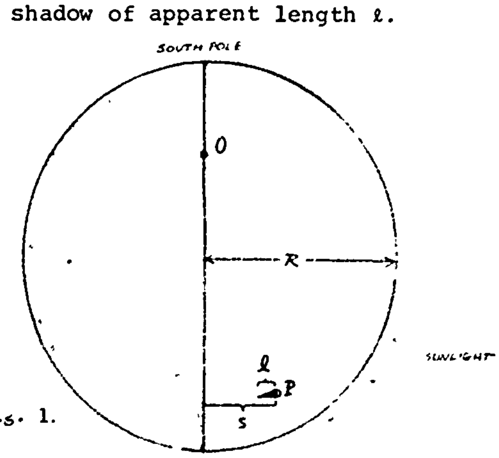


Fig. 1.

Figure 2 shows how the moon would appear if viewed from above the point O which is on the terminator and 90° from Piton. From this point of view Piton is seen on the edge of the moon's disc. Its size is exaggerated in the sketch.

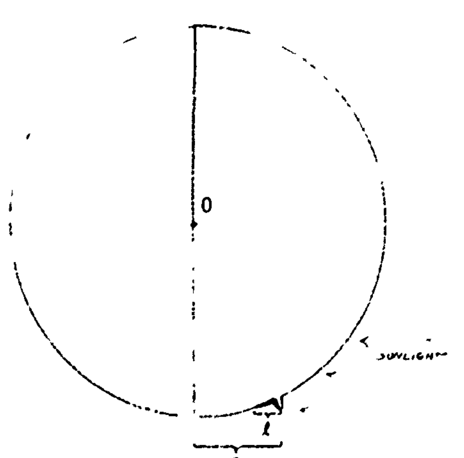


Fig. 2.

This change of the point of view can be made clear with a large ball (basket ball). Mark North pole, and central meridian (terminator). Use golf tee to represent Piton. Show to class so that they see it as in photograph and Figure 1. Now rotate by bringing north pole up towards you (away from class) until the class sees Piton on the edge of the moon's disc. This is the view seen in Figure 2.

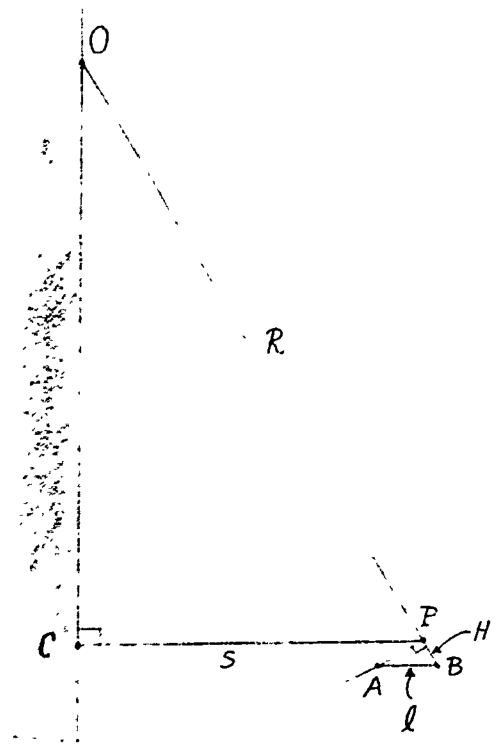


Figure 3

In fact this is not an essential assumption. Both s and ℓ change with the moon's phase and these changes tend to compensate each other.

This is not an easy question for students to answer. But they should agree that these are "reasonable" assumptions, and that the error introduced is less than 10%.

Some simplifying assumptions

You can readily derive the height of Piton from measurements made on the photograph if you assume that:

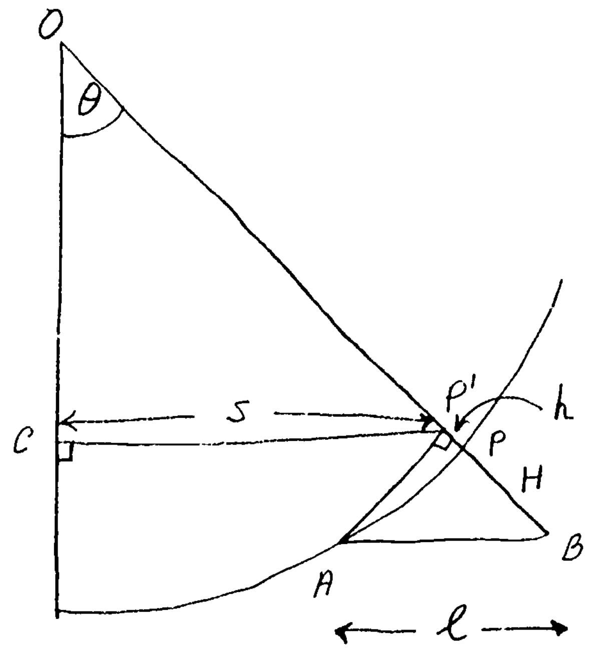
1) the shadow ℓ is short compared to the lunar radius R. This allows you to neglect the curvature of the moon under the shadow—you can approximate arc AP by a straight line.

2) in Fig. 1 you are looking straight down on the top of the peak.

3) the moon was exactly at third quarter phase when the photograph was made.

Q1 How big an error do you think these assumptions might involve?

Because the moon's surface does curve slightly under the shadow, Figure 3 is not exact. A more honest diagram looks like this (the height H and the angle θ have been exaggerated):



The geometric model

Look at the triangles OCP and APB in Fig. 2. Both have a right angle (at C and P respectively) and AB is parallel to CP. From this it can be shown that the angles $\hat{COP}$ and $\hat{PAB}$ are equal. The two triangles are therefore similar. Corresponding sides of similar triangles are proportional, so we can write $\frac{s}{R} = \frac{H}{\ell}$.

To determine H, you must measure ℓ, s and R from the photograph in arbitrary units, and then establish the scale of the photograph by comparing the measured diameter of the moon photo (in mm) with its given diameter (3476 km).

The line AP' is perpendicular to the radius OP. It is (part of a) chord), not a tangent as the simple treatment assumes.

The similar triangles are COP' and P'AB. Hence

$$\frac{s}{R} = \frac{H+h}{\ell} .$$

The simple formula <u>over estimates</u> H by the length h. This turns out to be a correction of perhaps as much as 25%. However the uncertainties in this experiment due to difficulty in measuring s and ℓ are of the same order.

A photograph of the moon taken at the third quarter phase. (Lick Observatory, University of California)

The best area to work in is probably southwest (above and to the left!) of Piton, between Piton and the crater Aristillus. We place the terminator through the Western (left) rim of the crater Cassini.

Encourage students to use the magnifier. Some may discover for the first time that a half tone reproduction is made up of dots.

Measurements and Calculations

The first problem is to locate the terminator. Because the moon has no atmosphere, there is no twilight zone. The change from sunlit to dark is abrupt. Those parts of the moon which are higher than others remain sunlit longer. Thus the shadow line, or terminator, is a ragged line across the moon's surface. To locate the terminator use a white thread or string stretched tight and seek an area close to Piton where the moon's surface is flat. Move the thread until it is over the boundary between the totally dark and partly illuminated areas.

Use a 10X magnifier to measure s and ℓ on the pictures provided. Or you can use a scale to measure the length of the shadow and the distance from Piton to the terminator on the 10X enlargement; the values of ℓ and s will be one tenth of these measured lengths. To measure R in millimeters find the moon's diameter along the terminator and divide by two. The diameter of the moon is 3476 km.

Q2 How much does one millimeter on the photograph represent on the moon's surface?

Use this scale factor and the equation given above to find the height of Piton, in km.

Below is a picture of the area enclosed by the white line, enlarged exactly ten times On these telescope pictures north is at the bottom, east is to the right, as in the map opposite.

Distance to Moon and Its Size

Some students will inquire how the size of the moon is known. In the sky we can see that the moon is about half a degree across. If then we can find the distance to the moon, we can determine its linear size. (The moon's linear diameter, for 0.5° angular diameter, will be $\frac{1}{2 \times 57.4}$ or about 1/115 of the distance from the earth to the moon.

The distance to the moon is derived by triangulation of the moon's position against the starry background with simultaneous observations made from places far apart on the earth. The geometry is easiest to visualize if the places are on a north-south line, but such orientation is not necessary (only the analysis is more complicated). If we could observe the moon simultaneously from the earth's north and south poles, we would find a difference of about 2° in the recorded positions. (This would be the angular size of the earth as seen from the moon.) An angle of 2 is subtended by an object when its distance ia about 30 times its linear size. (Students can easily check this.) Therefore we can conclude that the distance to the moon is about 30 earth-diameters, or 60 earth-radii. Since the earth's radius is about 4000 miles, or 6,400 kilometers, the approximate distance to the moon is 24,000/115, about 2090 miles, or 3,340 kms.

If students wish to attempt to derive this result from their own observations, remind them that (1) the baseline between observations must be as long as possible (but need not be north or south), (2) the moon moves continuously among the stars — so simultaneous observations are necessary, and (3) locating the bright moon accurately among a field of faint stars is difficult. Even so, we hope that some will try.

We found, using a magnifier

$$s = 4.0 \pm 0.5 \text{ mm}$$
$$\ell = 2.0 \pm 0.2 \text{ mm}$$
$$2R = 164 \pm 2 \text{ mm}$$

$$1 \text{ mm on photo} = \frac{3476}{164} = 21 \text{ km}$$
on moon

$$H = \frac{S\ell}{R} = \frac{4.0 \times 2.0}{82} = 0.98 \text{ mm}$$
$$= 0.98 \times 21 = 2.1 \text{ km}$$

Location of terminator, and measurement of s is the least certain measurement. Uncertainty is s may be $\pm$ 0.5 mm. This alone gives an uncertainty of $\frac{0.5}{3.5} \times 100 = 14\%$ in the final answer.

Another question that each student must answer for himself is: "From which point in the illuminater part of the mountain should I measure—from the center, or from the western (left hand) edge?" In fact is probably correct to measure from the left hand edge on the assumption that it is that part of the mountain that casts the longest shadow.

Uncertainty in ℓ is about 10%. Uncertainty in R is insignificant compared with these. Maximum uncertainty of final result is therefore 15% + 10% = 25%.

Discussion

1. What is your value for the height of Piton?

2. Which is the least certain of your measurements? What is your estimate of the uncertainty of your final result?

3. Astronomers, using methods considerably more sophisticated than those you

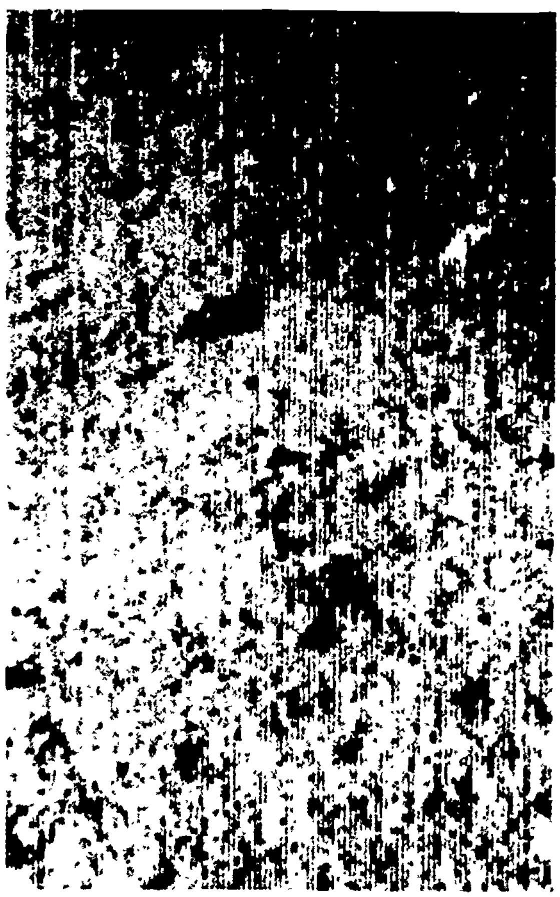
Tenth lunar surface picture taken b
shows a moon rock six inches high a
America's first lunar soft-landing
of Storms at 11:17 p.m., Pacific Da
spots at left are reflections of th
the National Aeronautics a.d Space
Laboratory.

Can you estimate the size of some o

Can you estimate the angle that the
(angle θ in Fig. 2)? (Photo credit

use here, calculate the height of Piton to be about 2.3 km (and about 22 km across at its base). How does your value compare with this?

4. Does your value differ from this by more than your experimental uncertainty? If so, can you suggest why?



y Surveyor I spacecraft on June 2, 1966,
nd twelve inches long. Surveyor I,
spacecraft, touched down in the Ocean
ylight Time, June 1, 1966. Bright
e sun. The picture was received at
Administration's Jet Propulsion

f the smaller objects?

sun's rays make with the moon's surface
: NASA).

Percentage error of our result is

$$\frac{2.3 - 2.1}{2.3} \times 100 = \frac{0.2}{2.3} \times 100 = 9\%$$

As the example above shows it is unlikely that students' results will differ from 2.3 km by more than the experimental uncertainty. The correct answer to "Can you suggest why?" is therefore probably "I under estimated the uncertainty in locating the terminator, etc."

One correction to the geometrical model is suggested above.

Another is that the shadow lies along the parallel of latitude through Piton and not along the great circle which is the edge of the section shown in Figure 2. This is a small factor.

Students may be disappointed with the rather high percentage error of their results. They should not be. To have measured the size of Piton even to within 30% is a worthwhile achievement.

This is also a good time to emphasize the importance and value of order of magnitude measurements in physics. This seems to contradict the cliche, "Physics is an exact science," but an order of magnitude value is often all that can be obtained, particularly if the quantity being measured is very large of very small. Of course it is essential to have an estimate of uncertainty.

Experiment 15*

THE SHAPE OF THE EARTH'S ORBIT

You may decide to run this experiment as a group activity in which the whole class participates in collecting the data. But each student should make his own plot. Once the data have been collected the plotting itself could be done at home. The only tools needed are protractor and ruler.

The experiment is based on the ▲ answers to these two questions. Ask students to think about them the day before you want them to plot the orbit. The answers are:

Q1. The sun moves 360° in one year. The rate is approximately one degree per day. (Actually $\frac{360.00^\circ}{365.24\text{ days}} = 0.98667^\circ/\text{day}$.)

Q2. The formula is: apparent size $\frac{1}{\text{distance}}$.

Q2 above should suggest the answer to this question. But you may get other suggestions. Perhaps the sun periodically expands and contracts, as many stars do. (The most famous type of periodic "variable stars" are called cepheid variables; one famous cepheid is Polaris which has a period of just less than four days.) You might ask that if the sun does vary in size, does it probably vary in other ways, e.g., in brightness? Could this be an explanation for the seasons? Does it seem reasonable that the sun's period of variation would coincide exactly with the earth's period of revolution about the sun? What other effects would we observe on Earth and on other planets if the sun varies in size or brightness?

*Based on an experiment developed by Dr. R.A.R. Tricker, formerly Chief Inspector of Schools for Science in England.

EXPERIMENT 15 The Shape of the Earth's Orbit

Two questions to think about before starting to plot the orbit

▲Q1 How long does it take the sun to make one complete cycle (360°) through the sky against the background of stars? (See Sec. 5.1 if you have difficulty answering this.) How fast does the sun move along the ecliptic in degrees per day (to the nearest degree)?

Q2 The further away you are from an object the smaller it appears to be. Suppose you photograph a friend at a distance of ten meters, and then again at a distance of twenty meters. Can you find a formula that relates his apparent size to his distance?

Ptolemy, and most of the Greeks, thought that the earth was at the center of the universe, and that the sun revolved around the earth. From the time of Copernicus the idea gradually became accepted that the earth and other planets revolve around the sun. You probably believe this, just as you believe that the earth is round. But from the evidence of our senses—how we see the sun move through the sky during the year—there is no reason to prefer one model over the other.

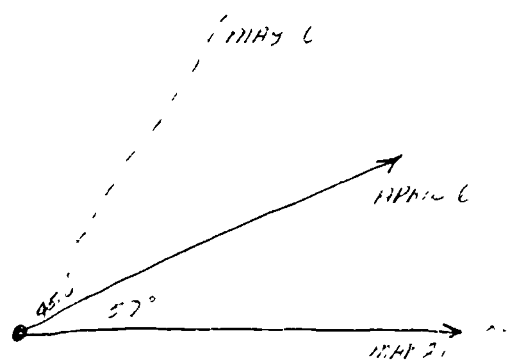
Plotting the orbit

The raw material for this experiment is a series of sun photographs taken at approximately one-month intervals and printed on a film strip.

Q3 The photograph in Frame 5 shows halves of the images of the January sun and July sun placed adjacent. How can you account for the obvious difference in size?

Assume that the earth is at the center of the universe. (This, after all, is the "common sense" interpretation; it is what our senses tell us.)

Take a large piece of graph paper ▲ (20" x 20" or four 8 1/2 x 11" pieces pasted together) and put a mark at the center to represent the earth. Take the 0° direction, the direction of the sun as seen from the earth on March 21st, to be along the grid lines toward the left.



The dates of all the photographs, and the direction to the sun measured from this zero direction are given in the table below. Use a protractor to draw a spider web of lines radiating out from the earth in these different directions. The angles are measured counterclockwise from the zero line.

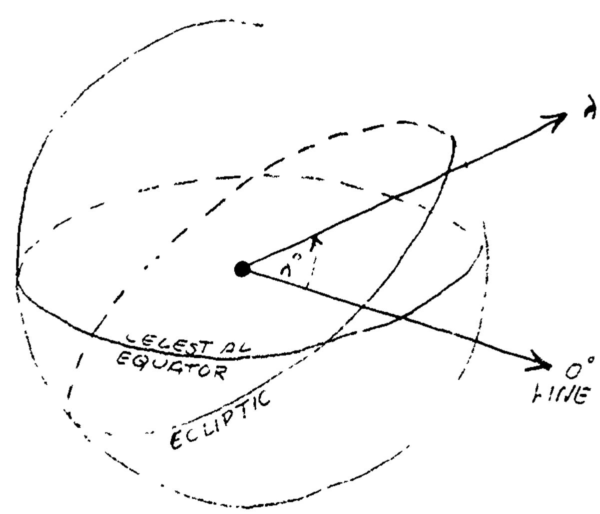
Table

Date	Direction from earth to sun
March 21	0°
April 6	15.7
May 6	45.0
June 5	73.9
July 5	102.5
Aug. 5	132.1
Sept. 4	162.0
Oct. 4	191.3
Nov. 3	220.1
Dec. 4	250.4
Jan. 4	283.2
Feb. 4	314.7
March 7	346.0

There are other possible sources of systematic or cyclic error to consider. The effective focal length (and therefore of magnification) of the telescope might vary with temperature—and therefore with the seasons. (A study of solar diameter vs. air temperature could be made. But a more conclusive study would involve similar photos from the southern hemisphere where the seasons differ from ours.)

After variation in the sun's physical size has been ruled out as a probable explanation, return to the idea that this is an apparent variation only and that probably it results from a variation in the distance from earth to sun.

▲ The point where the ecliptic crosses the equator on March 21 is called the Vernal Equinox (see Unit 2 text, page 9). The sun is then close to Pisces. A line from the earth of the Vernal Equinox is the reference line from which celestial longitudes are measured along the ecliptic. The angles are measured eastward from the 0^{o} line.

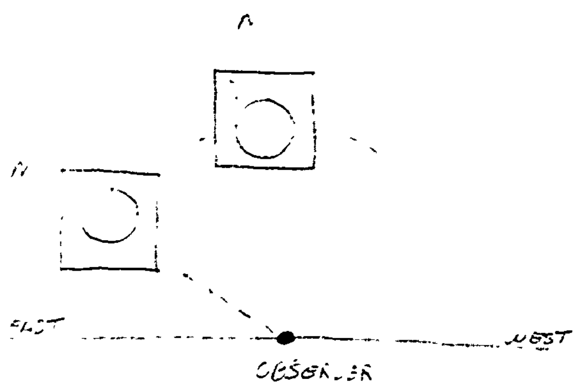


This system is used in several of the Unit 2 experiments (Experiments 17*, 18, 19, 21 as well as the present one) and students should be familiar enough with it to understand what they are doing in these experiments. They should appreciate that the "0^{o} line" is a line from the earth to a point (the Vernal Equinox) on the celestial sphere.

Part of Transparency T14 can be used to help explain the system.

Project the film strip onto a wall or other flat surface to give an image of the sun between 50 and 100 cm in diameter. Measure the diameters with a meter stick. Do NOT move or refocus the projector after the measurements start.

In frames 7-18 the directions marked N and E refer to the directions as seen in the sky and not to directions on the sun itself. Note that the direction marked "north" varies from frame to frame. This arises principally because the photographs were made at different times of the day. The frames always have the same alignment with respect to the horizon. Plates taken early in the morning have the north direction



tipped toward the left. We recommend measuring along the horizontal diameters which are parallel to the bottoms of the frames. This lessens the effect of atmospheric refraction, see Figs. 6.12 and 6.13 in the text. REMEMBER: Do not move the projector, or refocus.

If you have more film strips than projectors, some students can measure on the film directly, using a 10X magnifier.

A 10 cm radius give an orbit which is convenient to use as the starting point for plotting the Orbit of Mars (Experiment 17*).

An off-center circle is the best curve to draw.

Perihelion is January 3. Aphelion is July 5.

The ratio $\frac{\text{aphelion distance}}{\text{perihelion distance}}$ is about 1.04.

Experiments

Measure carefully the projected diameters of each of the frames in the film strip. What is the relationship between these measurements and the relative distance of sun from earth? Adopt a scale factor (a constant) to convert the diameters to distances from the earth in arbitrary units. Your plot of the earth-sun distance should have a radius of about 10 cm. If your measurement for the sun's diameter is about 50 cm, you should use a scale factor of 500 cm.

$$\text{Distance to sun} = \frac{\text{constant}}{\text{sun's diameter}} = \frac{500}{\text{diameter}}.$$

(For instance, if the measured diameter of the sun is 49.5 cm the relative distance will be $\frac{500}{49.5} = 10.1$ cm; and for a measured diameter of 51.0 cm the relative distance would be $\frac{500}{51.0} = 9.8$ cm.) Make a table of the relative distances for each of the twelve dates.

Along each of the direction lines you have drawn measure off a length corresponding to the relative distance to the sun on that date. Draw a smooth curve through these points using a compass or a set of French curves. This is the orbit of the sun relative to the earth. The distances are relative; we cannot find the actual distance in miles from the earth to the sun from this plot.

Q4 Is the orbit a circle? If so, where is the center of the circle? If the orbit is not a circle, what shape is it?

Q5 Locate the major axis of the orbit, through the points where the sun passes closest to and farthest from the earth! What are the approximate dates of closest approach (perihelion) and greatest distance (aphelion)? What is the ratio of aphelion distance to perihelion distance?

Students may be surprised to learn that the sun is actually closer to the earth in (northern hemisphere) winter than it is in summer. But this has nothing to do with seasons of course. They are caused by the inclination of the earth's axis of rotation to its plane of revolution.

A heliocentric system

Copernicus and his followers adopted the sun-centered model only because the solar system could be described more simply that way. They had no new data that could not be accounted for by the old model. If you are not convinced that the two models, geocentric and heliocentric, are equally valid descriptions of what we see, try one of the activities at the end of the notes on this experiment.

You can use the same data to plot the earth's orbit around the sun, if you make the Copernican assumption that the earth revolves above the sun. You already have a table of the relative distances of the sun from the earth. Clearly there's going to be some similarity between the two plots. The dates of aphelion and perihelion won't change, and the table of relative distances is still valid because you didn't assume either model when deriving it. Only the angles used in your plotting cha ge.

When the earth was at the center of the plot the sur was in the direction 0° (to the right) on March 21st.

Q6 What is the direction of the earth as seen from the sun on that date? What is the earth's longitude as seen from the sun? (The answer to this question is given at the end of the notes on this experiment. Be sure that you understand it before going on.)

180°.

At this stage the end is in sight. Perhaps you can see it already, without doing any more plotting. But if not, here is what you must do:

If the plot is turned through 180° the plot now represents the orbit of the earth around the sun.

Make a new column in your table giving the angle from sun to earth. (These are the angles given on the last frame of the film strip.) Place a mark for the sun at the center of another sheet of paper on each of the twelve dates. Use the

Some students may see this, many probably won't. Rather than try to convince them by intellectual arguments that the same plot can be used for either orbit we suggest that they begin to plot the motion of the earth around the sun. If they are encouraged to compare the new plot with their first one as they go along, they should soon realize that they do not need to make a whole new plot. Rotating the plot is all that is needed.

new angles to plot the orbit of the earth. When you have marked the earth's position on a few dates, compare the new plot with the first one you made around the sun. You may now see the relationship between the two plots. Before you finish the second plot you will probably see what the relationship is, and how you can use the same plot to represent either orbit. It all depends on whether you initially assume a geocentric or a heliocentric model.

Answer to Q6

The direction of the earth on March 21st, as seen from the sun, is to the left on your plot. The angle is 180°. If you do not understand the answer read the following:

I am standing due north of you. You must face north to see me. When I look at you I am looking south. If the direction north is defined as 0° the direction south is 180°. Any two "opposite" directions (north and south, south-west and north-east) differ by 180°.

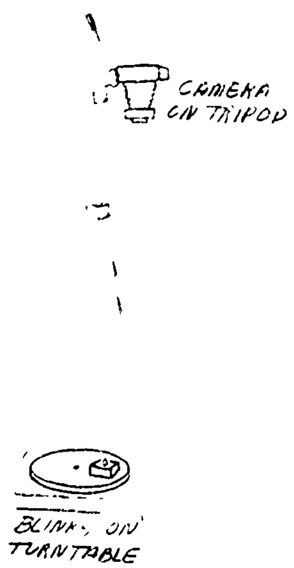
About the photographs in the film strip

Your students may comment that the solar images on the film strip are not uniformly bright. This phenomenon, called "limb darkening", (the edge of the sun is the "limb"), is evident because the layers of the sun are progressively cooler outward toward the surface. Light originating near the edge of the sun comes from a higher and cooler layer than does light from the center of the sun's image. Thus the limb appears darker than regions nearer the sun's center.

Two activities on frames of reference

(1) You and a classmate take hold of opposite ends of a meter stick or a piece of string a meter or two long. You stand still while he walks around you in a circle at a steady pace. You see him moving around you. But how do you appear to him? Ask him to describe what he sees when he looks at you against the background of walls, furniture, etc. You may not believe what he says; reverse your roles to convince yourself. In which direction did you see him move—toward your left or your right? In which direction did he see you move—toward his left or his right?

(2) The second demonstration uses a camera, tripod, blinky and turntable. Mount the camera on the tripod and put the blinky on a turntable. Aim the camera straight down.



Take a time exposure with the camera at rest and the blinky moving one revolution in a circle. If you do not use the turntable, move the blinky by hand around a circle drawn faintly on the background. Then take a second print, with the blinky at rest and the camera moved steadily by hand about the axis of the tripod. Try to move the camera at the same rotational speed as the blinky moved in the first photo.

Can you tell, just by looking at the photos, whether the camera or the blinky was moving?

The high contrast photos show few, if any, sunspots. The sun rotates, but not as a solid body. The equatorial regions rotate somewhat faster than do those at higher latitudes. In the equatorial regions the rotational period is about twenty-five days, but at the latitude of 30 degrees, the period is about two and one-half days longer. This rotation is too slow to produce any measurable oblateness.

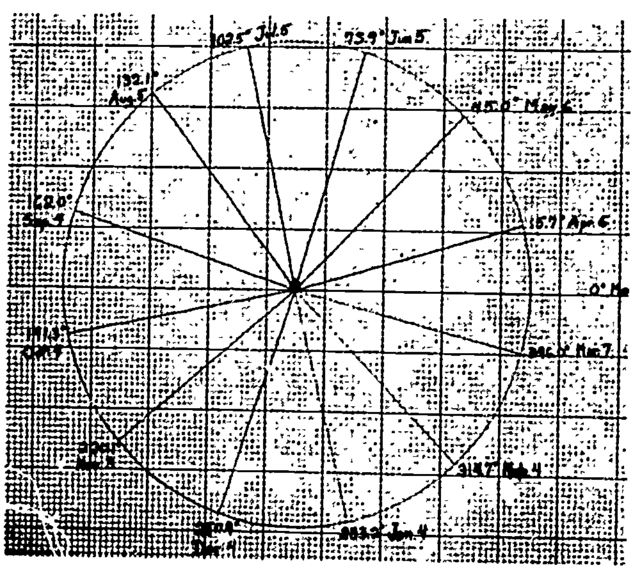
The orbit of the moon

The early Greek astronomers had no devices capable of detecting the small apparent variations of the sun's diameter. They did, however, detect the variation in the apparent diameter of the moon during the course of a lunar month. This is not surprising, since the eccentricity of the lunar orbit about the earth is relatively large ($e = 0.055$), and the resulting change in apparent size is about 10%, or 3 minutes of arc.

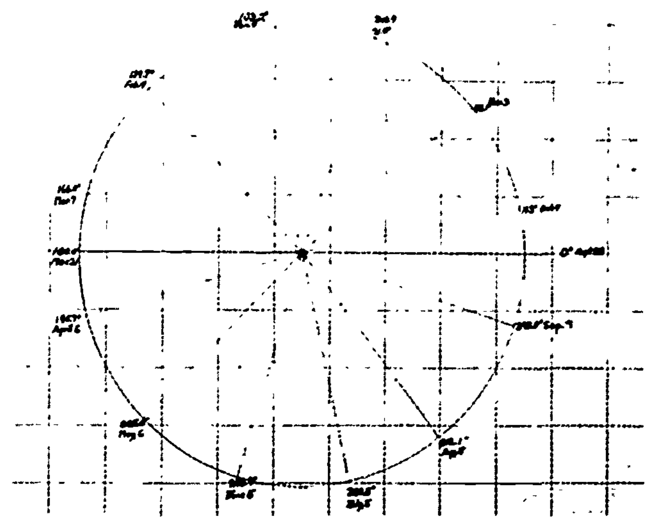
A similar experiment can be used for determining the orbit of the moon about the earth. The necessary materials were published by Owen Gingerich in the April, 1964, issue of "Sky and Telescope." Reprints are available from the Sky Publishing Corporation, 49 Bay State Rd., Cambridge, Massachusetts, 02138. Costs: 10-24 copies @ $0.15 ea.; 25-99 copies @ $0.10 ea.; 100-999 copies @ $0.08 ea.; minimum order is $1.00.

The following are typical values for this experiment.

Date	Solar Diam. (cm)
Jan. 4	50.0
Feb. 4	49.6
Mar. 7	49.5
Apr. 6	48.7
May 6	48.7
June 5	48.5
July 5	48.1
Aug. 5	48.6
Sept. 4	49.0
Oct. 4	49.5
Nov. 3	49.7
Dec. 4	49.9
Sept. 23	--



A) Plot of Sun's apparent motion about the earth. The orbit is represented by a circle whose center is slightly above the earth's position on the plot



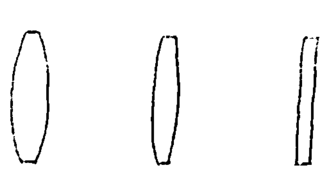
B) Plot of earth's motion about the sun. The "0°" direction is still to the right. This plot is equivalent to the plot of the sun's apparent motion about the earth rotated through 180°.

By working through some simple optical experiments students can learn enough optics to understand how a telescope works.

Use your own stock of assorted lenses. Include lenses which have the same diameter but different powers. Include some negative lenses (also some plane glass discs, if you have them). If necessary, use lenses from the telescope kits; otherwise, keep the kits in the background. CAUTION students to handle lenses by the edges and to keep their fingers off the surfaces. This method yields the maximum field of view and the minimum distortion. If the object distance is held constant at its value for maximum magnification, the angular size of the image will remain constant as the eye is moved away from the lens.

Magnifiers are thicker in the middle than at edges. ▲

Curvature is the important feature. ●



Increasing magnification

EXPERIMENT 16 Using Lenses, Making a Telescope, Using the Telescope

In this experiment you will first examine some of the properties of single lenses, then combine these lenses to form a telescope which you can use to observe the moon, the planets and other heavenly (as well as earth-bound) objects.

The simple magnifier

You certainly know something about lenses already—for instance, that the best way to use a magnifier is to hold it immediately in front of the eye and then move the object you want to examine until its image appears in sharp focus.

Examine some objects through several different lenses. Try lenses of a variety of shapes and diameters. Separate any lenses that magnify from those that don't. Describe the difference between lenses that magnify and those which do not.

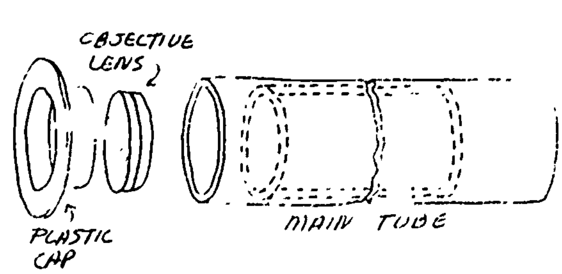
▲ Q1 Which lens has the highest magnifying power? Arrange the lenses in order of their magnifying powers.

● Q2 What physical feature of a lens seems to determine its power—is it diameter, thickness, shape, the curvature of its surface?

Sketch side views of a high-power lens, of a low-power lens and of the lowest power lens you can imagine.

Real images

With one of the lenses you have used, project an image of a ceiling light or a (distant) window on a sheet of paper.



Describe all the properties of the image that you can observe. Such an image is called a real image.

Q3 Do all lenses give real images? ▲

Q4 How does the image depend on the lens?

Q5 If you want to look at a real image ● without using the paper, where do you have to put your eye?

Q6 Why isn't the image visible from ★ other positions?

Q7 The image (or an interesting part of ◇ it) may be quite small. How can you use a second lens to inspect it more closely? Try it.

Q8 Try using other combinations of ○ lenses. Which combination gives the maximum magnification?

Making a telescope

Parts list

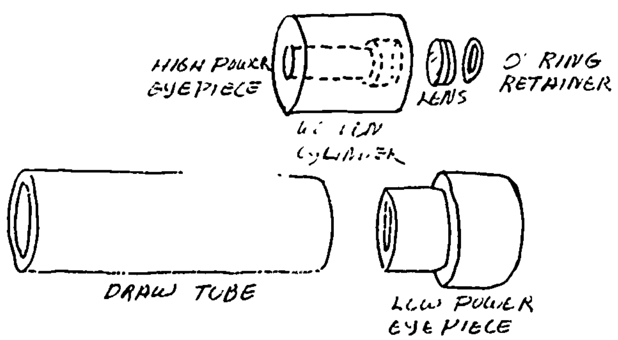
- 1 large lens
- 1 magnifier
- 1 small lens, mounted in a wooden cylinder
- 2 cardboard tubes
- 1 plastic cap

Note the construction of the largest lens. This lens is called the objective lens (or simply the objective).

Q9 How does its magnifying power compare with the other lenses in the kit?

Assembling the telescope

The sketches show how the various parts go together to make your telescope.



Colors reproduced correctly; image is inverted, two-dimensional, smaller than object.

▲ Only the magnifying lenses produce real images of objects. A high power lens produces a smaller image, closer to the lens, than does a lower power lens.

● Behind the image and at least 10 inches from it, so the eye can focus on it. To a student who has trouble finding image: "How far would your eye have to be from a real object to see it clearly?"

★ Light from the object that has gone through the lens is confined to a narrow cone, which the eye must intercept.

◇ Use magnifier, held at proper distance from the real image. If a student has difficulty in placing the magnifier in the right place, let him focus the image on a sheet of paper, then remove paper. (The student has constructed a telescope!)

○ Lowest power lens as objective, highest power as eyepiece.

Objective has lower magnifying power.

Experiments

Notes on assembly

1. If you lay the objective down on a flat clean surface, you will see that one surface is more curved than the other: the more curved surface should face outward.

2. Clean dust, etc., off lens (using lens tissue or <u>clean</u> handkerchief) <u>before</u> assembling telescope.

3. Focus by sliding the draw tube, not by moving the eyepiece in the tube.

4. To use high power satisfactorily, a firm support—tripod—is essential.

5. Be sure that the lens lies flat in the high-power eyepiece. Low power gives about 12X magnification. High power gives about 30X magnification.

Try out your telescope on objects inside and outside the lab. The next section suggests some astronomical observations you can make.

Telescope on Binocular Observations

Introduction

You should look at terrestrial objects with and without the telescope to develop handling skill and familiarity with the appearance of objects.

Mounting

Telescopic observation is difficult when the mounting is unsteady. If a swivel-head camera tripod is available, the telescope can be held in the wooden saddle by rubber bands, and the saddle attached to the tripod head by the head's standard mounting screw. Because camera tripods are usually too short for comfortable viewing from a standing position, it is strongly recommended that the observer be seated in a reasonably comfortable chair. The telescope should be grasped as far forward and as far back

as possible, and both hands rested firmly against a car roof, telephone pole, or other rigid support.

Aiming

Even with practice, you may have trouble finding objects, especially with the high-power eyepiece. Sighting over the top of the tube is not difficult, but making the small searching movements following the rough sighting takes some experience. One technique is to sight over the tube, aiming slightly below the object, and then tilt the tube up slowly while looking through it. Or, if the mounting is firm and the eyepiece tubes are not too tight, an object can be found and centered in the field with low power, and then the high-power eyepiece carefully substituted. Marks or tape ridges placed on the eyepieces allow them to be interchanged without changing the focus.

Focussing

Pulling or pushing the sliding tube tends to move the whole telescope. Use the fingers as illustrated in Fig. 1 to push the tubes apart or pull them together. Turn the sliding tube while moving it (as if it were a screw) for fine adjustment.

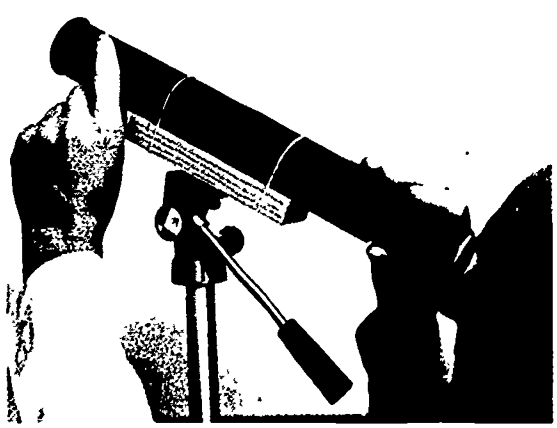


Fig. 1

An observer's eyeglasses keep his eye much farther from the eyepiece than the optimum distance. Far-sighted or near-sighted observers are generally able to view more satisfactorily by removing their eyeglasses and refocussing. Observers with astigmatism have to decide whether or not the astigmatic image (without glasses) is more annoying than the reduced field of view (with glasses).

Many observers find that they can keep their eye in line with the telescope while aiming and focussing if the brow and cheek rest lightly against the forefinger and thumb. When using a tripod mounting, the hands should be removed from the telescope while actually viewing to minimize shaking the instrument. (See Fig. 2.)

Fig. 2

Limitations

By comparing the angular sizes of the planets with the resolving power of the telescope, you can get some idea of how much fine detail to expect when observing the planets.

For a 30X telescope to distinguish between two details, they must be at least 0.001° apart. The low-power Project Physics telescope gives a 12X magnification, and the high power gives 30X. (Note: Galileo's first telescope gave 3X magnification, and his "best" gave about 30X. You should find it challenging to see whether you can observe all the phenomena mentioned in Sec. 7.7 of your text.)

The angular sizes of the planets as viewed from the earth are:

Mars:	0.002° (minimum)
	0.005° (maximum)
Saturn:	0.005° (average)
Uranus:	0.001° (average)
Venus:	0.003° (minimum)
	0.016° (maximum)
Jupiter:	0.012° (average)

Observations

The following group of suggested objects have been chosen because they are (1) fairly easy to find, (2) representative of what is to be seen in the sky, and (3) quite interesting. You should observe all objects with the low power first and then the high power. For additional information on current objects to observe, see the Handbook of the Heavens, the last few pages of each issue of "Sky and Telescope," "Natural History," or "Science News."

Venus: It will appear as a featureless disc, but you can observe its phases, as shown on page 68 of your text. When it is very bright you may need to reduce the amount of light coming through the telescope in order to tell the true shape of the image. A paper lens cap with a hole in the center will reduce the amount of light.

Saturn: It is large enough that you can resolve the projection of the rings beyond the disc, but you probably can't see the gap between the rings and the disc with your 30X telescope. Compare your observations to the sketches on page 69 of the text.

Jupiter: Observe the four satellites that Galileo discovered. If you use the low-power eyepiece with the ruled scale, you can record the relative radii of the orbits for each of the moons. By keeping detailed data over about 6 months' time, you can determine the period for each of the moons, the radii of their orbits, and

then the mass of Jupiter. See the notes for film loop 12, Jupiter Satellite Orbit, in Chapter 8 of the Student Handbook for directions on how to analyze your data.

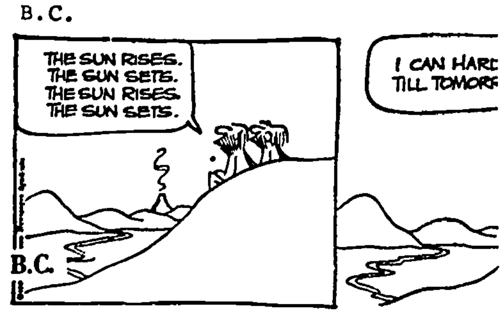
Jupiter is large enough that some of the surface detail—like a broad, dark, equatorial stripe—can be detected (especially if you know it should be there!)

Moon: Best observations are made between four days after new moon and four days after first quarter. Make sketches of your observations, and compare them to Galileo's sketch on page 66 of your text. Look carefully for walls, central mountains, peaks beyond the terminator, craters in other craters, etc.

The Pleiades: A beautiful little group of stars which is located on the right shoulder of the bull in the constellation Taurus. They are almost overhead in the evening sky during December. The Pleiades were among the objects Galileo studied with his first telescope. He counted 36 stars, which the poet Tennyson described as "a swarm of fireflies tangled in a silver braid."

The Hyades: This group is also in Taurus, near the star Aldebaran, which forms the Bull's eye. The high power may show several double stars.

B.C.



The Great Nebula in Orion: Look about half way down the row of stars which hangs like a sword in the belt of Orion. It is in the southeastern sky during December and January.

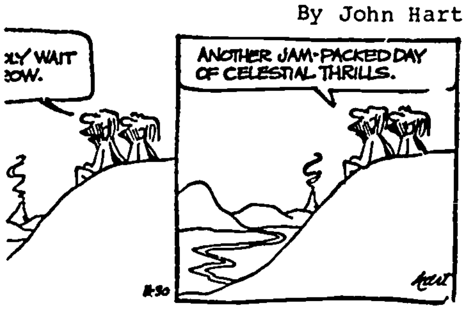
Algol, a famous variable star, is in the constellation Perseus, south of Cassiopeia. Algol is high in the eastern sky in December, and near the zenith during January. Generally it is a second-magnitude star, like the Pole Star. After remaining bright for almost 2 1/2 days, it fades for 5 hours and becomes a fourth-magnitude star, like the faint stars of the Little Dipper. Then it brightens during 5 hours to its normal brightness. From one minimum to the next the period is 2 days, 20 hours, 49 minutes.

Algol, The Double Cluster in Perseus: Look for the two star clusters at the top of the constellation Perseus near Cassiopeia. High power should show two magnificent groups of stars.

Great Nebula in Andromeda: Look high in the western sky in December, for by January it is on the way toward the horizon. It will appear like a fuzzy patch of light, and is best viewed with low power. The light from this nebula has been on its way for 1.5 million years.

The Milky Way: It is particularly rich in Cassiopeia and Cygnus (if air pollution in your area allows it to be seen).

By John Hart



EXPERIMENT 17 The Orbit of Mars

(See Student Activity, Three-Dimensional Model of Two Orbits, in this section, before starting this experiment.)

In this laboratory activity you will derive an orbit for Mars around the sun by the same method that Kepler used. Because the observations are made from the earth, you will need the orbit of the earth which you developed in Experiment 15.

Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not the sun around the earth.

If you did not do the earth's orbit experiment, you may use, for an approximate orbit, a circle of 10 cm radius drawn in the center of a large sheet of graph paper. Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in this experiment.

From the sun (at the center) draw a line to the right, parallel to the grid of the graph paper. Label the line 0°. This line is directed toward a point on the celestial sphere called the Vernal Equinox and is the reference direction from which angles in the plane of the earth's orbit (the ecliptic plane) are measured.

The earth crosses this line on September 23. On March 21 the sun is between the earth and the Vernal Equinox.

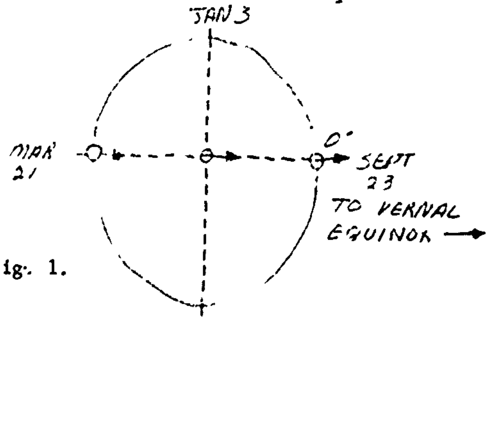


Fig. 1.

The photographs

You will use a booklet containing sixteen enlarged photographs of the sky showing Mars among the stars at various dates between 1931 and 1950. All were made with the same small camera used for the Harvard Observatory Sky Patrol. On some of the photographs Mars was near the center of the field. On many other photographs Mars was near the edge of the field where the star images are distorted by the camera lens. Despite these distortions the photographs can be used to provide positions of Mars which are satisfactory for this study.

Changes in the positions of the stars relative to each other are extremely slow. Only a few stars near the sun have motions large enough to be detected from observations with the largest telescopes after many years. Thus we can consider the pattern of stars as fixed.

Theory

Mars is continually moving among the stars but is always near the ecliptic. From several hundred thousand photographs at the Harvard Observatory sixteen were selected, with the aid of a computer, to provide pairs of photographs separated by 687 days—the period of Mars around the sun as determined by Copernicus. During that interval the earth has made nearly two full cycles of its orbit, but the interval is short of two full years by 43 days. Therefore the position of the earth, from which we can observe Mars, will not be the same for the two observations of each pair. But Mars will have completed exactly one cycle and will be back in the same position. If we can determine the direction from the earth toward Mars for each observation, the sight lines will cross at a point on the orbit of Mars. This is a form of triangulation in which we use the different

None of the photos are retouched; they may show lint or dust marks or the edges of dried watermarks which mar the originals. The center is often heavily exposed while the edge is barely exposed. This is due to vignetting (shadowing) at the edges of the field caused by camera design. The student is as close to the "data" of observational astronomy as are research astronomers themselves.

Some photographs show fewer or fainter stars than others because:

1. In some parts of the sky, well away from the Milky Way, star density is low.

2. Some of the photographs may have been made through chin cloud, smoke or haze, and will not show the fainter stars.

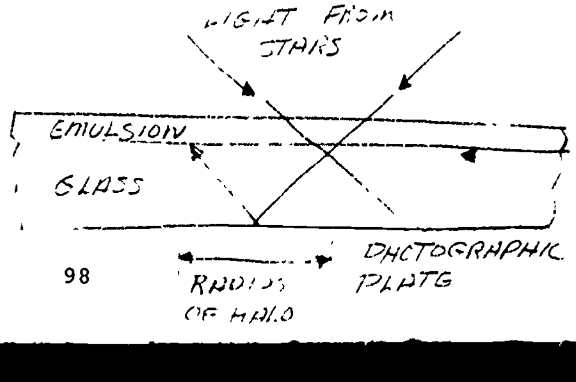
The size of a focused image of Mars on the original plate is about 1/1000 inch—as are the diffraction disks of star images. Consequently, the images of planets are theoretically virtually indistinguishable from those of stars. Actually, light scattered within the photographic emulsion and "twinkling" causes brighter images to grow larger than fainter images.

The best image formed by a lens is on the optical axis (i.e., in the center of the picture). Distortion becomes more pronounced toward the edges of the field. One kind of distortion which makes the shape of images seem triangular is clearly evident in some of the frames taken from the edge of a plate. Nothing can be concluded about real shapes and sizes from these photos; they are valuable because they record relative positions of Mars and the stars.

See Teacher Guide notes on experiment 15 for more on this celestial coordinate system. Transparency T14 could be used here.

Because of the changing attractions of the moon and sun on the oblate earth, the earth's rotation has several wobbles. All except one are too small to be important in this analysis. The one large change is called precession and has a period of 26,000 years. During this time the direction in which the north pole of the earth points, moves through a large circle on the sky. The result of this motion is to cause the location of the Vernal Equinox to slide westward about 50" each year. Since our coordinate grids are based upon the Vernal Equinox as the zero point, the longitudes we assign the stars change very slowly with the years. For this study we have adopted the coordinate system of 1950.0.

Almost surely some questions will arise about the "halo" around the bright object (Mars) in plates F and O. This occurs because the photograph emulsion is on a glass plate about $\frac{1}{32}$ inch thick. As the sketch indicates, light from a very bright object will penetrate through the thin emulsion, be reflected by the back of the glass, and strike the emulsion from below in a ring around the initial image.



positions of the earth to provide a baseline (Fig. 2). Each pair of photographs in the booklet (A and B, C and D, etc.) is separated by a time interval of 687 days.

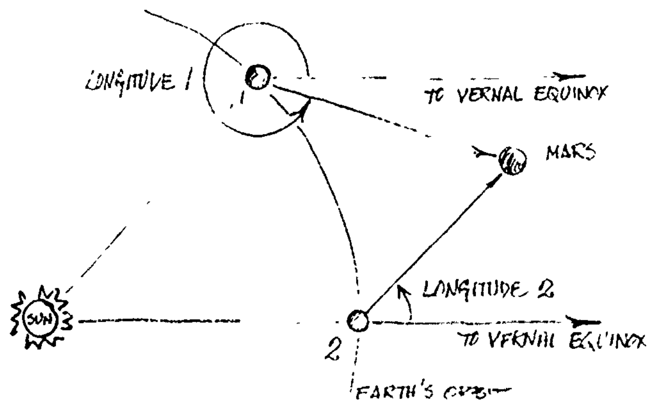


Fig. 2 Points 1 and 2 on the earth orbit are 687 days apart. This is the period of Mars. Mars is therefore back in the same position. Observations of the direction of Mars on these two dates enable us to find its position.

Coordinate system

When we look into the sky we see no coordinate system. We create coordinate systems for various purposes. The one we wish to use here centers on the ecliptic. Remember that the ecliptic is the imaginary line along which the sun moves on the celestial sphere during the year.

Along the ecliptic we measure longitudes always eastward from the 0° point, the direction towards the Vernal Equinox. Perpendicular to the ecliptic we measure latitudes north or south to 90°. The small movement of Mars above and below the ecliptic is considered in the next experiment, The Inclination of Mars' Orbit.

To find the coordinates of a star or of Mars we must project the coordinate system upon the sky. To do this you are provided with transparent overlays which show the coordinate system of the ecliptic for each frame, A to P. The positions of various stars are circled. Adjust the overlay until it fits the star posi-

tions. Then you can read off the longitude and latitude of the position of Mars. Figure 3 shows how you can interpolate between marked coordinate lines. Because we are interested in only a small section of the sky on each photograph, we can draw each small section of the ecliptic as a straight line. Since the values you obtain are to be used for plotting, an accuracy of 0.5° is quite sufficient. Record your results in the table provided.

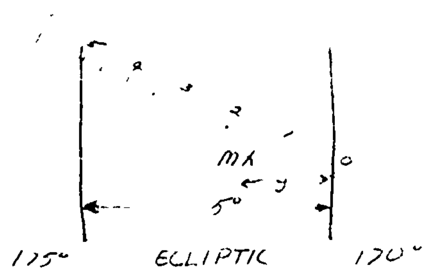


Fig. 3 Interpolation between coordinate lines. In the sketch Mars (M) is at a distance y° from the 170° line. The distance between the 170° line and the 175° line is 5°.

Take a piece of paper or card at least 10 cm long. Make a scale divided into 10 equal parts and label alternate marks 0, 1, 2, 3, 4, 5. This gives a scale in ½° steps. Notice that the numbering goes from right to left on this scale.

Place the scale so that the edge passes through the position of Mars. Now tilt the scale so that 0 and 5 marks each fall on a grid line. Read off the value of y from the scale.

In the sketch y = 1½° and so the longitude of M is 170° + 1½° = 171½°.

This technique is necessary because the plates are not all to the same scale.

For a simple plot of Mars' orbit around the sun you will only use the first column—the longitude of Mars. You will use the columns for latitute, Mars' distance from the sun, and the sun-centered coordinates if you derive the inclination, or tilt, or Mars' orbit in Experiment 18. Record the latitude of Mars; you might use it later on.

Longitude of Mars, as seen from Earth

Plate	Date	long.
A	Mar 21, 1931	118.6°
B	Feb 5, 1933	169.0°
C	Apr 20, 1933	151.4°
D	Mar 8, 1935	204.4°
E	May 26, 1935	186.7°
F	Apr 12, 1937	245.7°
G	Sep 26, 1939	297.5°
H	Aug 4, 1941	916.5°
I	Nov 22, 1941	012.1°
J	Oct 11, 1943	080.1°
K	Jan 21, 1944	065.6°
L	Dec 9, 1945	123.2°
M	Mar 19, 1946	107.6°
N	Feb 3, 1948	153.4°
O	Apr 4, 1948	138.3°
P	Feb 21, 1950	190.7°

Observed Pos

Frame	Date	Lon
A	Mar. 21, 1931	
B	Feb. 5, 1933	
C	Apr. 20, 1933	
D	Mar. 8, 1935	
E	May 26, 1935	
F	Apr. 12, 1937	
G	Sept.16, 1939	
H	Aug. 4, 1941	
I	Nov. 22, 1941	
J	Oct. 11, 1943	
K	Jan. 21, 1944	
L	Dec. 9, 1945	
M	Mar. 19, 1946	
N	Feb. 3, 1948	
O	Apr. 4, 1948	
P	Feb. 21, 1950	

Now you are ready to locate points on the orbit of Mars.

1. On the plot of the earth's orbit, locate the position of the earth for each date given in the 16 photographs. You may do this by interpolating between the dates given for the earth's orbit experiment. Since the earth moves through 360° in 365 days, you may use ±1° for each day ahead or behind the date given in the previous experiment. (For example, frame A is dated March 21. The earth is at 166° on March 7 and 195.7° on April 6. You now add 14° (14 days) to 166° or subtract 17° (17 days) from 196°.) Always work from the earth-position-date nearest the date of the Mars photograph.

2. Through each earth position point draw a "0° line" parallel to the line you drew from the sun towards the Vernal Equinox (the grid on the graph paper is helpful). Use a protractor and a sharp pencil to establish the angle between the 0° line and the direction to Mars as seen from the earth (longitude of Mars). Lines drawn from the earth's posi-

itıons of Mars

Mars		Mars'	Dıst	Heliocentrıc	
g.	Lat.	Earth	Sun	Long.	Lat.

tions for each pair of dates will intersect at a point. This is a poınt on Mars' orbit. Figure 4 shows one point on Mars' orbit obtained from the data of the first pair of photographs. By drawing the intersecting lines from the eight pairs of positions, you establish eight points on Mars' orbit.

3. Draw a smooth curve through the eight points you have established. Perhaps you can borrow a French curve or long spline (e.g., from the mechanical drawing department). You will notice that there are no points in one section of the orbit. But since the orbit is symmetrical about ıts major axis you can fill in the missing part.

Now that you have plotted the orbit you have achieved what you set out to do: you have used Kepler's method to determine the path of Mars around the sun.

If you have time to go on, it is worthwhile to see how well your plot agrees with Kepler's generalization about planetary orbits.

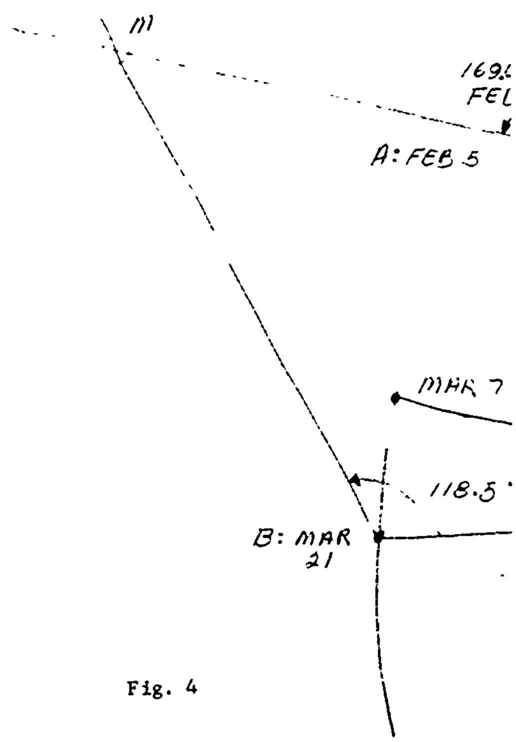


Fig. 4

Kepler's laws from your plot

Q1 Does your plot agree with Kepler's conclusion that the orbit is an ellipse?

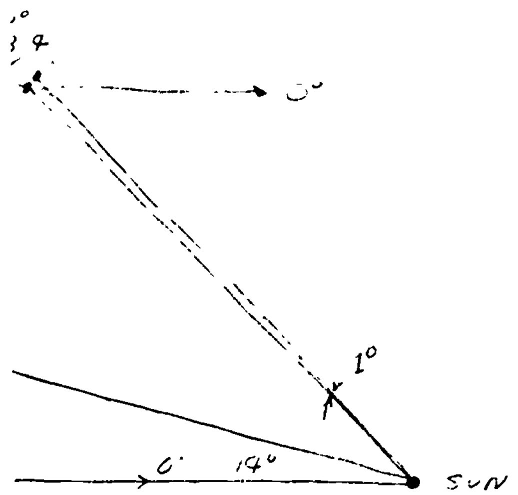
Q2 What is the mean sun-to-Mars distance in AU?

Q3 As seen from the sun, what is the direction (longitude) of perihelion and of aphelion for Mars?

Q4 During what month is the earth closest to the orbit of Mars? What would be the minimum separation between the earth and Mars?

Q5 What is the eccentricity of the orbit of Mars?

Q6 Does your plot of Mars' orbit agree with Kepler's law of areas, which states that a line drawn from the sun to the planet, sweeps out areas proportional to the time intervals? From your orbit you see that Mars was at point B′ on February 5, 1933, and at point C′ on April 20, 1933.



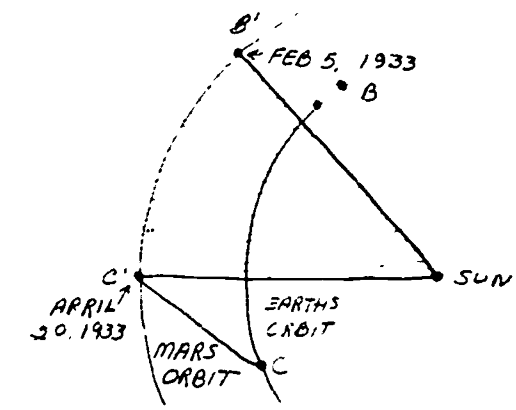


Fig. 5 In this example the time interval is 74 days.

There are seven such pairs of dates in your data. The time intervals are different for each pair.

Connect these pairs of positions with a line to the sun. Find the areas of these sectors by counting blocks of squares on the graph paper (count a square when more than half of it lies

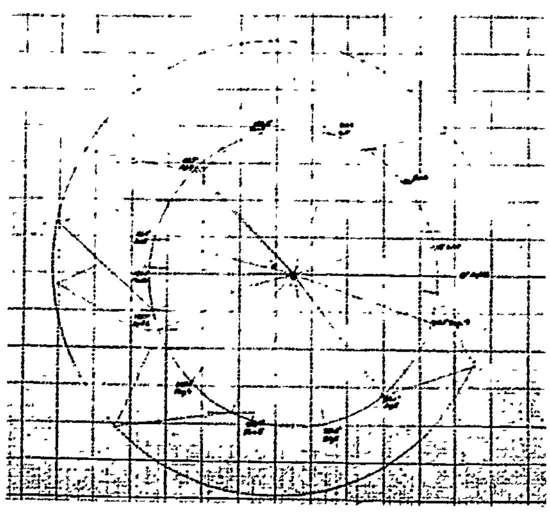
Two articles on measuring areas with planimeters have appeared in the Scientific American, 1958 Aug., and 1959 Feb. Mechanically minded students might wish to make and calibrate their own planimeters.

within the area). Divide the area (in squares) by the number of days in the interval to find an "area per day" value. Are these values nearly the same?

Q7 How much (what percentage) do they vary?

Q8 What is the uncertainty in your area measurements?

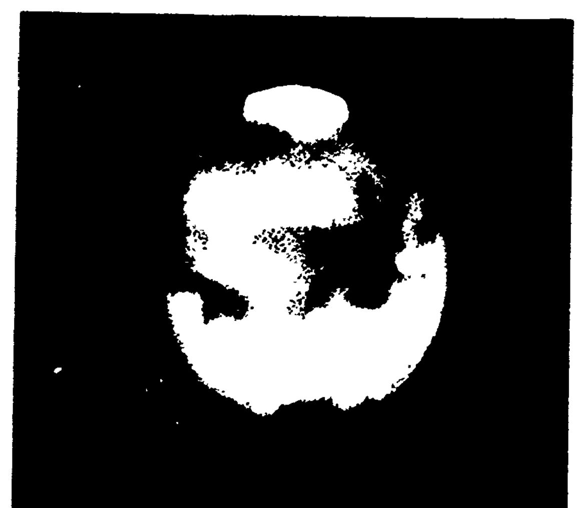
Typical Result



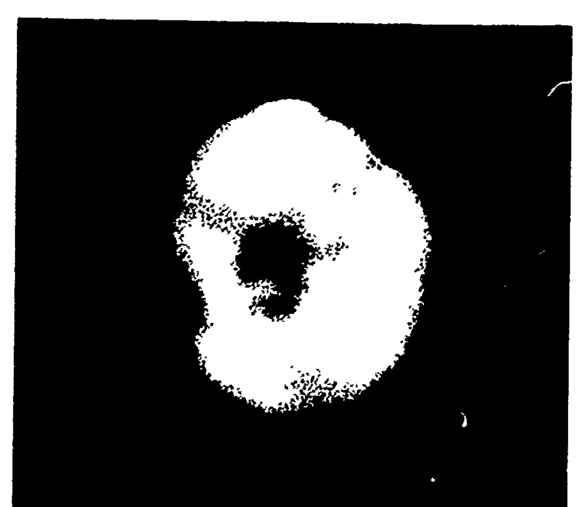
In this plot we have drawn a circle, radius 15.5 cm (1.55 A.U.) that passes through or close to most of the positions of Mars. The center of the circle is above and to the left of the sun's position.

Data for Mars' orbit

Mean distance a = 1.52 A.U.
Eccentricity e = 0.09



August ORANGE



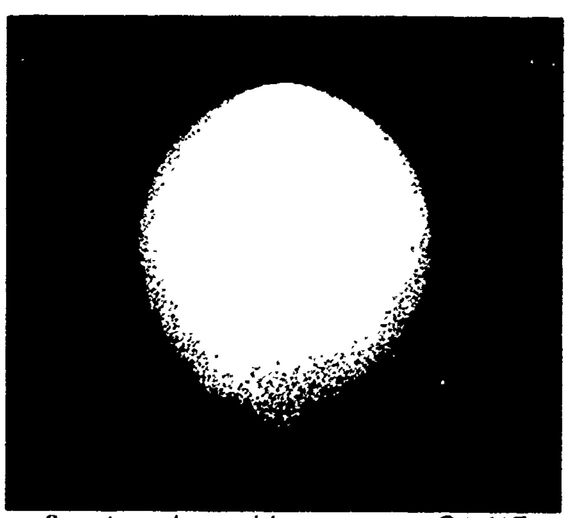
September 11 ORANGE

Q9 Is the uncertainty the same for large areas as for small?

Q10 Do your results bear out Kepler's law of areas?

This is by no means all that you can do with the photographs you used to make the plot of Mars' orbit. If you want to do more, look at Experiment 18.

RED



September 11 BLUE

Some items for discussion

1. As seen from the sun, what is the direction (longitude) of perihelion and of aphelion of Mars? (approximately 340°, 160° respectively)

2. The sun-to-earth distance is often referred to as "one astronomical unit" (A.U.). What is the distance from sun-to-Mars in A.U.? (Maximum distance = 1.7 A.U., minimum distance = 1.4 A.U., mean distance = 1.55 A.U.)

3. During what month is earth nearest the orbit of Mars? (September) What is its distance in A.U. at this time? (about 0.4 A.U.) This is the closest that the two planets can ever approach each other.

4. What is the eccentricity of the Mars orbit? (e = 0.09)

If you have some students who are interested in carrying further the analysis of Mars' Orbit (Experiment 17) they can use the same material (star-field photographs and coordinate overlays) to derive the inclination of Mars' orbit.

EXPERIMENT 18 The Inclination of Mars' Orbit

When you plotted the orbit of Mars in Experiment 17 you ignored the slight movement of the planet above and below the ecliptic. This movement of Mars north and south of the ecliptic shows that the plane of its orbit is slightly inclined to the plane of the earth's orbit. In this experiment you will measure the angular elevation of Mars from the ecliptic and so determine the inclination of its orbit.

Theory

From each of the photographs in the set of 16 you can find the latitude (angle from the ecliptic) of Mars as seen from the earth at a particular point in its orbit. Each of these angles must be converted into an angle as seen from the sun (heliocentric latitude).

Figure 1 shows that we can represent Mars by the head of a pin whose point is stuck into the ecliptic plane. We see Mars from the earth to be north or south of the ecliptic, but we want the N-S angle of Mars as seen from the sun. An example shows how the angles at the sun can be derived.

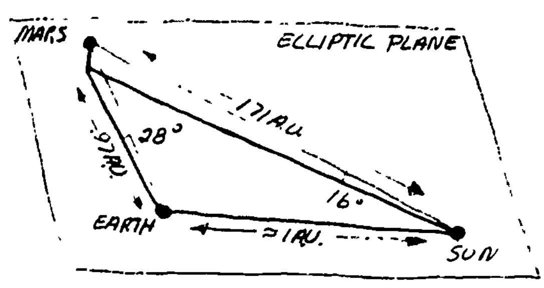


Fig. 1

In plate A (March 21, 1933) in the booklet of photographs Mars is about 3.2° north of the ecliptic as seen from the earth. But the earth was considerably closer to Mars on March 21, 193 than the sun was. The angular elevation of Mars above the ecliptic plane as seen from the sun will therefore be considerably less than 3.2°.

Measurement on the plot of Mars' orbit (Experiment 17) gives the distance earth-Mars as 9.7 cm (0.97 AU) and the distance sun-Mars as 17.1 cm (1.71 AU) on the date of the photograph. The heliocentric latitude of Mars is therefore

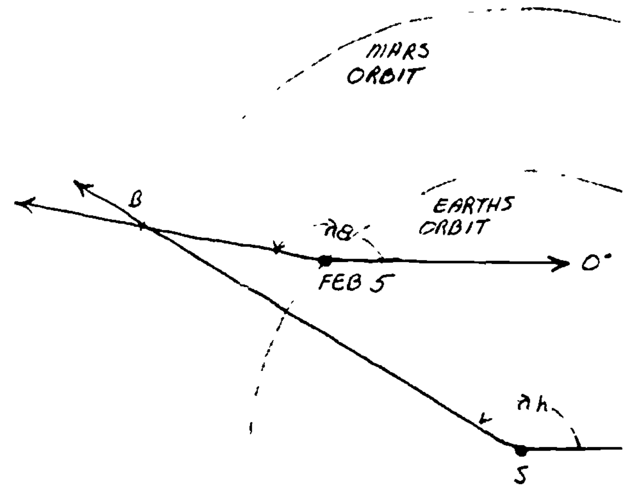
$$\frac{9.7}{17.1} \times 3.2°N = 1.8°N.$$

You can get another value for the heliocentric latitude of this point in Mars' orbit from photograph B (February 5, 1933). The earth was in a different place on this date so the geocentric latitude and the earth-Mars-distance will both be different, but the heliocentric latitude should be the same to within your experimental uncertainty.

Making the measurements

With the interpolation scale used in Experiment 17 measure the latitude of each image of Mars. If necessary, place the edge of a card across the 5° latitude marks. Remember that the scale factor is not the same on all the plates.

On your Mars orbit plot from Experiment 17 measure the corresponding earth-Mars and sun-Mars distances. From these values calculate the heliocentric latitudes as explained above. The values of heliocentric latitude calculated from the two plates in each pair (A and B, C and D, etc.) should agree within the limits of your experimental technique.



On the plot of Mars' orbit measure the <u>heliocentric longitude</u> λ_h for each of the eight Mars positions. Heliocentric longitude is measured from the sun, counter-clockwise from the 0° direction (direction towards Vernal Equinox), as shown in Fig. 2.

Complete the table given in Experiment 17 by entering the earth-to-Mars and sun-to-Mars distances, the geocentric and helio-centric latitudes, and the geocentric and and heliocentric longitudes for all six-teen plates.

Make a graph, like Fig. 3, that shows how the heliocentric latitude of Mars changes with its heliocentric longitude.

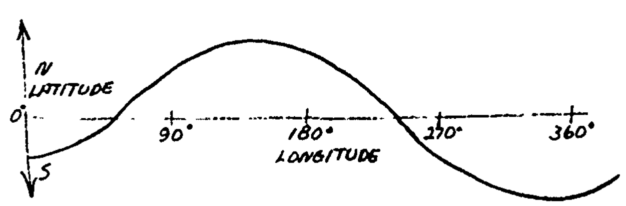
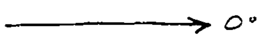


Fig. 3

Fig. 2 The heliocentric longitude (λ_h) of Point B on the Mars orbit is 150°; the geocentric longitude (λ_g) measured from the earth's position on February 5, the date of the photograph, was 169°.



From this graph you can determine two of the elements that locate the orbit of Mars with respect to the ecliptic. The point at which Mars crosses the ecliptic from south to north is called the ascending node, Ω. (The descending node is the point at which Mars crosses the ecliptic from north to south.) The planet reaches its maximum latitude above the ecliptic 90° beyond the ascending node. This maximum latitude equals the inclination of the orbit i, which is the angle between the plane of the earth's orbit and the plane of Mars' orbit.

Two angles, the longitude of the ascending node, Ω, and the inclination, i, locate the plane of Mars' orbit with respect to the plane of the ecliptic. One more angle is needed to locate the orbit of Mars in its orbital plane. This is the "argument of perihelion" ω and is the angle in the orbit plane between the ascending node and perihelion point. On your plot of Mars' orbit measure the angle from the ascending node Ω to the direction of perihelion to obtain the argument of perihelion, ω.

The elements of an orbit are discussed again in Experiment 21: Model of a Comet Orbit.

Parameters of an orbit

If you have worked along this far, you have done well. You have determined five of the six elements or parameters that define <u>any</u> orbit:

- a - semi-major axis, or average distance (determines the period) } From Experiment 17
- e - eccentricity (shape of orbit)
- i - inclination (tilt of orbital plane)
- Ω - longitude of ascending node (where orbital plane crosses ecliptic)
- ω - argument of perihelion (orients the orbit in its plane).

These elements fix the orbital plane of any planet or comet in space, tell

Transparency T18 shows this more clearly than a single drawing can.

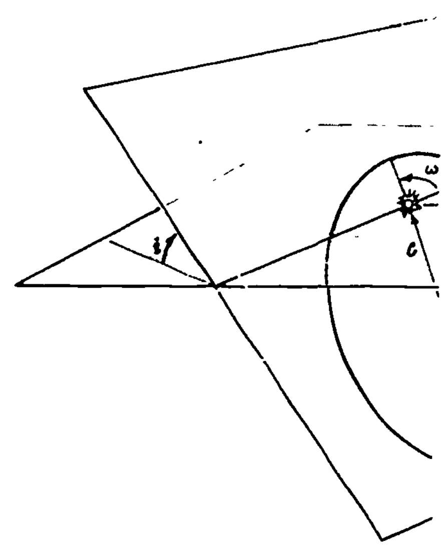


Fig.

This simple exercise provides additional experience with the concepts of orbit theory. Orbital eccentricity and Kepler's Second Law can both be studied. Although this is a relatively brief activity, the results can be surprisingly accurate. Some pupils have done it at home in less than 20 minutes.

Because the orbit of Mercury is not a circle, the tangent to the orbit is not perpendicular to the line joining sun to planet, i.e., the assumption suggested here is only an approximation. Students may find that they cannot draw a smooth curve to join all the points located in this way. In this case it is quite legitimate to move some of the points slightly. The final orbit should be a smooth curve that touches, without crossing, all the sight lines.

EXPERIMENT 19 The Orbit of Mercury

Mercury, the innermost planet, is never very far from the sun in the sky. It can only be seen at twilight close to the horizon, just before sunrise or just after sunset, and viewing is made difficult by the glare of the sun. Except for Pluto, which differs in several respects from the other planets, Mercury's orbit is the most eccentric planetary orbit in our solar system (e = 0.206). The large eccentricity of Mercury's orbit has been of particular importance, since it has led to one of the tests for the general theory of relativity.

Procedure

Let us assume a heliocentric model for the solar system. Mercury's orbit can be found from Mercury's maximum angles of elongation east and west from the sun as seen from the earth on various known dates.

The angle θ (Fig. 1), measured at the earth between the earth-sun line and the earth-Mercury line, is called the "elongation angle." Note that when θ reaches its maximum value, the elongation sight-lines from the earth are tangent to Mercury's orbit.

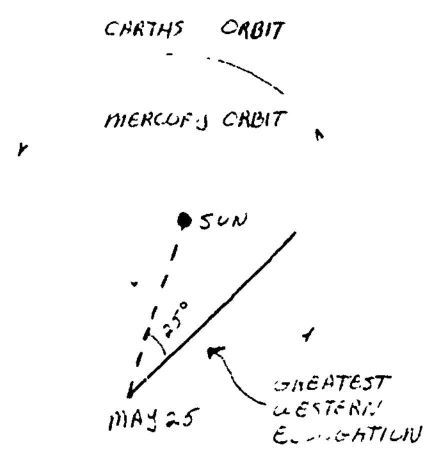
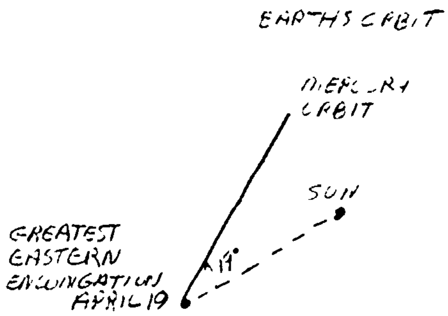


Fig. 1



Since the orbits of Mercury and the earth are both elliptical, the greatest value of θ varies from revolution to revolution. In Fig. 5.5(a) in the text, the 28° elongation angle given for Mercury refers to the maximum possible value of θ for that planet.

Plotting the orbit

Table 1

Some Dates and Angles of Greatest Elongation for Mercury (From the American Ephemeris and Nautical Almanac)

Date	θ
Jan. 4, 1963	19° E
Feb. 14	26° W
Apr. 26	20° E
June 13	23° W
Aug. 24	27° E
Oct. 6	18° W
Dec. 18	20° E
Jan. 27, 1964	25° W
Apr. 8	19° E
May 25	25° W

You can work from the plot of the earth's orbit that you established in Experiment 15. Make sure that the plot you use for this experiment represents the orbit of the earth around the sun, not of the sun around the earth.

If you did not do the earth's orbit experiment, you may use, for an approximate orbit, a circle of 10 cm radius drawn in the center of a sheet of graph paper. Because the eccentricity of the earth's orbit is very small (0.017) you can place the sun at the center of the orbit without introducing a significant error in the experiment.

Draw a reference line horizontally from the center of the circle to the right. Label the line 0°. This line points towards the Vernal Equinox and is the reference from which the earth's position in its orbit on different dates can be established. The point where the 0° line from the sun crosses the earth's orbit is the earth's position in its orbit on September 23.

The earth takes 365 days to move once around its orbit (360°). Use the rate of 1° per day, or 30° per month to establish the position of the earth on each of the dates given in Table 1. Remember that the earth moves around this orbit in a <u>counter-clockwise</u> direction, as viewed from the north celestial pole. Draw radial lines from the sun to each of the earth positions you have located.

Now draw sight-lines from the earth's orbit for the elongation angles. Be sure to note from Fig. 1 that for an <u>eastern</u> elongation, Mercury is to the <u>left</u> of the sun as seen from the earth. For a <u>western</u> elongation it is to the <u>right</u> of the sun.

You know that on a date of greatest elongation Mercury is somewhere along the sight line, but you don't know exactly where on the line to place the planet. You also know that the sight line is tangent to the orbit. A reasonable assumption is to put Mercury at the point along the sight line closest to the sun.

You can now find the orbit of Mercury by drawing a smooth curve through, or close to, these points. Remember that the orbit must <u>touch</u> each sight line without crossing any of them.

Calculating the semi-major axis a

To find the size of the semi-major axis of Mercury's orbit, relative to the earth's semi-major axis, you must first find the aphelion and perihelion points of the orbit. You can use your drawing compass to find the points on the orbit farthest from and closest to the sun.

Mercury's perihelion point occurs at a longitude of about 78° (in about the direction of the earth's position on December 10).

Measure the size of the orbit along the line perihelion-sun-aphelion. Since 10.0 cm corresponds to one AU (the semi-major axis of the earth's orbit), you can now obtain the semi-major axis of Mercury's orbit in AU's.

The major axis of Mercury's orbit is about 7.8 cm (0.78 AU) long.

$2a = 0.78$ AU
$a = 0.39$ AU

The accepted value for a is 0.387 AU.

Calculating orbital eccentricity

Eccentricity is defined as $e = c/a$ (Fig. 2). Since c, the distance from the center of Mercury's ellipse to the sun, is small on our plot, we lose accuracy if we try to determine e directly.

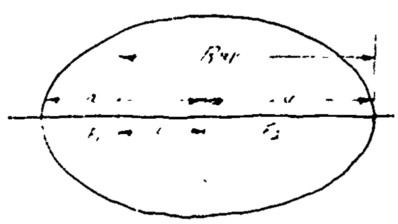


Fig. 2

From Fig. 2, R_{ap}, the aphelion distance, is the sum of a and c.

$$R_{ap} = (a + c);$$

but since $c = ae$,

$$R_{ap} = (a + ae) = a(1 + e).$$

Now, solving for e,

$$e = \frac{R_{ap}}{a} - 1.$$

From the plot $R_{ap} = 4.60$ cm, $a = 3.9$ cm therefore $e = \frac{4.60}{3.9} - 1 = 1.18 - 1$ therefore $e = 0.18$.

The accepted value of e is 0.206.

Label the sun S and the positions of Mercury as follows:

Jan.	4, 1963	A
Feb.	14, 1963	B
June	13, 1963	C
Aug.	24, 1963	D

Count the number of squares in the triangles SAB and SCD.

With out data:

$$\frac{SAB}{t_1} = \frac{274 \text{ sqs.}}{41 \text{ days}} = 6.7 \text{ sq/day}$$

$$\frac{SCD}{t_2} = \frac{525 \text{ sqs.}}{72 \text{ days}} = 7.3 \text{ sq/day}$$

$$\text{Avg. } \frac{274 + 525}{41 + 72} = \frac{799}{113} = 7.07 \text{ sq/day}$$

If the Law of Areas holds, these two ratios would be identically equal. Our experimentally determined ratios agree within 10%. Perhaps your students, using a smoother orbit, will obtain greater accuracy.

Sample Result

Kepler's second law

You can test the equal-area law on your Mercury orbit in the same way that is described in Experiment 17, The Orbit of Mars. By counting squares you can find the area swept out by the radial line from the sun to Mercury between successive dates of observation (e.g., January 4 to February 14, June 13 to August 24). Divide the area by the number of days in the interval to get the "area per day." This should be constant, if Kepler's law holds for your plot. Is it?

technique for solving problems. Modern high-speed digital computers use repeated steps to solve complex problems, such as the best path (or paths) for a Mariner probe to follow between earth and Mars.

Make these additional assumptions:

1) The force on the comet is a radial attraction toward the sun.

2) The force of the blow varies inversely with the square of the distance from the sun.

3) The blows occur regularly at the ends of equal time intervals, in this case 60 days. The magnitude of each brief blow has been chosen to equal the total effect of the continuous attraction of the sun throughout a 60-day interval.

The effect of the central force on the comet's motion

From Newton's second law you know that the gravitational force will cause the comet to accelerate towards the sun. If a force $\vec{F}$ acts for a time interval Δt on a body of mass m, w· know that

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\therefore \vec{F}\,\Delta t = m\,\Delta\vec{v}.$$

This equation relates the change in the body's velocity to its mass, the force, and the time for which it acts. The mass m is constant. So is Δt (assumption 3 above). The change in velocity is therefore proportional to the force $-\Delta\vec{v} \propto \vec{F}$. But remember that the force is not constant: it varies inversely with the square of the distance from comet to sun.

Q5 Is the force of a blow given to the comet when it is near to the sun greater or smaller than one given when the comet is far from the sun?

Q6 Which blow causes the biggest velocity change?

Q1 Ball will continue to move in a straight line with same velocity.

Q2 Path direction will change.

Q3 Speed may change depending on initial speed, and acceleration imparted by blow. (In circular motion only direction changes, not speed.)

Ball will move in a path made up of a series of straight line segments. If enough blows are given it will eventually return to somewhere near its starting point.

● In the Project Physics film loops numbers 13 and 14 (Program Orbit I and Program Orbit II) a computer works out the same orbit by iteration. In the first loop the time interval between blows is long and the result is close to what students should obtain. In the second loop a much shorter iteration interval is used; the orbit is smoother. Both loops (certainly the second one) should probably be used after the students have made their plots.

Q5 The force is greater if the comet is near to the sun

$$F \propto \frac{1}{R^2}.$$

Q6 The greater the force of the blow the greater the velocity change ($\Delta\vec{v} \propto F$). Use transparency T14, overlays 2 and 4, to help explain the addition of velocities.

There may be some confusion between the two times involved, and their significance:-

Because all blows have the same <u>duration</u> (Δt), Newton's Second Law $\vec{F} = m\frac{\Delta v}{\Delta t}$ can be simplified to $\Delta v \propto F$ (m also being constant).

Because the time interval <u>between blows</u> is constant (60 days) the comet's displacement along its orbit during a 60-day interval is proportional to its velocity.

$\Delta d = \vec{v} \times 60$ days, becomes $\Delta \vec{d} \propto \vec{v}$

In Fig. 1 the vector $\vec{v}_0$ represents the comet's velocity at the point A. You want to plot the <u>position</u> of the comet. You must use its initial velocity ($\vec{v}_0$) to derive its displacement (Δd_0) during the first sixty days: $\Delta d_0 = \vec{v}_0 \times 60$ days. Because the time intervals between blows is always the same (60 days) the displacement along the path is

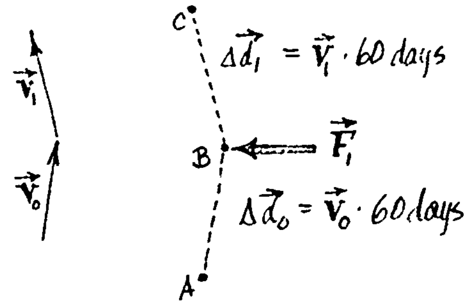


Fig. 1

proportional to the velocity—$\Delta \vec{d} \propto \vec{v}$. We can therefore use a length proportional to the comet's velocity in a given 60-day period to represent its displacement during that time interval.

During the first sixty days, then, the comet moves from A to B (Fig. 1). At B a blow provides a force F_1 which causes a velocity change $\Delta \vec{v}_1$. The new velocity after the blow is $\vec{v}_1 = \vec{v}_0 + \Delta \vec{v}_1$, and is found by compieting the vector triangle (Fig. 2).

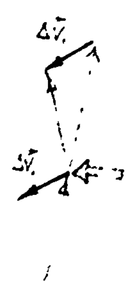


Fig. 2

The comet therefore leaves point B with velocity $\vec{v}_1$ and continues to move with this velocity for another 60-day interval. The displacement $\Delta\vec{d}_1 = \vec{v}_1 \times 60$ days establishes the next point, C, on the orbit.

The scale of the plot

The shape of the orbit depends on the initial position and velocity, and on the force acting. Assume that the comet is first spotted at a distance of 4 AU's from the sun. Its velocity at this point is $\vec{v} = 2$ AU per year (about 20,000 miles per hour) at right angles to the sun-comet radius.

The following scale factors will reduce the orbit to a scale that fits conveniently on a 20" × 20" piece of grapn paper.

1. Let 1 AU be scaled to 2.5 inches, so 4 AU becomes 10 inches (SA, in Fig. 3).

2. Since the comet is hit every 60 days, it is convenient to express the velocity in AU's per 60 days. We will adopt a scale factor in which a velocity of 1 AU/60 days is represented by a vector 2.5 inches long.

The comet's initial velocity of 2 AU per year can be given as $\frac{2}{365}$ AU per day, or $\frac{2}{365} \times 60 = 0.33$ AU per 60 days. This scales to a vector 0.83 inches long.

The <u>displacement</u> of the comet in the first 60 days ($\Delta d_0 = \vec{v}_0 \times 60$) is (0.33 AU/60 days) × 60 days = 0.33 AU. This displacement scales to 0.83 inches.

Notice that AB is perpendicular to SA, the line from the sun to S(Fig. 3).

The particular orbit chosen is similar to that of the short-period comets, like encke's Comet, which stay entirely within the orbit of Jupiter. These parameters, and the 60-day interval, give an orbit which is completed in about 25 steps. Half the orbit can be obtained in 12 steps.

The earth's average orbital speed is about 60,000 miles per hour.

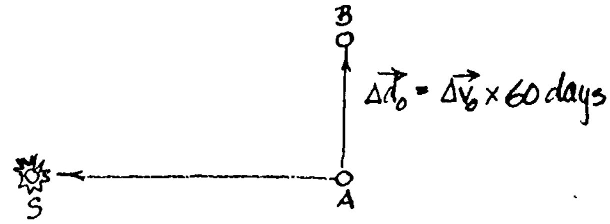


Fig. 3

Computing Δv

On the scale and with the 60-day iteration interval that we have chosen the force field of the sun is such that the Δv given by a blow when the comet is 1 AU from the sun is 1 AU/60 days.

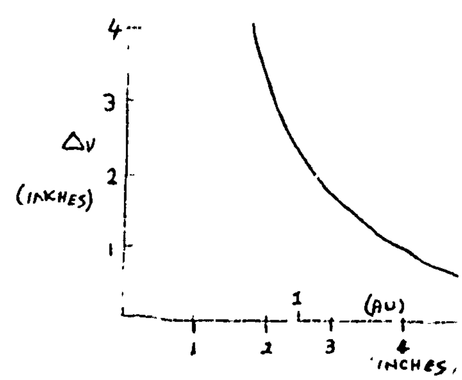
Q7 What will the Δv be at a distance of 2 AU from the sun?

Q7 At twice the distance from the sun the force, and therefore the Δv will be <u>one quarter</u>. If Δv = 1 AU/60 days for R = 1 AU, then for R = 2 AU Δv = 1/4 AU/60 days.

Values of Δv for other distances from the sun which have been calculated according to the inverse-square law are given in Table 1.

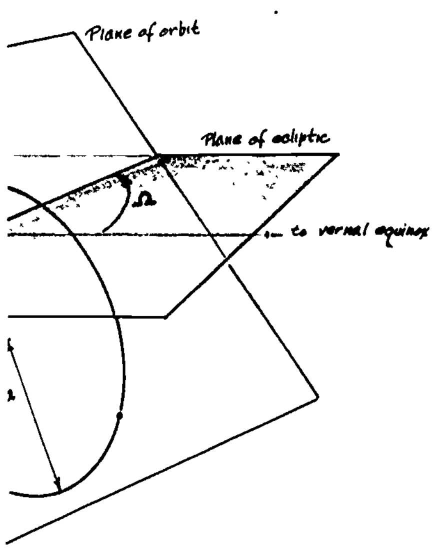
Table 1

R, from sun		$\Delta \vec{v}$	
AU	Inches	AU/60 days	Inches
0.75	1.87	1.76	4.44
0.8	2.00	1.57	3.92
0.9	2.25	1.23	3.07
1.0	2.50	1.00	2.50
1.2	3.0	0.69	1.74
1.5	3.75	0.44	1.11
2.0	5.0	0.25	0.62
2.5	6.25	0.16	0.40
3.0	7.50	0.11	0.28
3.5	8.75	0.08	0.20
4.0	10.00	0.06	0.16

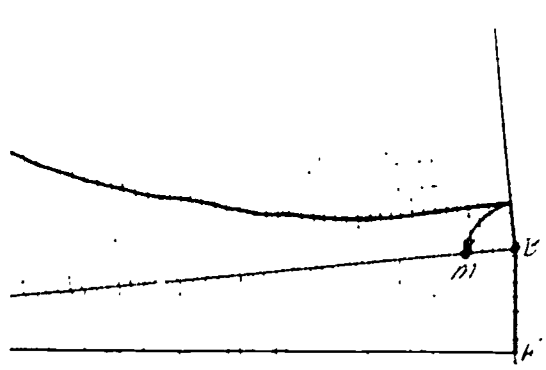


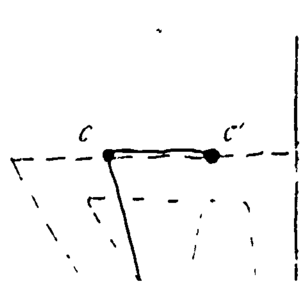
same procedures to make a model of the orbit of Mars. Cardboard, plastic sheets, or wire could be used.

E	May 26, 1935	186.7	0.6N	[illegible]	[illegible]	0.3N
F	Apr 12, 1937	245.7	0.7N	[illegible]	22[illegible]	0.3N
G	Sep 16, 1939	297.5	4.6S		335	1.7S
H	Aug 4, 1941	016.5	4.1S		335	1.7S
I	Nov 22, 1941	012.1	0.6S		043	0.2S
J	Oct 10, 1943	080.1	0.5S		043	0.2S
K	Jan 21, 1944	065.6	2.7N		096	1.3N
L	Dec 9, 1945	123.2	2.9N		096	1.3N
M	Mar 19, 1946	107.6	3.0N		142	1.9N
N	Feb 3, 1948	153.4	4.4N		142	1.8N
O	Apr 4, 1948	138.3	3.1N		169	1.6N
P	Feb 20, 1950	190.7	3.5N		169	1.6N



4





Twelve or thirteen steps should bring the comet to perihelion. If this is all they have time for students can complete the orbit by assuming that the halves are symmetrical.

Data from one plot:

Perihelion distance = 1.1 AU
Eccentricity = 0.54
Period of revolution = 24 × 60 days
= 4 years

The closer the comet is to the sun the greater its speed.

A student could draw a smooth ellipse by using the two-pin-and loop-of-string technique (Text Unit 2, page 55).

The two film loops, Program Orbit I and Program Orbot II, bring out the point that the shorter the iteration interval the closer the orbit is to a smooth ellipse.

8. Again the comet moves with uniform velocity for 60 days. Its displacement in that time is $\Delta d_1 = \vec{v}_1 \cdot 60$ days and because of the scale factor we have chosen, the displacement is represented by the line BC. C is therefore a point on the comet's orbit.

9. Repeat steps 1 through 8 to establish point D and so forth, for 14 or 15 steps (25 steps gives the complete orbit).

10. Connect points A, B, C... with a smooth curve. Your plot is finished.

Prepare for discussion

Since you derived the orbit of this comet, you may name the comet.

From your plot, find the perihelion distance.

Q8 What is the length of the semi-major axis of the ellipse?

Q9 Find the center of the orbit and calculate the eccentricity.

Q10 What is the period of revolution of your comet? (Refer to text, Sec. 7.3.)

Q11 How does the comet's speed change with its distance from the sun?

If you have worked this far, you have learned a great deal about the motion of this comet. It is interesting to go on to see how well the orbit obtained by iteration obeys Kepler's laws.

Q12 Is Kepler's first law confirmed? (Can you think of a way to test your curve to see how nearly it is an ellipse?)

The time interval between blows is 60 days, so the comet is at positions B, C, D..., etc., after equal time intervals. Draw a line from the sun to each of these points (include A), and you have a set of triangles.

Find the area of each triangle. The area of a triangle is given by $A = \frac{1}{2}ab$ where a and b are altitude and base, respectively. Or you can count squares to find the areas.

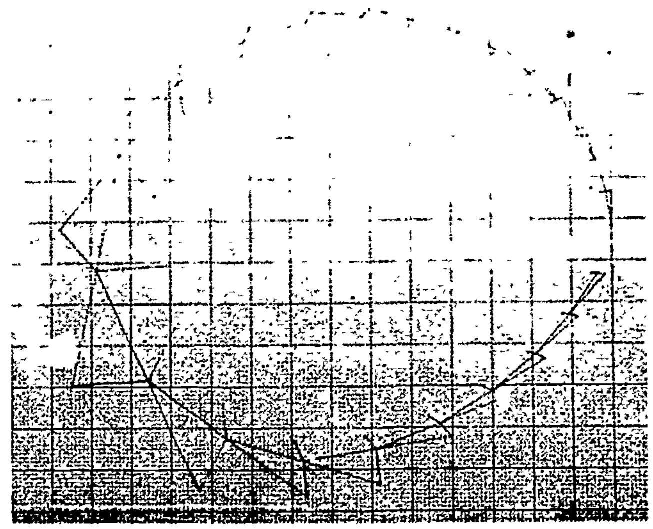
Q13 Is Kepler's second law (the Law of Equal Areas) confirmed?

More things to do

1. The graphical technique you have practiced can be used for many problems. You can use it to find out what happens if different initial speeds and/or directions are used. You may wish to use the $1/R^2$/force computer, or you may construct a new computer, using a different law (e.g., force proportional to $1/R^3$, or to $2/R$ or to R) to produce different paths; actual gravitational forces are not represented by such force laws of course.

2. If you use the same force computer (graph) but reverse the direction of the force (repulsion), you can examine how bodies move under such a force. Do you know of the existence of any such repulsive forces?

Based on similar experiment developed by Leo Lavatelli *Am. J. Phys.* 33, 605, 1967.



The shapes of these orbits will be different, of course, but the Law of Equal Areas applies to any central force, attractive or repulsive.

Forces between electrically charged bodies.

The transparency T17 illustrates the elements of an orbit more clearly than a single drawing can.

EXPERIMENT 21 Model of a Comet Orbit

The complete orbit of a comet can be derived from only three observations of its position, but this is quite an intricate process. Here we reverse the problem and make a three dimensional model of the orbit from the six elements that describe the comet's orbit.

From this model, you will be able to construct at least a rough timetable (ephemeris) for the apparent positions of the comet and can check these against reported observations. Halley's comet has been considered several times in the text and its orbit has several interesting features. When you have constructed the model you can compare it to the plot of observations across the sky during its last return, Fig. 6.10, p. 43 of your text.

The elements of a comet's orbit

Six elements are needed to describe a comet's orbit. Three of these—semi-major axis, eccentricity and perihelion date—are already familiar from planetary orbits.

The orbits of the earth and the other planets are all in or very close to the same plane—the plane of the ecliptic. (If you did Experiment 18, you will remember that Mars' orbit is inclined at about 1.8 degrees to the ecliptic.) But this is not so for comets. The orbit of a comet can be inclined to the ecliptic plane at any angle. Three more elements—inclination, longitude of ascending node and angle from node to perihelion are needed to describe the inclination. These elements are illustrated in Fig. 1.

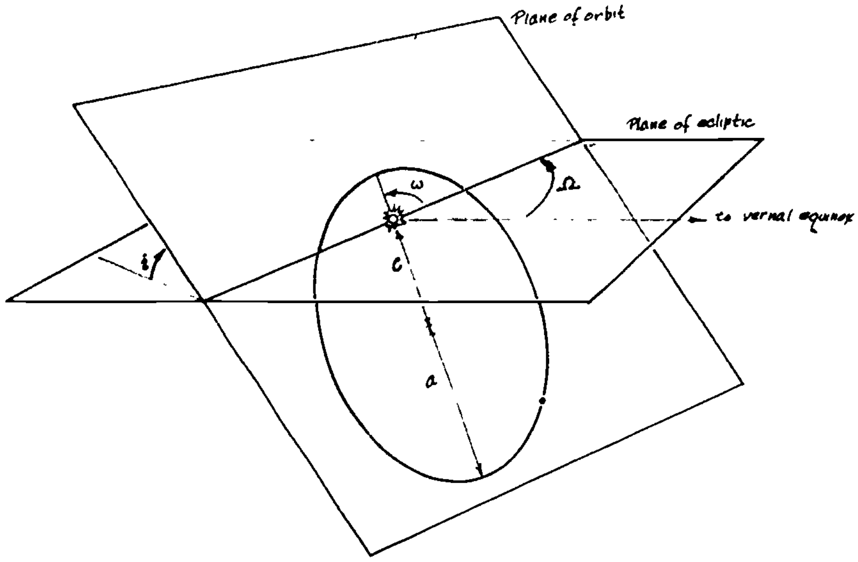


Fig. 1

The elements of Halley's comet are, approximately:

a (semi-major axis)	17.9 AU
e (eccentricity)	0.967
i (inclination)	162°
Ω (longitude of ascending node)	57°
ω (angle from ascending node to perihelion)	112°
T (perihelion date)	April 20, 1910

From these data we also know that period $P = 76$ years, and perihelion distance $q = a(1-e) = 0.59$ AU.

Plotting the orbit

In the center of a large sheet of stiff cardboard draw a circle 10 cm in radius for the orbit of the earth. Also draw approximate (circular) orbits for Mercury (radius 0.4 AU) and Venus (radius 0.7 AU). For this plot, you can consider all of the planets to lie roughly in the one plane. Draw a line from the center, the sun, and mark this line as 0° longitude. Now, as in Experiments 15 and 17, you can establish the position of the earth on any date. You will need these positions of the earth later in the experiment.

A parabola is the conic section having eccentricity of 1. For Halley's Comet e = 0.967.

Use another large sheet of stiff card-board to represent the orbital plane of the comet. Down the middle draw a line for the major axis of the orbit. Choose a point for the sun on this line about 15 cm from one edge of the sheet.

For this experiment, we can consider the oribt of Halley's comet as being essentially a parabola. In fact the small near-sun section of the large ellipse dces have almost exactly that shape. Now you want to construct a parabola.

You have an orbital plane with the major axis drawn and the position of the sun marked. Use the same scale as for the earth's orbit (1 AU equals 10 cm), and mark a point on the major axis at a distance q from the sun. This is the perihelion point, one point on the orbit. The orbit will be symmetrical around the major axis and will flare out and away from the perihelion point (see Fig. 2).

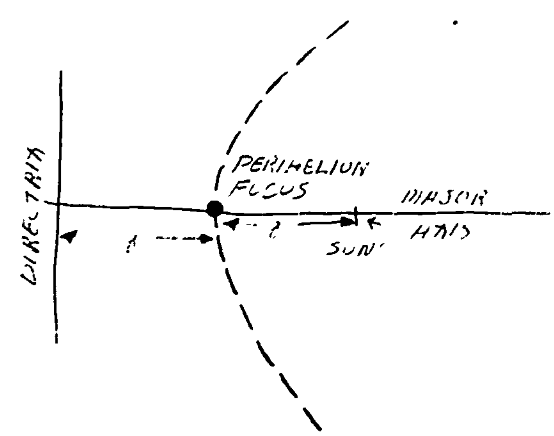
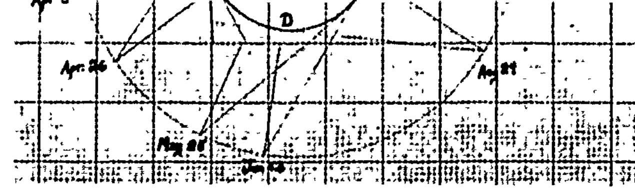


Fig. 2

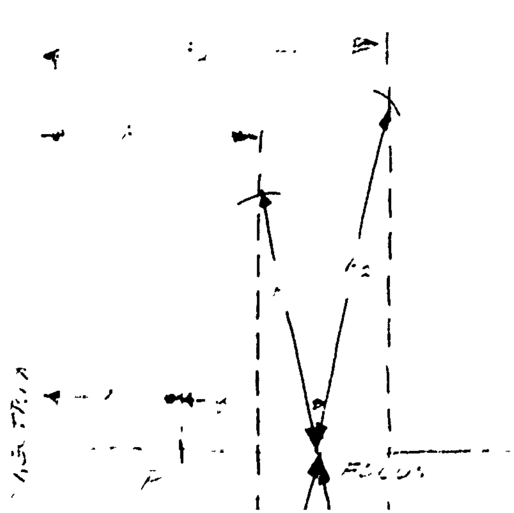
Mark another point on the major axis, at a distance q beyond the perihelion point, or at a distance 2q from the sun. Draw a line perpendicular to the axis at this point: this construction line is known in analytical geometry as the "directrix." A parabola has the property that each point on it is equidistant



from a straight line (the directrix) and from a fixed point (the focus).

Here is one way to draw a parabola: if you know another, try it. Draw a line parallel to the directrix and at a distance R from it. Use a drawing compass centered at the focus to swing two arcs of radius R, one above and one below the major axis.

The intersections of the arcs and the line are two points on the parabola. Repeat the process with arcs of different sizes to locate more points on the parabola.



prediction of the orbit of a body in a varying force field is complicated. However, if one assumes that the force acts intermittently at equal time intervals, like hammer blows, an orbit can be approximated rather quickly. This is the activity of this experiment.

▲ This "thought experiment" could form the basis of a demonstration in which the teacher applies repeated lateral (sideways) blows all directed towards the same point, to a heavy ball, or air puck. Alternatively try a demonstration involving all the students—each one gives a centrally directed blow to the ball or puck, as it passes him. Ball's initial velocity must be fairly high, and the blows not too strong.

[1] Read Feynman's Lectures on Physics, Vol. I, Chapter 9-7, for a mathematical equivalent of this geometric method for computing orbits. Newton used this method to prove that Kepler's Second Law follows from a central force hypothesis: Principia, p. 40 etc., in paperback edition, (see text, Chapter 8.4, and the Unit II Reader). The experiment described here is based on that developed by Dr. Leo Lavatelli, University of Illinois (Am. J. Phys., Vol. 33, p. 605, 1965).

level surface such as a piece of plate glass.

Q1 What would you predict for the path of the ball, based on your knowledge from Unit 1 of Newton's laws of motion?

Q2 Suppose you were to strike the ball from the side. Would the path direction change?

Q3 Would the speed change? Suppose you gave the ball a series of "sideways" blows as it moves along, what do you predict its path might be?

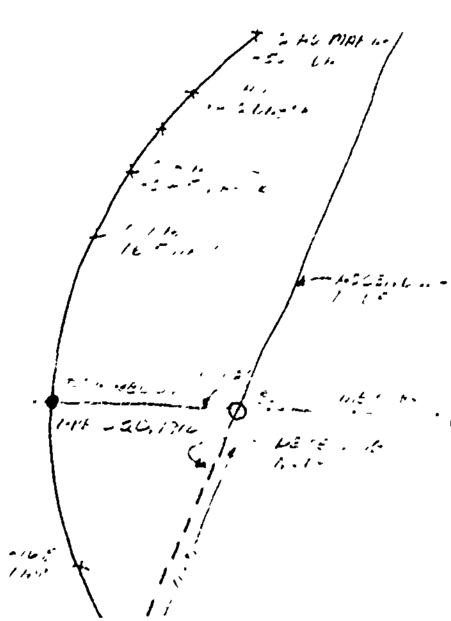
Reread Sec. 8.4 if you have difficulties answering these questions.

A planet or satellite in orbit has a continuous force acting on it. But as the body moves, the magnitude and direction of the force change. To predict exactly the orbit under the application of this constantly changing force requires advanced mathematics. However, you can get a reasonable approximation of the orbit by plotting a series of separate points. In this experiment, therefore, you will assume a series of sharp "blows" acting at 60-day intervals on a moving comet and explore what orbit the body would follow.

The application of repeated steps is known as "iteration." It is a powerful

Now we have the two orbits, the comet's and the earth's, in their planes, each of which contains the sun. You need only to fit the two together.

The line along which the orbital plane cuts the ecliptic plane is called the "line of nodes." Since you have the major axis drawn, you can locate the ascending node, in the orbital plane, by measuring ω from perihelion in a direction opposite to the comet's motion (see Fig. 4).



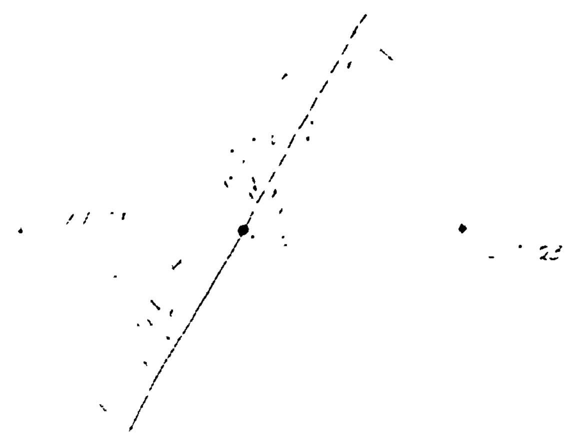


Fig. 5

To establish the model in three dimensions you must now fit the two planes together at the correct angle. Remember that the inclination i is measured upward (northward) from the ecliptic from the longitude Ω + 90° (see Fig. 1).

You can construct a small tab to support the orbital plane in the correct position. In the ecliptic plane draw a line in the direction Ω + 90°. From this line measure off the angle of inclination i towards the descending node.

If the inclination is less than 90°, draw a line from the sun at the angle of inclination. Then with a razor blade cut a section of the tab as shown in the sketch (Fig. 6). The model will be

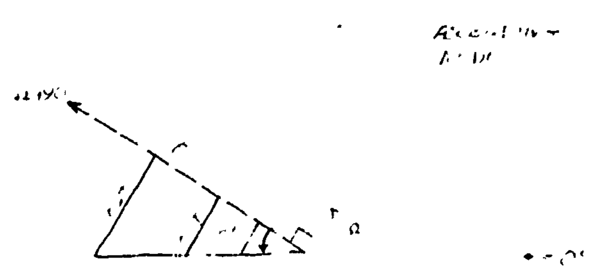


Fig. 6

Halley's comet moves in the opposite sense to the earth and other planets. Whereas the earth and planets move counterclockwise when viewed from above (north of) the ecliptic, Halley's comet moves clockwise.

stronger if you do <u>not</u> cut the triangular wedge all the way into the sun's position. Fold the tab up vertically along the line (Ω + 90°) and tape it into place.

If the inclination exceeds 90°, as it does for Halley's Comet, cut the tab and bend it up along the line Ω + 270° (see Fig. 7).

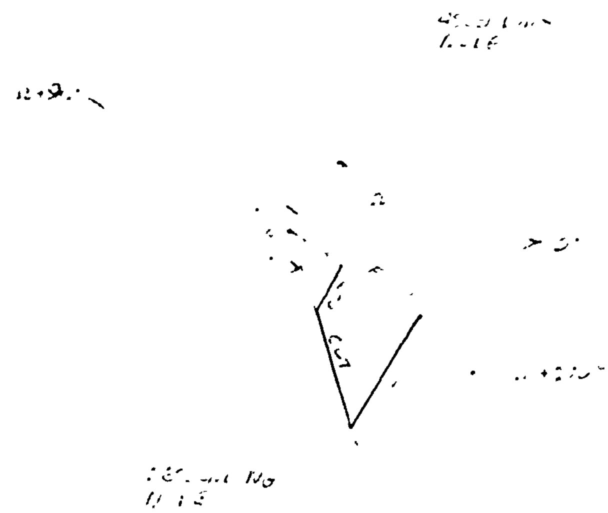


Fig. 7

When you fit the two planes together you will find that the comet's orbit is on the underside of the cardboard. The simplest way to transfer it to the top is to prick through with a pin at enough points to draw a smooth curve.

Finally, you can develop the timetable, or ephemeris, of the comet in its parabolic orbit. Because all parabolas have the sam shape and eccentricity (e = 1) this calculation is simple. The time t (days) required for a body to move from a solar distance r (AU) to perihelion is given by:

$$t = 27.4(r + 2q)(r - q)^{\frac{1}{2}}$$

where q is the perihelion distance.

Times for certain values of r and q are given in Table 1. If plotted, these data produce an interesting set of curves.

Table 1

Time (days) to perihelion passage from different solar distances (r) for parabolic orbits of different perihelion distances (q)

(From Between the Planets, F.G.Watson, pp. 215-216)

Solar dist. r (AU)	Perihelion distance, q (AU) 0.0	0.2	0.4	0.6*	0.8	1.0	1.2
2.0	77.5	88.1	97.1	103.8	108.0	109.6	107.8
1.8	66.1	76.2	84.3	90.0	93.2	93.0	88.6
1.6	56.1	64.8	72.0	76.7	78.2	76.0	69.4
1.4	45.4	54.0	60.3	63.6	63.4	59.0	46.6
1.2	36.0	43.9	48.9	50.7	48.5	38.0	0.0
1.0	27.4	34.3	38.0	38.2	31.9	0.0	
0.8	19.6	25.4	27.8	24.5	0.0		
0.7	16.1	21.3	22.5	16.5			
0.6	12.8	17.3	17.2	0.0			
0.5	9.7	13.5	11.3				
0.4	6.9	9.8	0.0				
0.3	4.5	6.1					
0.2	2.5	0.0					
0.1	0.9						

*This column was used in determining dates on Fig. 4.

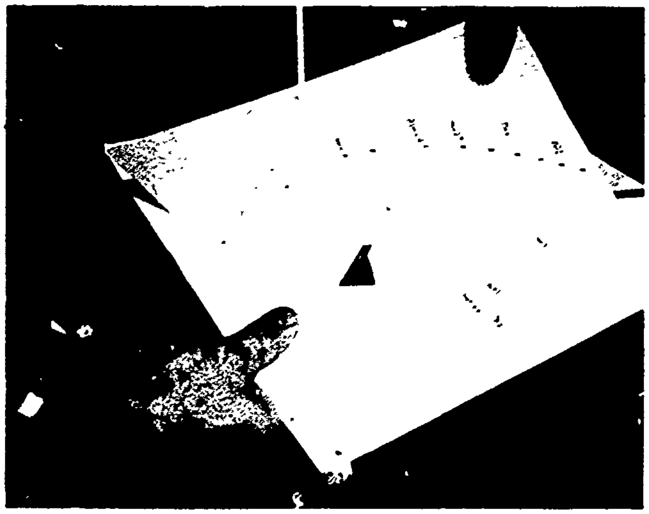
Since the date of perihelion passage of Halley's Comet T (April 20, 1910) is given, you can use this table to find the dates at which the comet was at various solar distances before and after perihelion passage. Mark these dates along the orbit.

Finally locate the earth's position in its orbit for each of these dates.

For each date make a sightline from the earth to the comet by stretching a thread between the two points.

You can make a timetable for the apparent positions of the comet in the sky by measuring the longitude of the comet around the ecliptic plane as seen from the earth on each date. With a protractor estimate the latitude of the comet (its angular height from the ecliptic).

Plot these points on a star map having ecliptic coordinates or plot them roughly relative to the ecliptic on a map having equatorial coordinates, such as the constellation chart SC-1.



Picture showing how the two planes go together.

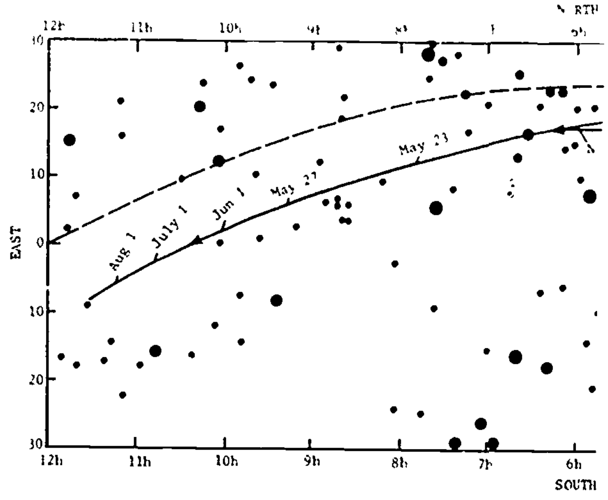


Fig. 6.10 Motion of Halley s Comet i

If you have persevered this far, and your model is a fairly accurate one, it should be easy to explain the comet's motion through the sky shown in Fig. 6.10. The dotted line in the figure is the ecliptic.

With your model of the comet orbit you can now answer some very puzzling questions about the behavior of Halley's Comet in 1910, as shown in Fig. 6.10.

1. Why did the comet appear to move westward for many months?

2. How could the comet hold nearly a stationary place in the sky during the month of April 1910?

3. After remaining nearly stationary for a month, how did the comet move nearly halfway across the sky during the month of May 1910?

4. What was the position of the comet in space relative to the earth on May 19th?

5. If the comet's tail was many millions of miles long on May 19th, is it likely that the earth passed through part of the tail?

6. Were people worried about the effect a comet's tail might have on life on the earth? (See newspapers and magazines of 1910!)

so that the horizontal (R) axis passes through the point where the blow is applied (e.g., point B). Read off the value of R at B. Pick off the value of Δv corresponding to this R from the computer with dividers. Lay off this distance (Δv) inwards along the radius line towards the sun (see Fig. 5 on next page).

Making the plot

1. Mark the position of the sun S halfway up the large graph paper and 12 inches from the right edge.

2. Locate a point 10 inches (4 AU) to the right from the sun S. This is point A where we find the comet.

3. Draw vector $\vec{AB}$ 0.83 inches (0.33 AU) long through point A, perpendicular to SA. This vector represents the comet's

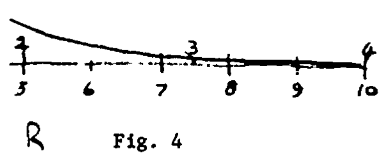
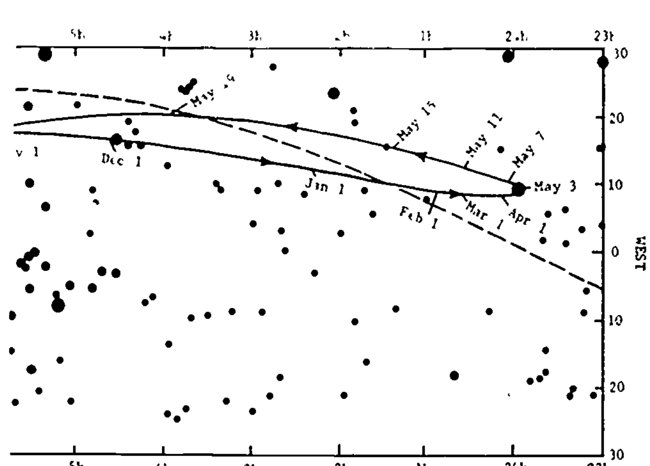


Fig. 4



n 1909-10.

7. Did anything unusual happen? How dense is the material in a comet's tail? Would you expect anything to have happened?

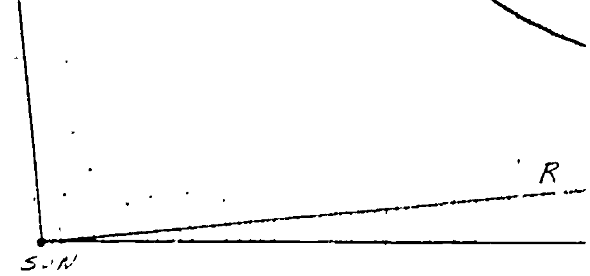


Fig. 5

velocity (0.33 AU/60 days), and B is its position at the end of the first 60-day interval. At B a blow is struck which causes a change in velocity $\Delta\vec{v}_1$.

4. Use your Δv computer to establish the distance of B from the sun at S, and to find $\Delta\vec{v}_1$ for this distance (Fig. 5).

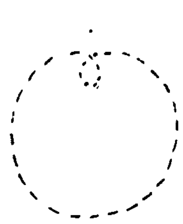
5. The force, and therefore the change in velocity, is always directed towards the sun. From B lay off $\Delta\vec{v}_1$ towards S. Call the end of this short line M.

6. Draw the line BC' which is a continuation of AB and has the same length as AB.

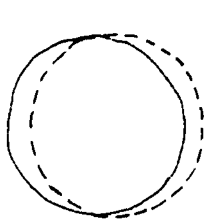
7. To find the new velocity $\vec{v}_1$ use a straightedge and triangle to draw the line C'C parallel to BM and of equal length. The line BC represents the new velocity vector $\vec{v}_1$, the velocity with which the comet leaves point B (Fig. 6).

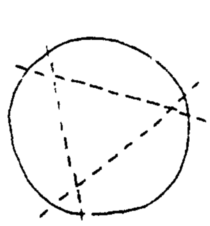
Epicycle Machine

This has been redesigned slightly. To produce "inside" loops the drive band



should not be crossed. If it is crossed the epicycle rotates in the opposite sense to the deferent and the resulting figure will either (a) be an eccentric circle (for 1:1 gear ratio) or (b) will have "outside" loops (for other ratios).





To attach the machine to a phonograph turntable (see "Epicycles and Retrograde Motion" in the Unit 2 Student Handbook) tape an extra piece of wood (1" × 3/4" × 8") to the handle. Drill a 9/32" hole in the middle to take the turntable spindle.

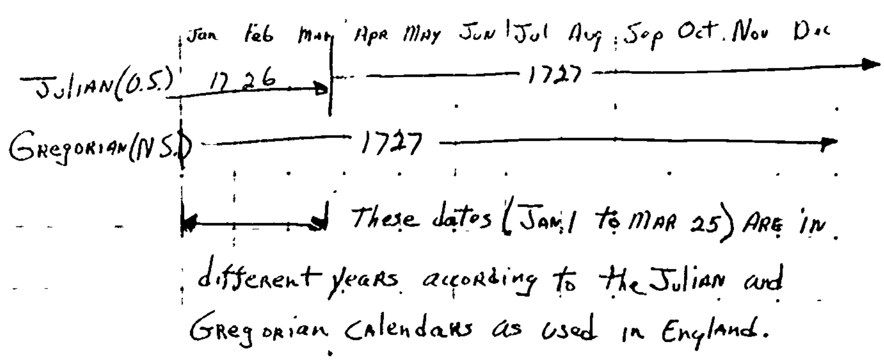
Background Information on Calendars
(modifying Secs. 5.1 and 3.1)

In 45 B.C. Julius Caesar decreed a new civil calendar of 365¼ days, based only upon the motion of the sun. As the text of Unit 2 indicates, this "Julian" year exceeded the actual motion of the sun by 11 minutes and 14 seconds. As a result the Julian calendar was slow by one day in 128 years. By 1582 A.D. the Julian calendar was off by ten days and the sun passed the vernal equinox on March 11 rather than on March 21, as required by church canons. In 1582 Pope Gregory XIII abolished the old calendar and replaced it with a new civil calendar now known as the Gregorian or New Style Calendar. October 4, 1582 was followed by October 15th. The new calendar was immediately adopted by all Catholic countries, but England and some other non-Catholic countries would not adopt the new calendar because it was established by Catholics. Not until 1752 did the Gregorian calendar (New Style) finally become official in England.

When the change from Julian to Gregorian calendars was made in England in 1752, September 2 was followed by September 14 for a correction of 11 days. Many peasants are reported to have claimed they "wanted their eleven days back." George Washington was actually born on February 11, 1732. Scholars have to be careful to distinguish Julian (Old Style) dates from Gregorian (New Style) dates on original documents from the latter half of the eighteenth century.

Some further examples of the confusions presented to historians by the change between the Julian and Gregorian calendars can be illustrated by the birth and death dates of Newton. Often Newton is said to have been born in the year that Galileo died. Galileo died in Italy on January 8, 1642 (New Style) and Newton was born in England on December 25, 1642 (Old Style). When the date of Newton's birth is changed to New Style (Gregorian) it becomes January 5, 1643.

Newton's death is generally reported as occurring on Ma[illegible] 20, 1727; yet the year carved on his [illegible]b in Westminster Abbey is 1726. When the Calendar Act of 1750 went into effect in 1752, not only were eleven days dropped from the time record, but also the date of New Year's Day was changed from March 25 to January 1. Actually Newton died on March 20, 1726 (Old Style), but on the new calendar, which was adopted later, this became March 20, 1727 (New Style). Schematically the change looked like this:



Articles
Armillary Sphere
Size and Dist.—Sun and Moon
Epicycles

Armillary Sphere

(modifying Secs. 5.1 to 5.3)

An armillary sphere is a mechanical device which shows the various coordinate systems used in the sky. Metal arcs are used to represent the horizon, the celestial equator and the ecliptic as well as north-south and east-west coordinates. You will find such a device very helpful as you try to visualize these imaginary lines in the sky.

Armillary spheres, and plastic spheres which can serve the same function, are available from several scientific equipment companies. (For example, item #6881A from the Welch Scientific Company, listed 1965. sells for $49.90.) However, you can make a reasonably satisfactory substitute for $2.00 or less from a hemispherical hanging-plant basket purchased from a garden supply store.

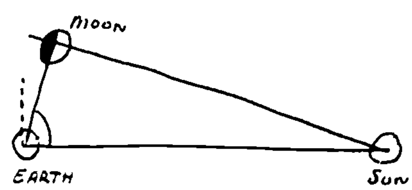
These wire baskets come in various sizes; 10 or 12 inches in diameter would probably be most useful. Two would make a sphere. They will have a great circle with ribs going toward the bottom (pole). One or two small circles of wire paralleling the great circle help support the ribs (meridians). You can add wire circles for other coordinates; for example, if the great circle represents the equator, add another great circle tipped at 23½° to show the ecliptic. Use bits of paper to locate some of the brighter stars.

Note on the Sizes and Distances to the Sun and Moon, by Aristarchus.

(modifying Sec. 5.6)

This summary is based on the section in A Sourcebook in Greek Science, M. R. Cohen and I. E. Drabkin, McGraw Hill Co., New York City, 1948.

Aristarchus assumed that the moon was a sphere shining by reflected sunlight. As the figure shows in an exaggerated manner, when the moon appeared to be just half illuminated, it would be located less than 90° from the sun. Aristarchus measured the angle at the earth between the sun and moon when the moon appeared to be exactly at first quarter—half illuminated, as 87° (actually the angle is about 89° 50'). By a complicated geometrical analysis he concluded that the sun must be between 18 and 20 times farther from the earth than is the moon. But the distance to the moon was known approximately to be several hundred thousands of miles. Therefore the distance to the sun must be several million miles.



The analysis also provided information on the sizes of the moon and sun. The moon was found to have a diameter about one third of that of the earth. Then the sun, having the same anuglar size at 18 times the moon's distance, must be at least 18/3 or 6 times the diameter of the earth, and 216 times the volume of the earth. To some philosophers, this raised a question whether the larger body would move around the smaller one. Note that there was no evidence of concern for the masses of these bodies.

Epicycles

(modifying Sec. 5.7)

The epicycle photographed for Fig. 5.14 has a radius about half that of the deferent. Ptolemy's values for the planets are almost the same as those used by Copernicus and shown in tables 6.1 to 6.3.

The rates of angular motion obtained by the use of epicycles did not agree well with the observations at certain sections of the orbits. As Fig. 5.16 indicates, in a series of oppositions of Mars no two occurrences were identical. To provide a better fit between theory and observations, Ptolemy introduced another geometrical device, called the equant. As Fig. 5.17 of an equant indicates, the point P moved at a constant distance from the center, O. (Epicycles around P could also be added.) P moved at a uniform angular rate about an off-center point C, while the earth and observer were located at E, offset equally but oppositely to C. The search for stability and uniformity, or predictability, required increasingly complex descriptions.

There is little evidence that anyone believed that the planets actually moved through space in paths described by Ptolemy; his analysis was strictly mathematical for the prediction of precise positions of each planet separately.

Figure 5.15 is a simplified scale diagram of the Ptolemaic system. The simplification results from the omission of the eccentrics and equants, and of the several motions of the moon. Notice

that each planet had only one epicycle. All other cyclic motions were represented by eccentrics and equants. The very large epicycle for Venus occupies about three-fourths of the space between the earth and sun. To Copernicus this was of special interest. With a protractor, students can check the angles subtended at the earth by the epicycles of Mercury and Venus to see if they agree with those shown in Fig. 5.5.

The lower part of Fig. 5.15 shows that the radii of the epicycles for Mars, Jupiter and Saturn had a period of one year and were always in line with the earth-sun line. This diagram will become important in Chapter 6 when we discuss how Copernicus replaced all these large epicycles by one annual motion for the earth, and derived distances to the orbits of the planets. In the Ptolemaic system, each planet was considered to be at a distance such that its motion did not quite overlap that of the adjacent planets. Actually, to Ptolemy these planetary distances were not important.

Our awareness of the advanced degree of Greek mathematics and technical skill was sharply increased a few years ago by the discovery of the so-called Antikythera machine, named for an island near which it was found. About the size of a large book, this device apparently contained at least twenty gears and a crown wheel, as well as pointers moving over dials. While the use of this complex, but badly corroded, machine is still to be unravelled, it is suspected that it was used to compute the positions of the sun and moon, and possibly of the planets too. The machine was recovered from the remains of a ship which sank about 65 B.C. Some students may want to read "An Ancient Greek Computer" by Derek J. De Solla Price in Scientific American, June, 1959.

Note on the "Chase Problem"

(modifying Sec. 6.2)

The motions of the hands of a clock provide a commonplace illustration of the "chase problem" described in the text and teacher guide, Sec. 6.2. Because the hands are moving in the same direction and because the hour hand continually moves ahead, the minute hand must "chase" the hour hand in order to overtake and pass it. Th questions listed below might be used to stimulate class discussion before or during a demonstration with a clock.

1. How many times does the minute hand overtake and pass the hour hand during an elapsed time of 12 hours? (Eleven times.)

2. Starting with the hands in the 12 o'clock position, at what time will the minute hand overtake the hour hand? ($1^h\ 05^m\ 27^s$.)

3. Can we derive an expression to show the relationship between the period of the minute hand, the period of the hour hand and the synodic period (the time between successive overtakes)?

Observe the hands of a clock or watch through 12 revolutions of the minute hand. Students may predict 11, 12 or 13 "overtakes," but there will be only 11.

Now find the relationship between the period of the hands and the synodic period; let T_m be the "sidereal period" of the minute hand. (The "sidereal period" is the time required for the revolving object to make one complete revolution—60 minutes in this case.) Let T_h be the "sidereal period" of the hour hand (12 hours or 720 minutes) and let T_s be the "synodic period" of the hands. In one minute, the minute hand advances $\frac{1}{T_m}$ revolution; its rate of motion is $\frac{1}{T_m}$ revolution per minute. Therefore, in time T_s, it must make $\frac{T_s}{T_m}$ revolutions. In the same time, the hour hand must make $\frac{T_s}{T_h}$ revolution at a rate of $\frac{1}{T_h}$ revolution per minute.

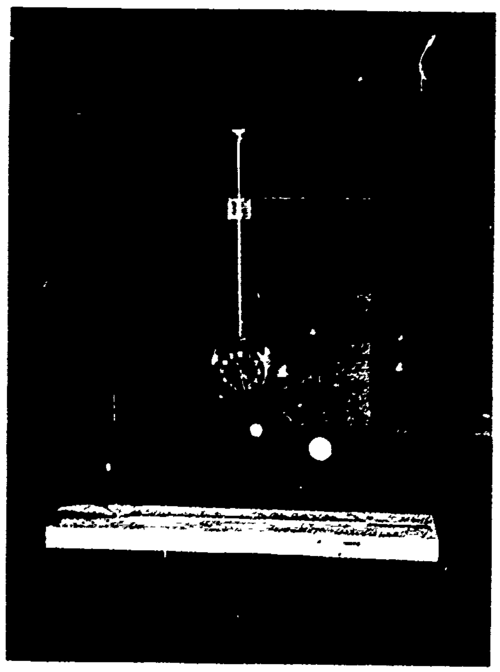
Turn the hands of the clock through one synodic period. Note that the minute hand makes one complete revolution, then goes an additional fraction of a revolution or angle, which is the same as that traversed by the hour hand. In symbols, $\frac{T_s}{T_m} = \frac{T_m}{T_m} + \frac{T_s}{T_h}$. This expression can be rearranged to find the value of any one of the quantities. Solving for T_s gives the synodic period: $T_s = \frac{T_m T_h}{T_h - T_m}$.

Substituting 60 minutes for T_m and 720 minutes for T_h gives T_s = 65.45 minutes, $1^h\ 05^m\ 27^s$, or 1.019 hours.

Ask students to imagine that they are riding on the minute hand of a huge clock, and that they can see the hour hand, the center of the clock, and various landmarks in the room around them. By observing the center of the clock against the background, they can measure T_m. By observing the hour hand each time they see it line up with the center of the clock (the hour hand is shorter than the minute hand), they can measure T_s. Solving the expression derived above for T_h, they can determine the sidereal

period of the hour hand. This is exactly the procedure astronomers use to determine the periods of the planets. The clock analogy is very close to the situation for earth and Jupiter; the synodic period is about 1.092 years, and Jupiter's sidereal period is then about 11.8 years.

A simple device to demonstrate these revolutionary relationships on an overhead projector can be constructed from a "dollar" pocket watch, a piece of 1/8-inch plastic and a few bits of wire. Remove the crystal from the watch, and cement the watch, face up, to the center of a 12" × 12" sheet of plastic. Cement a 2" length of wire to the minute hand and a 4' length of wire to the hour hand. Cement small discs of paper to the ends



of the wires to represent planets. Solder or braze a 7" length of heavy wire to the winding stem of the watch. Place the plastic on the stage of an overhead projector and rotate the hands slowly by twisting the wire soldered to the winding stem.

With a little more effort, the device described here can be used to show how the retrograde motion of the planets occurs. Instead of paper discs on the wires, cement thumbtacks with their points up. Then cut a very thin pointer from balsa wood so that it will ride on the two thumb tacks as the hands revolve. The pointer will show clearly the apparent backward motion of the outer "planet" as seen from the inner one, each time the minute hand overtakes the hour hand.

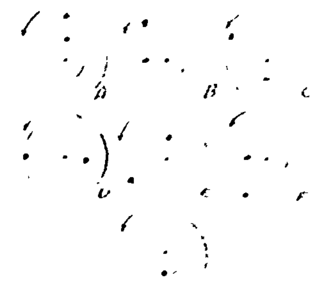
Another analysis of this problem is given below.

<u>Derivation of planetary periods</u>

Because some students may request a more precise solution for the period of a planet, the following may be useful. Assume that

1. the earth E and an outer planet P move around the sun in circular orbits.

2. the periods of the revolutions are T_E for the earth (one year), T_P for the planet, and T_P is greater than T_E.



Their rates of angular motion as seen from the central sun are $\frac{1}{T_E}$ and $\frac{1}{T_P}$.

As seen from the earth this difference in angular rate will result in the earth gaining on the planet. After an interval T_S, called the synodic period, the earth, planet and sun will again come to the same relative positions. For example, the planet might be seen from the earth to be in opposition at intervals of 680 days. During this interval, as Fig. 1 indicates, the earth will have made one more revolution about the sun than has the planet. This synodic period can be between oppositions, or any other identifiable configuration such as maximum elongation. Then the synodic rate, the rate of overtake in the chase problem is

$$\frac{1}{T_s} = \frac{1}{T_E} - \frac{1}{T_P} .$$

But the synodic period T_S can be found by observation, while the earth's period T_E is known as one year. Then

$$\frac{1}{T_P} = \frac{1}{T_E} - \frac{1}{T_s} ,$$

which becomes

$$T_P = \frac{T_E}{\left(1 - \frac{T_E}{T_s}\right)} .$$

For Mars T_S = 780 days, while T_E = 365 days. Then

$$T_{Mars} = \frac{365 \text{ days}}{\left(1 - \frac{365}{780}\right)} = \frac{365 \text{ days}}{1 - .469} = 687 \text{ days}.$$

If the planet moves inside the earth's orbit, the planet gains on the earth and the equation becomes

$$\frac{1}{T_P} = \frac{1}{T_E} + \frac{1}{T_S}, \text{ or } T_P = \frac{T_E}{\left(1 + \frac{T_E}{T_P}\right)}$$

Consider Venus, which comes to maximum elongations at an average interval of 584 days. Then

$$T_{Venus} = \frac{365 \text{ days}}{\left(1 + \frac{365}{584}\right)} = \frac{365}{1 + 0.626} = 225 \text{ days}.$$

Atmospheric Refraction
(modifying Sec. 6.7)

Any ray of light entering the earth's atmosphere at a slant is bent downward, with the result that we see the source to be higher above the horizon than it really is. The farther the body is from the observer's zenith (straight overhead), the greater is the length of the air path and also the angle at which the ray enters the atmosphere. As a result the amount of deviation by refraction increases rapidly near the horizon. The curved atmosphere is acting like a thin lens.

Because the amount of the refraction increases rapidly near the horizon, the observed image of the setting sun is distorted. The bottom limb of the sun is half a degree farther from the zenith than is the upper limb of the sun. Light from the lower limb is refracted upward more, and the sun takes on an elliptical or oval appearance, as seen in Fig. 6.12.

An interesting consequence of this refractive effect is that the actual sun (no atmosphere) has set below the horizon before the lower limb of the apparent (refracted) sun touches the horizon. Notice also that most of the blue and much of the green light is scattered from sunlight by the atmosphere. The only light not strongly scattered (Rayleigh scattering) is red, the color of the setting sun.

If you wished to expand on this refractive effect and the coloring due to scattering, consider the appearance of the moon in total eclipse. Students may be surprised to learn, and perhaps can confirm from their own observations sometime, that the moon does not "go black." Instead it appears coppery red, even in the middle of the earth's shadow. With a bit of suggestion, students can conclude that the thin edge of the earth's atmosphere perpendicular to sunlight is acting like a thin lens. Thus some sunlight is refracted into the earth's shadow. The path of this light through the earth's atmosphere is twice as long as the light we see from the setting sun. Therefore, only red light remains in the rays refracted into the earth's shadow.

Clearly atmospheric refraction would result in errors in star positions unless corrected. Yet long sequences of careful observations, such as those made by Tycho, are needed before the corrections can be determined.

About Mass
(modifying Sec. 8.7)

In Sec. 3.7 the concept of mass is introduced as something that is measured by inertia, the resistance to a change in motion. This sort of mass is called inertial mass, and it is the one used in Newton's second law, $F = ma$.

We measured the inertial mass of a body by seeing how its motion changes under the action of a known force.

In Sec. 3.8 the concept of mass is connected with gravitational forces of attraction. Here the mass of a body is a measure of the gravitational force that other bodies exert on it, or that it exerts on other bodies. Assigning a value to the mass of one particular body, we can, in principle, find the relative mass of any other by measuring the gravitational force between the two. That sort of mass is called gravitational mass.

The question now arises whether the inertial and gravitational masses of a body are linearly proportional to each other. If they are, we can set them equal to each other by adjusting the units in which we measure one or the other.

If we know how bodies accelerate when experiencing just a gravitational force, then we can tell whether inertial and gravitational masses are proportional. Consider two bodies A and B with inertial masses m_{A_i} and m_{B_i} and gravitational masses m_{A_g} and m_{B_g}. We put body A a distance R from a fixed third body with gravitational mass M, and we ask for the acceleration of body A when it experiences only the gravitational attraction due to M. Using Newton's second law, we have

$$\frac{Gm_{A_g}M}{R^2} = m_{A_i}a_A$$

or

$$a_A = \left(\frac{m_{A_g}}{m_{A_i}} \right) \frac{GM}{R^2} \qquad (1)$$

If we do the same with body B, we get

$$a_B = \left(\frac{m_{B_g}}{m_{B_i}} \right) \frac{GM}{R^2} \qquad (2)$$

Now if a body's gravitational mass is linearly proportional to its inertial mass, i.e., $m_g = km_i$ in general, where k is a universal constant, then

$$m_{A_g} = km_{A_i}$$

and

$$m_{B_g} = km_{B_i}.$$

If we put the first of these expressions into Eq. (1) and the second into Eq. (2), then we get

$$a_A = a_B = k \frac{GM}{R^2},$$

i.e., the two accelerations will be equal. To check this analysis all we need do is perform the experiment and see if the accelerations are equal.

If we use the earth as M, we would conclude that, at a given distance from the earth's center, all freely falling bodies should have the same acceleration. Therefore, experimental verification that this is true would be proof that inertial mass is proportional to gravitational mass. Unfortunately, the experiment is very difficult to perform with high precision.

Isaac Newton devised an experiment that tested the proposition in an indirect way. A pendulum bob is not a freely falling object, but in its motion to and fro it does accelerate, and the value of its acceleration governs the rate of oscillation. Newton was able to

$$T = 2\pi \sqrt{\frac{x}{a}} = 2\pi \sqrt{\frac{\ell}{g}}$$

$$\text{But } \frac{m_i a}{m_g g} = \frac{x}{\ell}, \therefore \frac{x}{a} = \frac{\ell}{g}$$

$$\underline{\underline{\text{if}}}\ m_i = m_g$$

One can test for <u>each</u> bob separately, then test for various materials.

show that only if inertial mass is proportional to gravitational mass will the rate of oscillation be the same for pendulum bobs of different mass. Newton made a hollow pendulum bob in the form of a thin metal shell into which he put different materials, always being careful to see, by using an equal arm balance, that the weight of the material was the same each time. Since weight is a measure of gravitational mass, any difference in the rate of oscillation of the pendulum would be due to a difference in inertial mass. No such difference appeared, and Newton concluded that inertial and gravitational masses are equivalent.

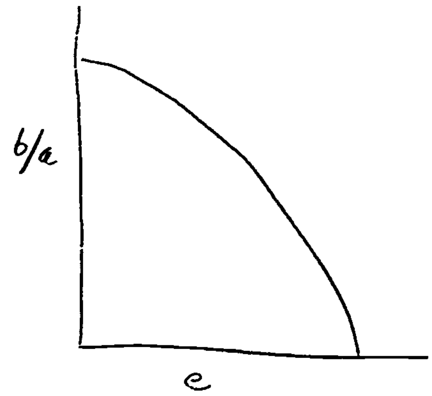
For more recent studies of this topic, see reader article R18. Professor Dicke and his co-workers at Princeton University believe they have shown the equivalence of inertial and gravitational mass to within 1 part in 10^{11}.

Relations in an Ellipse

a = major axis
b = minor axis
c = focus from center
e = eccentricity

$e = c/a$	$b/a = (1 - e^2)^{1/2}$
0.1	0.995
0.2	0.98
0.3	0.955
0.4	0.916
0.5	0.866
0.6	0.800
0.7	0.715
0.8	0.600
0.9	0.435
0.95	0.313

Give the equation, $(b/a)^2 = (1 - e^2)$ and suggest that someone work out this table and graph it. Students will be surprised to see how rapidly e changes with shape—or how "round" is an ellipse of large e.



The Moon's Irregular Motion
(modifying Secs. 8.14 and 8.17)

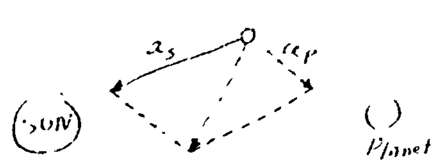
The observed motion of the moon contains many small variations which cannot be predicted by the simple assumption of a gravitational force between two mass points m_E and m_M.

Newton's investigations accounted for some of these discrepancies, but he studied only a few. Nevertheless, his theoretical results were reasonably close to the observed values of his time.

Though the process of applying the law of universal gravitation to separate sets of two-mass systems may seem to allow relatively easy solutions for motions, what happens when a third body gets involved? Thus, the sun-earth and earth-moon systems appear to be simple gravitational phenomena; but the reality is a single sun-moon-earth system that becomes so complex that a solution of its motions by gravitational theory becomes possible only under very limited conditions (that is, if based upon very special assumptions).

Perhaps this kind of complication becomes more real for the student, if you associate it with Fig. 8.10, page 97, the diagram on page 89 and its application in Experiment E20.

What if another large body (like Jupiter at 5 AU from the sun) were on the other side of the comet (see the figure below)? The student can then realize the prediction of orbital motion must be painstakingly (and painfully) worked out by adding up all the acceleration vectors concerned. This suggests the real problem



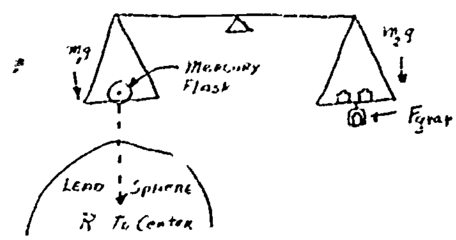
of computing paths or orbits for real space probes, moon-missions, and Mars and Venus fly-bys, where all the planets are attracting the space ship (or another planet). As with Gemini 10, the computations are so lengthy and complex that precision orbits, docking, etc., would be impossible without high-speed computers.

You might refer your students to the article, "The Earth's Gravity," by Weikko A. Heiskanen, in the September 1955 Scientific American. (Reprints are available from W. H. Freeman & Co., 660 Market Street, San Francisco, California.)

Measuring G
(modifying Sec. 8.15)

It might also be easier for some students to understand another method of measuring G, which was designed and carried out by a German physicist, Von Jolly, in the mid-nineteenth century. He used an equal-arm balance instead of the rather complicated torsion apparatus of Cavendish. On one side, Von Jolly put a spherical flask filled with mercury and balanced this with weights in the other pan. Then he put a large lead sphere below and close to the flask of mercury. He could determine the distance between the two spheres. The gravitational force between the two spheres caused the side with the flask to dip down slightly. Then the weights necessary to rebalance the equipment were a measure of the F_{grav} between the spheres.

Here is a set of figures typical of the Von Jolly experiment which your students can use to calculate G for themselves:



m_1 (mass of mercury) = 5 kg

m_2 (mass of lead sphere) = 5775 kg

R (between sphere centers) = 0.57 m

F_{grav} = extra mass added for balance, 0.59 milligrams, or 5.9×10^{-7} kg $\times$ 9.8 = 57.8×10^{-7} newtons.

Then

$$F_{grav} = \frac{Gm_1m_2}{R^2}$$

and

$$G = \frac{FR^2}{m_1m_2}$$

$$G = \frac{57.8 \times 10^{-7} \times (5.7 \times 10^{-1})^2}{5 \times 5.775 \times 10^3}$$

$$G = \text{about } 6.5 \times 10^{-11}\ m^3/kg\cdot sec^2.$$

You might ask the students to indicate which of the above measurements present probable sources of error, and why.

(For example, does the lead ball also attract the weights in the other pan?) Another important point that could be made here is that the Cavendish experiment represents an inertial method of measuring G, while the Von Jolly experiment uses the gravitational method. You might wish to refer back to Sec. 3.8.

A "True" Scale of Sun, Moon, and Earth*

Charles K. Arey, Professor of Education, University of Alabama, University

I have seen many schoolroom scale models of the solar system or parts thereof, but none which employs the same scales for both planetary distances and diameters. The following partial model avoids this difficulty and thus may provide a truer and more readily understandable picture of the immensity of space.

On a scale of one million miles to an inch, the sun, being about three-quarters of a million miles in diameter, would be represented by an object 3/4 inch in diameter. The earth, 93 inches away, would be about eight thousand millionths or 1/125 of an inch in diameter. The moon, one quarter of a million miles away, would be located at 1/4 inch from the earth.

Measure off 93 inches on the blackboard or a convenient wall. Tape a five-cent piece, which is about 3/4 inch in diameter, to the board to represent the sun. From ordinary newspaper print, cut one period with a small amount of white paper around it, and paste it 93 inches away from the nickel to represent the earth. From the smallest print to be found, cut another period. Paste this second period 1/4 inch from the first. This period represents the moon, and completes the model. Other dots can be placed between "earth" and "sun" to represent the inner planets, but these probably add little effectiveness to the model.

This model can bring home to the learner how very large and empty space really is, even within the solar system. We talk glibly about man exploring space, but so far men have not yet gone the first quarter-inch, and it may be quite a while before they do.

*Reprinted from The Science Teacher, September, 1967, page 63.

Theories (AN EXTENSION)
(modifying Sec. 8.19)

Theories often have important practical applications. This is less apparent in the astronomical context, although the development of instruments and mathematics were influenced. Many other examples of more direct practical consequences will appear through the course. Currently rocket development is having a major impact upon the design of many commercial products; this often occurs without public notice.

As human creations, theories are produced, developed, judged and applied by men who have personal prejudices and frailties. Therefore the combined judgment of many scientists is safer than the reaction of one. Yet history may show that the one might be right and the majority wrong. It is important here to try to replace the all-too-common snap-judgment, good-or-bad evaluation of new ideas by developing a critical interest in theories as possibilities. One can be informed about and interested in a new theory without necessarily accepting or rejecting the theory. Suspended judgment is often a mark of maturity.

Can the students suggest other theories in science, government, economics, etc. which at first seemed shocking, yet have become commonly accepted? Perhaps impressionistic art now commonly used in advertising would be an example. What was the public reaction to Manet, Picasso, etc.? Or have someone interested in music report on the initial reception of the compositions of Wagner, Brahms, or Stravinsky. (The latter's ballet, The Rites of Spring, was loudly booed when first performed in Paris in 1913.) Or perhaps consider the acceptance of Ulysses by James Joyce.

Contest among ideas, the trial by combat, is essential in science and every other field of human creation. Supine acceptance or adulation of the great man's creation or its casual rejection marks a decadent subject or society.

Theories are changed over time. They are not fixed and permanent to be idolized, but rather are working tools to be used and resharpened. Rarely is a theory completely abandoned. Most are modified, but some are replaced. Scientists, like other people, cannot tolerate a complete absence of some sort of explanation. They will not completely abandon an old theory, even if it is known to have serious limitations. At least it worked in some cases, and still satisfies some phenomena.

In many ways scientists are artists. Each is a specialist in the study and interpretation of some set of phenomena.

Each brings to his work a general sense of what type of theories and explanations is satisfying. That is, scientists have personal styles. Some are mainly concerned about the precision of measurement and the design of equipment. Others look at theories as the bases for predictions. Still others try to imagine a variety of possible explanations; and some are more daring than others. Einstein and Fermi are revered because they were very imaginative and would play with possibilities, turning them this way and that to see what consequences might result. In this way the individual characteristics of the scientists are most apparent. At least initially, the possible line of a theory is always qualitative and often pictorial. The "sort of like this" imagery comes first and reveals the basic aesthetic approach of the individual, his vision of the world in which he lives.

Bibliography

KEY:

(T) Recommended for teacher background.
(S) Student supplementary material.
(T,S) Teacher should read first.
(S,T) Student can read; teacher would find useful.
* Highly recommended.
** Essential.

Good texts for general use in astronomy are those by: Abell, Dreyer, Hoyle, Pannekoek and Reichen.

Recommended strongly for teachers and students are the books by: Andrade, Armitage, Asimov, Bernhard (et al.), Beveridge, Butterfield, Caspar, Drake, Dreyer, Galileo, Newton and Rosen. The rest are titles worth knowing about. Many will make good reading for interested students.

(T,S) *"Sky and Telescope," magazine published by Sky Publishing Company, 49-51 Bay State Rd., Cambridge, Mass. Recommend for your school library. Will enable you to quickly locate planets, meteor showers, comets, etc.

(T) Abell, George O. — *Exploration of the Universe, Holt, Rinehart, and Winston, New York, 1964. College-level text. Good for teacher's reference.

(S) Alter, Dinsmore; Clemenshaw, Clarence; Phillips, John G. — Pictorial Astronomy, Thomas Y. Crowell Co., New York, 1963. Good diagrams and pictures. (Chapter 5)

(S,T) Andrade, E. N. da — *Sir Isaac Newton, Doubleday Anchor Books, Garden City, New York, 1958. Biography. (Chapter 8)

(S) Armitage, Angus — *Sun, Stand Thou Still, Mentor Paperback. Very readable on Copernican system. (Chapter 6)

(S,T) Asimov, Isaac — Asimov's Biographical Encyclopedia of Science and Technology, Doubleday & Co., Inc., Garden City, New York, 1964. A very readable source of information on over a thousand men of science. Highly recommended for students and teachers.

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(T,S) Beveridge, W. I. B. — The Art of Scientific Investigation, Random House, New York, 1951. Good discussion, readable, includes some biological and medical examples.

(S) Bixby, William — The Universe of Galileo and Newton, Harper and Row, 551 Fifth Ave., New York, N. Y., 1964. Easy reading. (Chapters 7, 8)

(S,T) Bondi, Hermann — The Universe At Large, Wesleyan Univ. Press, Inc., Columbus, Ohio, 1960. Good reference for unit.

(S,T) Brecht, Bertoldt — Galileo, Grove Press, 1959.
Brecht's last play. For students interested in the theatre.

(T) Bronowski, J. — The Common Sense of Science, Vintage Paperback, V-168.
Interesting discussion from a practical viewpoint.

(T,S) Butterfield, Herbert — *The Origins of Modern Science, Collier Books, New York, 1951.
Good book for introductory philosophy. Teacher should read first.

(T,S) Caspar, Max — Kepler, 1571-1630, Collier Books, New York, New York, 1962 (transl. C. D. Hellman).
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(S) Cellini, Benevenuto — Autobiography, Dodd and Mead, New York, New York, 1961.
Report of life in time of Galileo. Good reading for students. (Chapter 7)

(T,S) Clagett, Marshall — Greek Science in Antiquity, Collier Books, New York, New York, 1963.
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(T,S) Cohen, I. Bernard — *The Birth of a New Physics, Wesleyan Univ. Press, Inc., Columbus, Ohio, 1960.
Teacher should read and then assign passages for interested students.

(T) Crombie, A. C. — Medieval and Early Modern Science, Vols. I and II, Doubleday-Anchor Books, Garden City, New York, 1959.
Good supplement to teacher's library.

(T,S) Dampier, William C., Dampier, Margaret — Readings in the Literature of Science, Part I: Cosmogony, Harper Torchbooks, 1959.
Brings out some interesting facets relating science and literature.

(T) de Santillana, Giorgio — The Crime of Galileo, University of Chicago Press, Chicago, Illinois, 1959.
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(T) Drake, Stillman (translator) — **Discoveries and Opinions of Galileo, Doubleday Anchor Books, Garden City, New York, 1957.
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(T) Dreyer, J. L. E. — A History of Astronomy, Dover Publishing, Inc., 180 Varick St., New York, New York, 1953.
Difficult reading but will be helpful. (Chapters 5, 6)

(T,S) Dreyer, J. L. E. — Tycho Brahe, Dover Publishing, Inc., 180 Varick St., New York, New York, 1963.
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(S) Fermi, Laura; Bernardini, Gilberto — Galileo and the Scientific Revolution, Basic Books, Inc., New York, New York, 1961.
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(S,T) Frank, Philipp — Philosophy of Science, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1957, Chapters 1-4.
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(T,S) Gamow, George — Gravity, Science Study Series, Anchor Books, 1962. (Chapter 8)

(T,S) Hall, A. R. — *The Scientific Revolution, The Beacon Press, Boston, Mass., 1960.
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(T) Holton, Gerald — *Introduction to Concepts and Theories of Physical Science, Addison Wesley, Reading, Mass., 1952.
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(T) Hoyle, Fred — Astronomy, Doubleday & Co., Inc., Garden City, N. Y., 1962.
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(T,S) Hoyle, Fred — The Nature of the Universe, Harper and Bros., N. Y., N. Y., 1950.
Easy reading. Good supplement for some students.

(T,S) Hurd, D. L.; Kipling, J. J. — The Origins and Growth of Physical Science, Parts I, II, and III. Penguin Books, Baltimore, Md., 1964.
Interesting supplementary source for teachers and some students.

(S) Jackson, Jos. H. — Pictorial Guide to the Planets, Thomas Y. Crowell Co., New York, New York, 1965.
Good pictures. Some students would enjoy this book.

(T,S) Koestler, Arthur — The Watershed, Wesleyan Univ. Press, Inc., Columbus Ohio, 1960.
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(T) Koyré, Alexandre — *From the Closed World to the Infinite Universe, Harper and Bros., New York, New York, Chapters 1-4.
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(T) Kuhn, Thomas S. — *The Copernican Revolution, Modern Library Paperbacks, Random House, New York, New York.
Good for teachers. (Chapter 6)

(S,T) Lodge, Sir Oliver
Pioneers of Science, Dover Publications, Inc., New York, New York, Part I, 1960. (Originally published in 1893.) Highly recommended for students. Includes Ptolemy, Copernicus, Kepler.

(S,T) Moore, Patrick
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(T) Newton, Sir Isaac
**Mathematical Principles of Natural Philosophy
Vol. I: The Motion of Bodies
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Translated into English by Andrew Motte in 1729, revised by Florian Cajori, University of California Press, Berkeley and Los Angeles, California, 1962. A must for the desk of any physics teacher. (Chapter 8)

(T) Neugebauer, O.
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(S,T) Nicolson, M.
Science and Imagination, Great Seal Books, Ithaca, New York, 1962.

(S,T) Noyes, Alfred (Lord)
The Watchers of the Skies, Blackwood Press, London, 1933. Poem re: Copernicus, Kepler, Galileo, etc. Good literature, might encourage English classes to use it.

(T) Palter, Robert M.
Toward Modern Science, Vols. I and II, Farrar, Straus and Cudahy, New York, New York, 1961. Vol. I, pp. 35-37. Vol. II, pp. 115-131; 192-216. Difficult, but valuable to teacher's background.

(S) Pannekoek, A.
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Making of the Modern Mind, Houghton Mifflin, 1940. Useful source on intellectual history.

(S) Reichen, Charles-Albert
A History of Astronomy, Hawthorne Books, Inc., 70 Fifth Ave., New York, New York, 1963. Good account for early development of Unit 2 for students.

(T) Rosen, Edward
*Three Copernican Treatises, Dover Publications, Inc., 180 Varick St., New York, New York. If the teacher has not read this book, he should. Part I is Copernicus' "Commentariolus." Introduction very useful. (Chapter 6)

(S) Rosen, Sidney
The Harmonious World of Johann Kepler, Little Brown, 1962. Easy reading, fictional biography. (Chapter 7)

(S) Sullivan, Gavin
Pioneers In Astronomy, George G. Harrap & Co., Ltd., London, 1964. Chapters 1-3. Good, easy reading, covers scientists in Unit 2.

Toulmin, S.
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*The Fabric of the Heavens, Harpers, 1961. Historical discussion of the development of astronomy. Parallels the emphasis in this unit.

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(S,T) Watson, Fletcher G. *Between the Planets, Doubleday Paperback N-17, Garden City, New York. Comets—Chapters 4 and 5. A help on understanding comets and their paths. (Chapters 7, 8)

(S,T) Weisskopf, Victor F. *Knowledge and Wonder, Wesleyan Univ. Press, Inc., Columbus, Ohio, 1963. Early chapters important to Unit 2. Plan to read all of this after you once start.

(S) Whipple, Fred L. *Earth, Moon, and Planets, Harvard University Press, Cambridge, Mass., 1963. Popular treatment, easy reading, students will like it. (Chapters 7, 8)

(T) Whitehead, A. N. Essays In Science and Philosophy, Philosophical Library, New York, New York, 1948. Part IV: Science. Fascinating addition to teacher's library.

(T,S) Wolf, A. A History of Science, Technology and Philosophy in the 16th and 17th Centuries, (2 volumes), Harper Torchbooks, 1959.

Resource Letters:

Teachers should know of and obtain copies of resource letters reprinted from the American Journal of Physics. Resource letters may be obtained free by sending a self-addressed, stamped envelope to the American Institute of Physics, 335 East 45th Street, New York City, 10017. Name the resource letter requested.

Resource Letter SL-1 on Science and Literature, prepared by Marjorie Nicolson, has many references that are appropriate to Unit II. Another scheduled for publication in 1966 or 1967 is Resource Letter on Collateral Reading in Physics Courses, prepared by Alfred Bork of Reed College.

Periodicals

Education

Journals with articles on experiments or apparatus for education.

American Journal of Physics. Published for the American Association of Physics Teachers by the American Institute of Physics, New York Monthly; $10.00 annually.
A journal devoted to the instructional and cultural aspects of physical science. Articles on curriculum, theory, experiments and demonstrations; book reviews; AAPT meetings. Includes Apparatus Notes monthly and occasionally Apparatus Reviews.

Journal of Chemical Education. Chemical Education Publishing Company, 20th and Northampton St., Easton, Pa. Monthly. Annual subscription $4.00
One of its monthly features is a series of articles on the theory, design and availability of laboratory instrumentation.

Nuclear Energy—Journal of the Institution of Nuclear Engineers. U.S. Representative: TPR Inc., 261 Madison Ave., N.Y. 16, N.Y. Subscription $10.00 annually.
Classroom and Laboratory Section publishes short reports on simple apparatus and methods useful for courses in nuclear physics. Through May 1964, 265 items have been published.

The Physics Teacher. A Journal of the American Association of Physics Teachers, 1201 16th Street, N.W., Washington, D.C. 20036. Subscription of eight issues $7.50.
Dedicated to the enhancement of physics as a basic science in the secondary schools.

The School Science Review. The Association for Science Education, 52 Bateman Street, Cambridge, England. Published November, March, June. Annual subscription 41s, 3d or $5.75.
Each issue contains articles of interest to science teachers. Many illustrated experiments and demonstrations on all levels.

Scientific American, 2 West 45th Street, New York 17, N.Y. Monthly. Subscription $6.00 annually.
Written by scientists for teachers, students and the lay reader. Covers entire field of science. Offprints of certain articles are available from W. H. Freeman and Company, San Francisco 4, California.

Fiction about Science and Scientists

Buck, Pearl S., Command the Morning, Day, 1959.

This book uses fictitious characters but traces actual events in its portrayal of the scientists who, in Chicago on December 2, 1942, ushered in the atomic age by producing the first self-sustaining chain reaction. Ranging from Chicago to Oak Ridge and from Washington to Los Alamos, the novel emphasizes the changes in the private and professional lives of persons burdened with awesome responsibility and gnawing guilt.

Colby, Merle E., Big Secret, Viking, 1949.

A picture of Washington life and politics, as an idealistic young scientist, Daniel Upstead, fights for scientific freedom against the pressure groups threatening to forbid or defeat atomic research.

Cronin, A. J., The Citadel, Little Brown, 1959.

The story of the career of a doctor, from his start in a mining town in Wales, to a London practice. After years of struggle against mediocrity and indifference, he decided to capitalize on personal charm and make money. But success meant forgetting honor and ideals, and brought estrangement from a gallant wife, until a tragic error brought him to his senses.

Ehrlich, Max S., The Big Eve, Doubleday, 1949.

1960 is marked by strange happenings throughout the world for which Russia is blamed and war is a question of minutes. At Palomar, Dr. Dawson, checked by internationally famous astronomers, announces the coming of a new planet Y which will overtake the earth and the world in 1962 at Christmas. The story of how our universe adjusts to the idea, of how peace becomes, for a little while, permanent, and prosperity and global good will come into their own, packs a surprise finish.

Lewis, Sinclair, Arrowsmith, Harcourt, 1949.

Sinclair Lewis follows Martin Arrowsmith, a born seeker and experimentalist, from medical school through experiences as a general country practitioner, as health officer and clinician, as fighter of a plague on a West Indian island, and finally as director of a medical institute. His first wife is both playmate and helpmate, who ministers to his genius and puts up with his egotism. His second wife, rich and exacting, tries to make him into a fashionable scientist. The book leaves Arrowsmith in the Vermont

woods with a fellow-spirit working as an independent researcher.

Master, Dexter, The Accident, Knopf, 1955.

This novel about the making and using of the atomic bomb takes place during the eight days it took Louis Saxl to die. He didn't want to die, but an experiment had gone wrong, and there was an accident. Was it avoidable? In the search for an answer, Louis's life is laid bare.

Menen, Aubrey, The Fig Tree, Scribner, 1960.

A Nobel Prize-winning scientist goes to Italy to continue his experiments on a substance that will increase the size and yield of plant life. The plant becomes an aphrodisiac when injected with this substance. The moral issue involves government ministers and the Vatican Sacred College of Cardinals before being resolved.

Shelley, Mary W., Frankenstein, Associated Booksellers, 1964.

Not really a "monster" story, but a troubling inquiry into some serious issues relating to science.

Snow, C. P., The Affair, Scribners, 1960.

The Affair depicts the struggle between two physicists, both of whom are claiming credit for an important discovery. The dispute is complicated by the fact that one of them holds unpopular political views.

Snow, C. P., The New Men, Scribners, 1955.

The "new" men are a group of nuclear scientists and government officials working in England during World War II. The author recounts the excitements and dangers and the conflicts between the scientists and bureaucrats. The increasing moral concern is of particular interest.

Snow, C. P., The Search, Scribners, 1959.

The story of a crystallographer, of his rise from indigence to eminence and of his eventual turning away from science.

Stevenson, Robert F., Dr. Jekyll and Mr. Hyde, Dutton, 1767.

The classic tale of a scientist who "splits" the human psyche.

Walter, William G., Curve of the Snowflake, Morton, 1956.

The novel is concerned with an adorable woman and three men who are in love with her, and with their experiences as members of an extraordinary team of eminent scientists who live and work together for the greater glory of mankind. It is concerned with a submarine that flies, the first manned earth satellite and the snowflake curve. But the heart of the matter is a journey into the future, as far as the year 2056, in a time machine that owes much to the possibilities inherent in the snowflake diagram.

Wilson, Mitchell, Meeting at a Far Meridian, Pocket Books, 1962.

An American scientist is invited to participate in a crucial experiment with a Soviet scientist.

Plays about Science and Scientists

Brecht, Bertolt, The Life of Galileo, 1952.

Durrenmatt, Friedrich, The Physicists, Grove, 1964.

Kingsley, Sidney, Yellowjack.

Shaw, George Bernard, The Doctor's Dilemma, Penguin, 1906.

Film Strip - Retrograde Motion of Mars

When to View This Strip: Because photographs are the most honest evidence we have of the actual retrograde motions of Mars (and Jupiter), the strip should be shown as soon as motions of the planets are mentioned. It should be seen before the film locp (10a) on retrograde motions (made by animation) is presented.

The Photographs: The frames were made from unretouched 4 x 5 inch contact print of sections of the original (8 × 10 inch) photographs. The photographs were taken with the short-focus camera (focal length 6 inches) shown in one of the first frames. Because Mars was never in the center of the field, but sometimes almost at the edge, the star images show distortions from limitations of the camera lens. During each exposure the camera was driven by a clockwork to follow the western motion of the stars and hold their images fixed cn the photographic plate. Because the sky was less clear on some nights and the exposures varied somewhat in duration, the images of the stars and planets are not of equal brightness on all pictures. However, some of the frames show beautiful pictures of the Milky Way in Taurus (1943) and Gemini (1945).

These three series of photographs were selected as the most extensive available for recent oppositions of Mars. The photographs were taken a part of the routine Harvard Sky Patrol, and were not made especially to show Mars. The planet just happened to be in the star fields being photographed.

FILM STRIP — Retrograde Motion of Mars

You should view this film strip before viewing Film Loop 10a on retrograde motions. The film loop is done by animation, but the film strip shows actual photographs of the night sky.

Photographs of the positions of Mars, from the files of the Harvard College Observatory, are shown for three oppositions of Mars, in 1941, 1943, and 1946.

The first series of twelve frames shows the positions of Mars before and after the opposition of October 10, 1941. The series begins with a photograph on August 3, 1941 and ends with one on December 6, 1941.

The second series shows positions of Mars before and after the opposition of December 5, 1943, beginning October 28 and ending February 19, 1944.

The third set of eleven pictures, showing Mars during 1945-46 around the opposition of January 14, 1946, begins with October 16, 1945 and ends with February 23, 1946.

Uses:

a) The star fields for each series of frames have been carefully positioned so that the background star positions are nearly identical in each frame. If you flick the frames of each series through the projector in rapid succession, the stars will be seen as stationary on the screen, while the motion of Mars among the stars is quite apparent.

b) You can project the frames on a paper screen and mark the positions of various stars and Mars. If you adjust the star positions for each frame to match the positions of the previous frame, the positions of Mars can be marked for the various dates. By drawing a continuous line through

▲ This loop helps define retrograde motion. The term stands for the westward motion (to the right on the screen) of the planets. Note that Mercury, and Venus also have retrograde motions similar to those of the other planets.

The representation of a point source by a disk is not entirely without reason. Because of the wave nature of light, the image of a star is spread out into a diffraction disk which appears larger for brighter stars.

Although the enlarged images of the animation shows Mercury's motion to be across the lower part of the sun's disk, this did not actually happen in 1963. However, such transits of Mercury across the face of the sun are not uncommon; they always occur in May or November, at the times when inferior conjunction (Mercury between the earth and sun) comes at the same time that Mercury is in ascending node or descending node. Transits of Mercury occurred, and will occur, on the following dates:

1953	Nov. 14
1957	May 5
1960	Nov. 7
1970	May 8
1973	Nov. 9
1986	Nov. 12

Opposition occurs about 11 time intervals after the start of the sequence; this is 110 days later than Oct. 17, i.e., about Feb. 4, 1963.

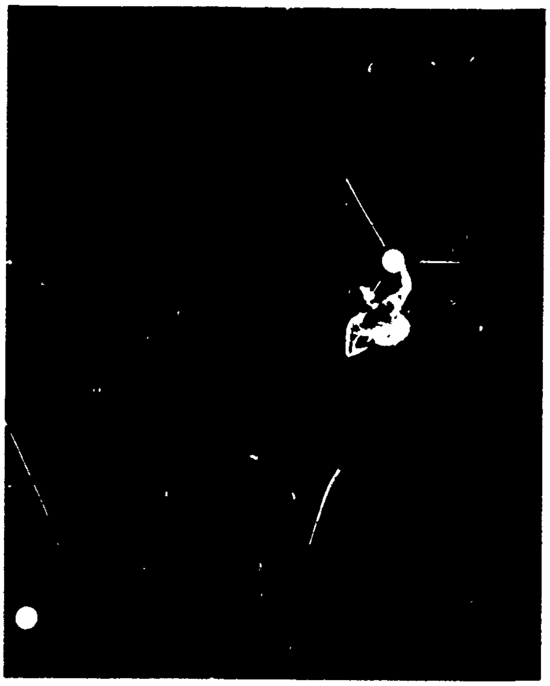
its retrograde motion, Mercury passes between the earth and the sun (inferior conjunction).

2. Motion of Mars starting October 17, 1962, with time markers at 10-day intervals. The retrograde motion occurs around the time when Mars passes through opposition. The field of view includes parts of the constellations Leo and Cancer; the cluster at the upper right is Praesepe (the Beehive), faintly visible to the naked eye on a moonless night.

You can use the time markers to determine the approximate date of the opposition of Mars (center of the retrograde portion).

FILM LOOP 10 Retrograde Motion — Geocentric Model

Using a specially-constructed large "epicycle machine" as a model of the Ptolemaic system, the film shows the motion around the earth of a planet such as Mars.



Note the changes in apparent brightness and angular size of the globe as it sweeps close to the camera. While the actual planets show no disk to the unaided eye and appear as points of light, certainly a marked change in brightness would be expected. This was, however, not considered in the Ptolemaic system, which focussed only upon the timetable of the angular motions and positions in the sky.

FILM LOOP 11 Retrograde Motion – Heliocentric Model

The machine used in Loop 10 was reassembled to give a heliocentric model with the earth and the planet moving in concentric circles around the sun. The earth (represented by a light blue globe) is seen to pass inside a slower moving outer planet such as Mars (represented by a white globe). The sun is represented by a yellow globe.

Then the earth is replaced by a camera, having a field about 25° wide, which points in a fixed direction in space. The arrow attached to the camera shows this fixed direction. (As in Loop 10, we are ignoring the daily rotation of the earth on its axis and are concentrating on the motion of the planet relative to the sun and the fixed stars.)

As it revolved about the center of motion, the camera was kept pointing in a fixed direction by means of a system of gears and belts.

Several scenes are shown. Each scene is viewed first from above, then viewed along the plane of motion. Retrograde motion occurs whenever Mars is in opposition; this means that Mars is opposite the sun as viewed from the earth. But not all these oppositions take place when Mars is in the sector toward which the camera points.

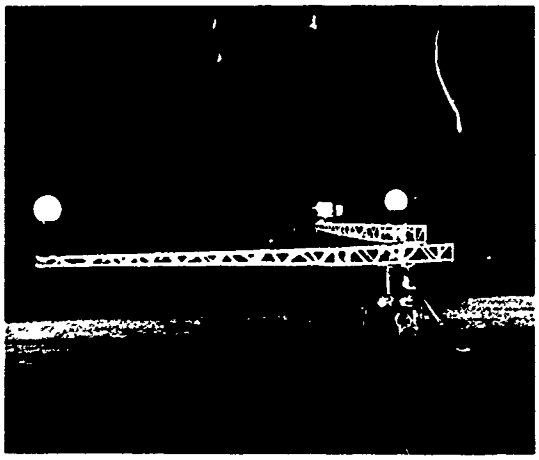
1. Mars is in opposition; retrograde motion takes place.

2. The time between oppositions averages about 2.1 years. The film shows that the earth moves about 2.1 times around its orbit (2.1 years) between one opposition and the next one. You can, if you wish, calculate this value, using the length of the year (sidereal period) which is 365 days for the earth and 687 days for Mars. This is the "chase problem" discussed on page 31 of Unit 2.

The actual interval between oppositions varies between 767 days and 798 days, because the orbits of Mars and the earth are ellipses, not circles.

In one day the earth moves 1/365 of 360°, Mars moves 1/687 of 360°, and the motion of the earth relative to Mars is

Film Loops



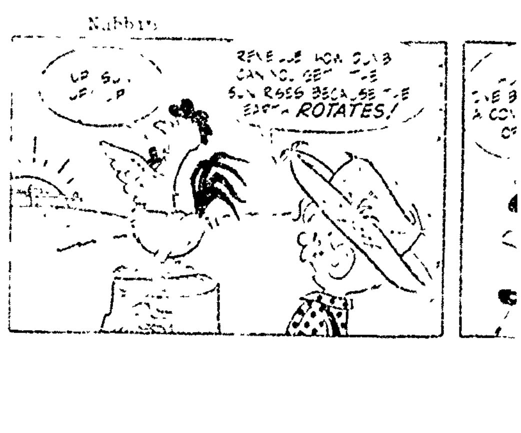
(1/365 - 1/687) of 360°. But 1/365 - 1/687 = 0.00274 - 0.00146 = 0.00128 = 1/780. Thus in one day the earth gets ahead of Mars by 1/780 of 360°; it will take 780 days for the earth to catch up to Mars again. The "phase period" of Mars is, therefore, 780 days, or 2.14 years. This is an average value.

3. The view from the moving earth is shown for a period of time greater than 1 year. First the sun is seen in direct motion, then Mars comes to opposition and undergoes a retrograde motion loop, and finally we see the sun again in direct motion.

Note the changes in apparent size and brightness of the globe representing the planet when it is nearest the earth (in opposition). Viewed with the naked eye, Mars does in fact show a large variation in brightness (ratio of 50:1). The angular size also varies as predicted by the model, although the disk of Mars, like that of all the planets, can be seen only with telescopic aid. The heliocentric model illustrated in this film is simpler than the geocentric model of Ptolemy, and it does give the main features observed for Mars and the other planets: retrograde motion and variation

Some of the finer details of the motion of Mars are related to the planet's rather strongly elliptical orbit (eccentricity 0.093 compared with 0.017 for the earth's orbit). Some oppositions are more "favorable" (i.e., closer) than are others. Fig. 1 shows that the closest oppositions occur if the earth is at A (in late August); a little over 2 years later the next opposition is not so close (earth a B, in November). The following distances illustrate these points: Most favorable opposition AA' = 35,000,000 miles; least favorable opposition, CC' = 63,000,000 miles' least favorable superior conjunction, AC' = 249,000,000 miles. The model used in the film is based on the approximation of circular orbits.

in brightness. However, detailed numerical agreement between theory and observation cannot be obtained using circular orbits. With the proposal by Kepler of elliptical orbits, better agreement with observation was finally obtained, using a modified heliocentric system.



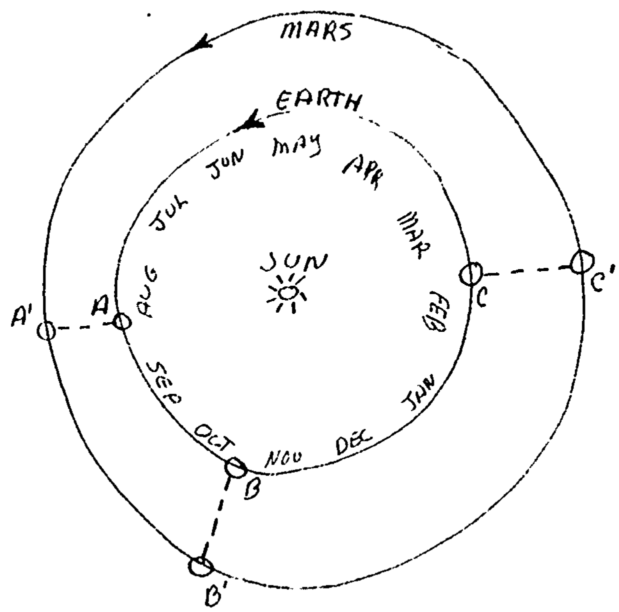


Fig. 1
Earth is at A in August of each year, but Mars is not necessarily at A' at that time.

The purpose of the loop is to give students a feeling for the motion of a celestial body—in this case, a satellite of Jupiter moving under the influence of gravitational force. The loop is primarily intended for qualitative use. However, some simple measurements of period and size of orbit can help a student appreciate the nature of astronomical observations in a "real" situation.

Jupiter was in opposition on Jan. 20, 1967, and was therefore closest to the earth (at about 4.2 astronomical units) and maintaining a relatively constant size as viewed from day to day. The entire orbit could not be photographed, because on Feb. 8, 1967 (when the missing part of the orbit was being traversed by Io) the image was blurred because of high-altitude jet streams in the earth's atmosphere. The next return of Io to this part of its orbit during darkness in Arizona was on Feb. 24, and by then Jupiter would have been farther away and its image would have been smaller. Also, use of a large telescope must be tightly scheduled, and our project had already used major amounts of telescope time on 7 nights. For all these reasons, we settled for a film showing 84% of a complete orbit, including all the portions needed for calculations to be made by students.

Exposures were for 4 sec on 35mm black and white film of the type used for aerial mapping and reconnaissance. A green filter (Wratten 58) was used to give maximum sharpness of the images. A decision was made to use the best exposure to show the satellites, thus overexposing the image of the disk of Jupiter. For this reason, the surface markings due to atmospheric storms on Jupiter are only glimpsed occasionally, during moments of haze or cloudiness.

FILM LOOP 12 Jupiter Satellite Orbit

The innermost of the four largest satellites of Jupiter, discovered by Galileo in 1610, is Io, which moves in a circular orbit with a period of $42^{h}\ 28^{m}$. The film shows most of the orbit of Io in time-lapse photography done at the Lowell Observatory in Flagstaff, Arizona, using a 24-inch refractor (Fig. 1). Exposures were made at 1-minute intervals

Fig. 1

during seven different nights early in 1967. The orbit had to be photographed in segments for an obvious reason: the rotation of the earth caused Jupiter to rise and set, and also, of course, caused interruptions due to daylight periods.

First, the film shows one segment of the orbit just as it was photographed at the focal plane of the telescope; a clock shows the passage of time. Due to small errors in guiding the telescope and also atmospheric turbulence, the highly mag-

nified images of Jupiter and its satellites dance about. To remove this unsteadiness, each of the images of Jupiter—over 2100 of them—was optically reprinted at the center of the frame, and the clock was masked out. The films with stabilized images were then joined together to give a continuous record of the motion of satellite I (Io). Some variation in the brightness of the satellites was caused by occasional light haze or cloudiness.

Table 1

Satellites of Jupiter

	Name	Period	Radius of Orbit (mi)	Eccentricity of orbit	Diameter (mi)
I	Io	$1^d\ 18^h\ 28^m$	262,000	0.0000	2,000
II	Europa	$3^d\ 13^h\ 14^m$	417,000	0.0003	1,800
III	Ganymede	$7^d\ 3^h\ 43^m$	666,000	0.0015	3,100
IV	Callisto	$16^d\ 16^h\ 32^m$	1,171,000	0.0075	2,800

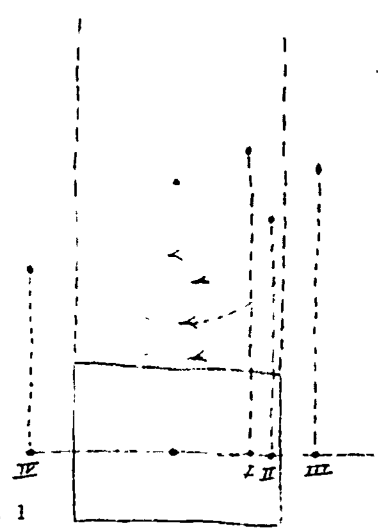


Fig. 1

The four Galilean satellites are listed in Table 1. On Feb. 3, 1967, they had the configuration shown in Fig. 2. The satellites move nearly in a plane which we view almost edge-on; thus they seem to move back and forth along a line. The field of view is large enough to include the entire orbits of I and II, but III and IV are outside the camera field when they are farthest from Jupiter.

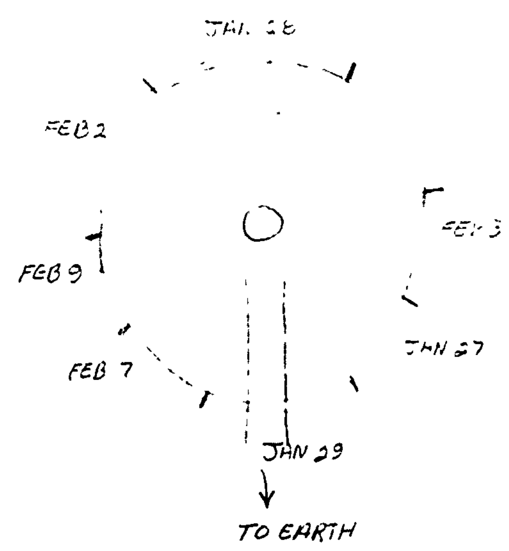


Fig. 3

The seven segments of film were spliced together as indicated in Fig. 3 to give a synthesis of the motion of satellite I (Io). For example, the position of I in the last frame of the Jan. 29 segment matches the position of I in the first frame of the Feb. 7 segment. However, since these were photographed 9 days apart, the other three satellites had moved varying distances around their orbits. Therefore, when viewing the film you will see satellites II, III and IV "pop in" and "pop out" while the image of I remains in a continuous path. Marking lines have been added to help you identify Io when each new section of film begins. Fix your attention on the steady motion of I and ignore the comings and goings of the other satellites.

Here are some interesting features that you can see when viewing the film:

a) At the start of the film, I is almost stationary at the right side of the camera field (it is almost at its greatest eastern elongation—see Fig. 2); satellite II is moving toward the left and overtakes I.

b) As I moves toward the left it passes in front of Jupiter and becomes

invisible for a while. This is called a transit. Satellite III (Ganymede, the largest of the satellites) also has a transit at about the same time. Also, II moves toward the right and disappears behind Jupiter (this is called an occultation). It is a very active scene! Figure 4 shows these three satellites at the start of this segment; satellite IV is out of the picture, far to the right of Jupiter.

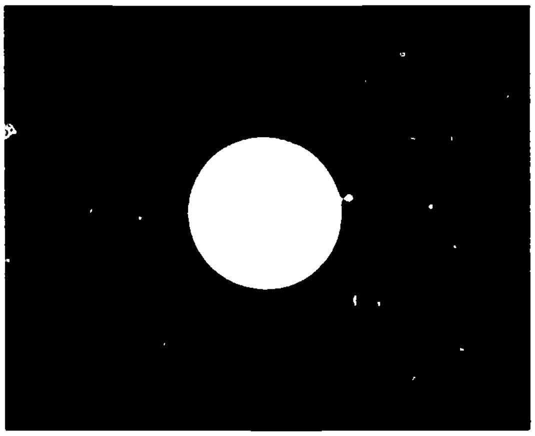


Fig. 4

If you look closely during the transit period, you can see the shadow of Ganymede and perhaps that of Io, on the left part of the surface of Jupiter.

c) Near the end of the film loop, I (moving toward the right) disappears at D in Fig. 5 as an occultation begins. Look for its reappearance—it emerges from an eclipse and suddenly appears in space at a point E to the right of Jupiter.

d) The image of Jupiter is not a perfect circle. Just as for the earth, the rotation of the planet on its axis causes it to flatten at the poles and bulge at the equator. The effect is quite noticeable for Jupiter, which is large and has a rapid rotation period of about $9^h\ 55^m$. The equatorial

During transit the satellites are invisible to us in this film; but they actually remain visible if a large telescope is used.

The flattening of Jupiter is about 1/15, compared with the flattening of 1/330 for the earth. The centripetal force at the equator is $mv^2/r = mr\omega^2$, so the effect depends on r as well as on ω, the angular velocity of rotation of the planet. For Jupiter, r is 11.2 times that of the earth, and ω is 2.42 times that of the earth. The centrifugal field (artificial gravity tending to lift a mass off the surface) is, therefore, $(11.2)(2.42)^2$ as much, i.e. 64 times as great on Jupiter as on the earth.

In measurement of T and R, a circular orbit is assumed. Io's orbit is perhaps the most nearly circular one known in all of astronomy (see Table 1). A student may raise the point that the earth is not infinitely far away, hence the points B and D are not on parallel lines tangent to Jupiter. This makes the observed time interval slightly less than half a revolution. The effect is negligibly small, as seen from the following analysis (for clarity, Fig. 1 has not

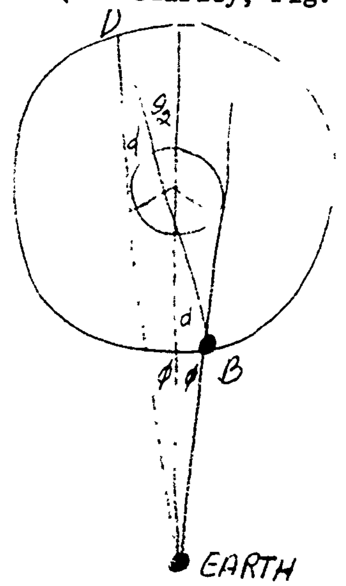


Fig. 1 (not to scale)

been drawn to scale): The two angles α are equal (similar right triangles). Then we have

$\theta_1 = \alpha - \phi$; $\theta_2 = \alpha + \phi$;

hence

$$\theta_2 - \theta_1 = 2\theta = 2\left(\frac{89 \times 10^3 \text{ mi}}{4.2 \times 93 \times 10^6 \text{ mi}}\right)$$

$= 4.6 \times 10^{-4}$ radian. The discrepancy in time is found, using the period of revolution which is 42.5 hours, or 1.53×10^5 sec:

$$\Delta T = (4.6 \times 10^{-4} \text{ rad})\left(\frac{1.53 \times 10^5 \text{ sec}}{2\pi \text{ rad}}\right)$$

= 11 sec of real time. In the film, this corresponds to only 0.01 sec of apparent time.

By going through these calculations, even approximately, the student can see how powerful Newton's law of gravitation and the laws of motion are; his own observations from the film tell him something about the density of Jupiter and the sun.

The simple measurement of orbit radius outlined in the student notes is even

diameter is 89,200 miles and the polar diameter is 83,400 miles. Occasionally you may notice the broad bands of clouds on Jupiter, but generally the pictures are too overexposed to show the bands.

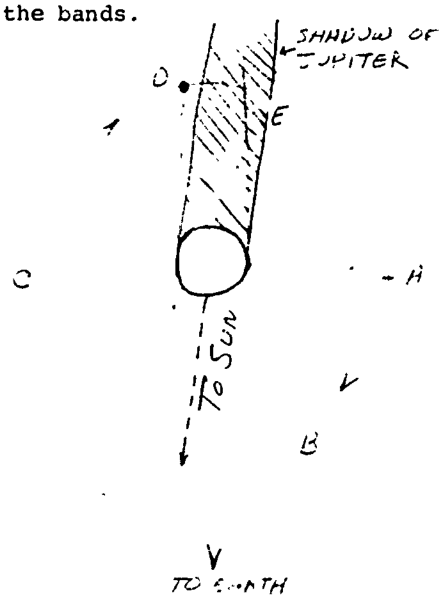


Fig. 5

Measurements

You can make two measurements from this film and from them find your own value for the mass of Jupiter.

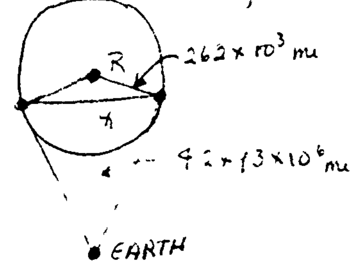
1. Period of orbit. Use a clock or watch with a sweep second hand to time the motion of the satellite between points B and D (Fig. 5). This is half a revolution, so in this way you can get the period, in apparent seconds. To convert to real time, use the speed-up

factor. Since film was exposed at 1 frame/minute and is projected at 18 frames/sec, the speed-up factor is 18 × 60, or 1080. (For a more precise value, calibrate your projector. A punch mark at the start of the loop gives a flash of light as it passes the lens. Measure the time interval between two flashes. Divide the total number of frames in the loop by the time interval to get the number of frames/sec for your projector.) In this way obtain the period T for one complete revolution of the satellite. How does your result compare with the value listed in Table 1?

2. Radius of orbit. Project the film on a piece of paper and mark the two extreme positions of the satellite, when it is farthest to the right (at A) and farthest to the left (at C). This gives the diameter of the orbit. To convert to miles, use the fact that Jupiter's equatorial diameter is 89,200 miles (about 11 times that of the earth). Now you can find the radius R of the satellite's orbit, in miles. How does your measurement compare with the value listed in Table 1?

3. Mass of Jupiter. You can use your ▲ previous calculations to find the mass of Jupiter relative to that of the sun (a similar calculation based on the satellite Callisto is given in Sec. 8.15 of the text). How does your experimental result compare with the accepted value, which is $m_J/m_S = 1/1048$?

Fig 2



better: in Fig. 2, x = R to better than 1 part in 10^6.

▲ As discussed in Sec. 8.15 of the text, the mass of the sun and the mass of Jupiter are related as follows:

$$\frac{m_S}{m_J} = \left(\frac{R_{\text{earth's orbit}}}{R_{\text{Io's orbit}}}\right)^3 \times \left(\frac{T_{\text{earth (1 year)}}}{T_{\text{Io around Jupiter}}}\right)^2$$

The student knows the values for the earth's orbit, and he has measured the values for Io's orbit. Hence he can calculate the ratio of the mass of the sun to that of Jupiter. Using values from Table 1 (similar to the ones the student will obtain by measurement of the film), we have

$$\frac{m_S}{m_J} = \left(\frac{93 \times 10^6 \text{ mi}}{262 \times 10^3 \text{ mi}}\right)^3 \times \left(\frac{42.5 \text{ hr}}{365 \times 24 \text{ hr}}\right)^2$$

$$= (355)^3 \times (0.00486)^2 = 1050$$

There is still more meat in these student measurements. If a student wishes to go further, he can now calculate the density of Jupiter relative to that of the sun, or that of the earth. The volumes are proportional to the cubes of the diameters, and the ratio of masses has been found. The average diameter of Jupiter is 86,300 mi; that of the sun is 864,000 mi. The result is that Jupiter's density relative to that of the sun is $(1/1048)(864{,}000/86{,}300)^3 = 0.95$. Thus Jupiter is only slightly less dense than the sun. The actual densities in gm/cm^3, based on knowledge of the gravitational constant G (see Sec. 8.16) are: earth, 5.52; sun, 1.42; Jupiter, 1.34.

Some teachers or students will have access to a computer. For their interest and possible use, we reproduce the actual prototype program used in the two Program Orbit loops. The program is written in Fortran.

See Student Handbook p.57 for the actual program.

FILM LOOP 13 Program Orbit I

This film is the first in a series in which a computer is used to help us understand the applications of Newton's laws to planetary motions. We use a computer for two reasons. First, the burden of calculation is removed, so we can immediately see the effect of a change in the data or in the assumed force law. Second, computers are a part of contemporary culture, so it is important to learn what a computer can do and cannot do.

A student is plotting the orbit of a planet, as in Experiment 20, Stepwise Approximation to an Orbit (Fig. 1). While the student is working, his teacher is preparing the computer program for the same problem by punching a set of cards. Then the computer is fed the program and instructed to solve the problem.

The computer's output can be presented in many ways. A table of X and Y values can be typed or printed. For a more easily interpreted display, the computer output is fed to an X-Y plotter, which prints a graph from the table of values. This X-Y plot is similar to the hand-constructed graph made by the student. The computer output can also be shown visually on a cathode-ray tube (CRT). The CRT display will be used in Loop 14.

Let us compare the work of the student and that of the computer. The student chooses an initial position and velocity. Then he calculates the force on the planet from the inverse-square law of gravitation; then he imagines a "blow" to be applied toward the sun, and uses Newton's

the duration and size of its retrograde loop. Average values are listed in Table 5.1 of the text for comparison.

The opposition dates for Jupiter were:

1941 - Dec. 8	1944 - Feb. 11
1942 - None	1945 - Mar. 13
1943 - Jan. 11	1946 Apr. 13

▲ FILM LOOP 10a Retrograde Motion of Mercury and Mars, shown by Animation

This film shows the retrograde motions of Mercury and Mars which are pictured in Fig. 5.7 of the text. The animation shows a background of fixed stars. Although no star except the sun is close enough for us to see as a visible disk even with the largest telescopes, to show the differences in magnitude small disks are used whose sizes are related to the brightness of the stars. (The same is true in Fig. 5.7.)

1. Motion of Mercury and Sun starting April 16, 1963, with time markers at 5-day intervals. The field of view includes portions of the constellations of Aries and Taurus; the familiar group of stars called the Pleiades cluster is at the upper left of the picture. The sun's motion is steadily eastward (to the left) due to the earth's orbital motion. During

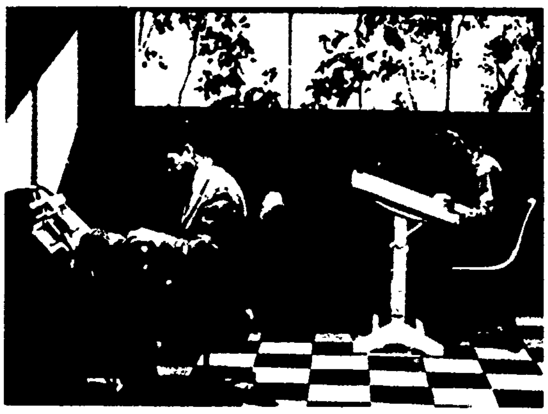


Fig. 1

laws of motion to calculate how far and in what direction the planet moves.

The computer does exactly the same steps. The initial values of X and Y are selected and also the initial components of velocity XVEL and YVEL. (We are beginning to use computer terminology here; XVEL is the name of a single variable, rather than a product of four! The computer "language" we will use is FORTRAN.) The FORTRAN program (represented by the stack of punched cards) consists of the "rules of the game"—which are the laws of motion and the law of gravitation. The "dialogue" between operator and computer takes place after the program has been stored in the com-

are concentrating on the motion of the planet relative to the fixed stars.) For an observer viewing the stars and planets from a stationary earth, this would be equivalent to looking always toward one constellation of the zodiac (ecliptic); for instance, toward Sagittarius or toward Taurus. With the camera located at the center of motion the planet, represented by a white globe, is seen along the plane of motion. A planet's retrograde motion does not always occur at the same place in the sky, so not all retrograde motions are visible in any chosen direction.

Several examples of retrograde motion are shown. In interpreting these scenes, imagine that you are facing south, looking upward toward the selected constellation. East is on your left, and west is on your right. The direct motion of the planet, relative to the fixed stars, is eastward, i.e., toward the left. First we see a retrograde motion which occurs at the selected direction (this is the direction in which the camera points). Then we see a series of three retrograde motions; smaller bulbs and slower speeds are used to simulate the effect of viewing from greater distances.

The dialogue for trial 1 is as follows: (the series of dots at the end of a machine statement represent a request for data):

(machine)

```
PROGRAM HAS NOW BEEN TRANSLATED INTO
MACHINE INSTRUCTIONS

PROGRAM ORBIT
SUBROUTINE GRAPH
READY....

                (operator)

                YES

GIVE ME INITIAL POSITION IN AU....

                X = 4.
                Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

                XVEL = 0.
                YVEL = 2.

GIVE ME CALCULATION STEP IN DAYS....

                60.

GIVE ME NUMBER OF STEPS FOR EACH POINT
PLOTTED....

                1.

GIVE ME DISPLAY MODE....

                X-Y PLOTTER.
```

We see that the orbit displayed on the

FILM LOOP 14 Program Orbit II

This is a continuation of Loop 13. We were left with the feeling that the orbit of trial 1 failed to close because the blows were spaced too far apart. A direct way to test this would be to calculate the orbit using many more blows—but to do this by hand would require much more pencil-pushing and a lot of time. Now we see one way in which a computer quickly solves a complex problem. The operator merely needs to change the dialogue slightly, giving a smaller interval between the calculated points. The laws of motion are the same as before, so the same program is used; only the dialogue is different. A portion of the new dialogue for trial 2 is as follows:

```
READY....

                YES

GIVE ME INITIAL POSITION IN AU....

                X = 4.
                Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

                XVEL = 0.
                YVEL = 2.

GIVE ME CALCULATION STEP IN DAYS....

                3.

GIVE ME NUMBER OF STEPS FOR EACH POINT
PLOTTED....

                7.

GIVE ME DISPLAY MODE....

                X-Y PLOTTER.
```

Note that only minor changes in the dialogue have been made. Points are now calculated every 3 days (20 times as many calculations as for trial 1), and only 1 out of 7 of the calculated points are plotted (to avoid a graph that is crowded with too many points).

The final instruction can also be modified to obtain a display on the face of the cathode-ray tube which exactly duplicates the X-Y plotter display:

GIVE ME DISPLAY MODE....

CRT

The CRT display has the advantage of speed and flexibility; plotted points can be erased if desired (as in Loop 17, on perturbations). On the other hand, the permanent record afforded by the X-Y plotter is more convenient and has better precision than a photographic record of a CRT display.

We will use the CRT display in the other films in this series, Loops 15, 16 and 17.

FILM LOOP 15 Central Forces (Computer Program)

Section 8.4 of the text shows that a body acted on by a central force will move in such a way that Kepler's law of areas applies. It doesn't matter whether the force is constant or variable, or whether the force is attractive or repulsive. The law of areas is a necessary consequence of the fact that the force is central, directed toward or away from a point. The proof in Sec. 8.4 follows that of Newton in the Principia.

The initial scene in the film shows a dry-ice puck bouncing on a circular bumper. This is one way in which a central force can be visualized as demonstrating the motion of a body under the action of repeated blows of equal duration, all directed toward a center. The rest of the film is made by photographing the face of a CRT which displays the output of a computer.

It is important to realize the role of the computer program: it controls the change in direction and change in

speed of the "mass" as a result of a "blow." This is how the computer program uses Newton's laws of motion to predict the result of applying a brief impulsive force, or blow. The program remains the same for all parts of the loop, just as Newton's laws remain the same during all experiments in a laboratory. However, at one place in the program the operator must specify how he wants the force to vary with the distance from the central point. The basic program (laws of nature) remains the same throughout.

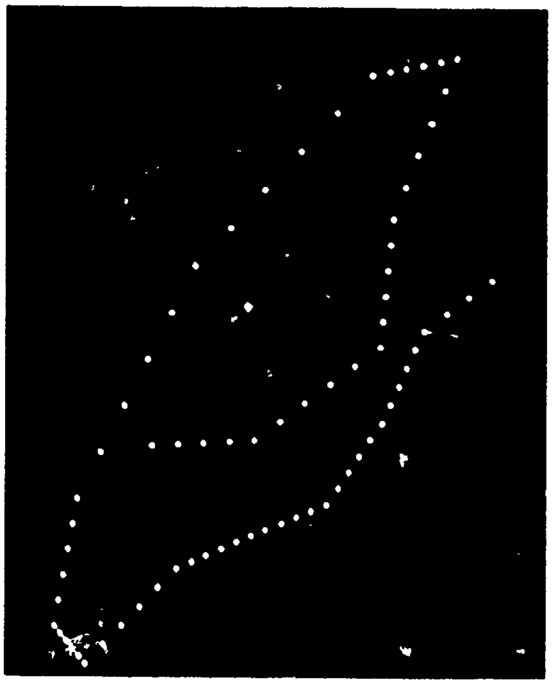


Fig. 1

1. Random blows. The photograph (Fig. 1) shows part of the motion of the mass as blows are repeatedly applied at equal time intervals. No one decided in advance which blows to use; the program merely told the computer to select a number at random to represent the magnitude of the blow. The directions toward or away from the center were also selected at random, although a slight preference for attractive blows was built in so the pattern would, on the whole,

stay on the face of the CRT. Study the photograph. How many blows were attractive? How many were repulsive? Were any blows so small as to be negligible?

You can see if the law of areas applies to this random motion. Project the film on a piece of paper, mark the center and mark the points where the blows were applied. Now measure the areas of the triangles. Does the moving mass sweep over equal areas in equal time intervals?

An easy way for the student to test the constancy of the areas is to calculate the products, (base) × (height), which are twice the areas. There is no need to divide by 2 each time.

2. Force proportional to distance. Perhaps your teacher has demonstrated such a force by showing the motion of a weight on a long string. If the weight is pulled back and released with a sideways shove, it moves in an elliptical orbit with the force center (lowest point) at the center of the ellipse. Notice in the film how the force is largest where the distance from the center is greatest. The computer shows how a relatively smooth orbit is built up by having the blows come at shorter time intervals. In 2a, only 4 blows are used to describe an entire orbit; in 2b there are 9 blows, and in 2c, 20 blows give a good approximation to the ellipse that would be obtained if the force acted continuously.

3. Inverse-square force. Finally, the same program is used for two planets simultaneously, but this time with a force which varies inversely as the square of the distance from the center of force. Notice how the force on each planet depends on the distance from the sun. For these ellipses, the sun is at one focus (Kepler's first law), not at the center of the ellipse.

Be sure the student understands the two ways in which elliptical orbits can arise from a central force. The ellipse with the sun at the center (instead of at one focus) would require a force which increases with distance ($F = kR$), obviously not a reasonable form for a law of gravitation. As pointed out in the student notes for Loop 17, the only inverse power law which gives closed (repeating) orbits not passing through the sun is an inverse-square law of gravitational force.

In this loop, the computer has done for us what we could do for ourselves (using Newton's laws) if we had great patience and plenty of time. The computer reacts so quickly that we can change

the conditions rather easily, and thus investigate many different cases and display the results as a diagram. And it makes fewer errors than a person!

FILM LOOP 16 Kepler's Laws for Two Planets (Computer Program)

The computer program described in the notes for Loop 15 was used to display the motion of two planets. According to this program, each planet was acted on at equal time intervals by an impulsive force of "blows" of equal duration, directed toward a center (the sun). The force exerted by the two planets on each other is ignored in this program. In using the program, the operator selected a force law in which the force varied inversely as the square of the distance from the sun (Newton's law of universal gravitation). Figure 1 (taken from Loop 15 on central forces) shows the two planets and the forces acting on them. For

Fig. 1

clarity, the forces are not shown in this loop. The initial positions and initial velocities for the planets were selected, and the positions of the planets were shown on the face of the cathode-ray tube at regular intervals. (Only representative points are shown, although many more points were calculated in between those that were displayed.) This procedure is illustrated in more detail in the notes for Loops 13 and 14. The film is spliced into an endless loop,

In order to obtain an endless loop, the periods had to be commensurable. Therefore the initial positions and velocities had to be related in a certain way. In making this loop, the operator set the initial positions; the computer calculated and set the initial velocities in such a way that the periods would be in a 3:2 ratio.

Both orbits are considerably more eccentric than the orbits of any of the major planets. The eccentricities of the program orbits are: inner planet, 0.69; outer planet, 0.43. Since the inner planet"s orbit is the most eccentric, it may offer a more interesting test of Kepler's first law. A simple way for the student to measure the eccentricity e is from the equation $e = \frac{R_{ap} - R_p}{R_{ap} + R_p}$ where R_{ap} is the aphelion distance from the sun, and R_p is the perihelion distance from the sun.

each planet's motion being repeated indefinitely.

You can check all three of Kepler's laws by projecting this film on a sheet of paper and marking the position of each planet at each of the displayed orbit points. The law of areas is checked immediately, by drawing suitable triangles and measuring their areas. For example, you can check the constancy of the areas swept over at three places: near perihelion, near aphelion and at a point approximately midway between perihelion and aphelion.

To check Kepler's law of periods (third law), use a ruler to measure the distances of perihelion and aphelion for each orbit. To measure the periods of revolution, use a clock or watch with a sweep second hand; an alternative method is to count the number of plotted points in each orbit.

To check the first law, you must see if the orbit is an ellipse with the sun at one focus. Perhaps as good a way as any would be to use a string and two thumb tacks to draw an ellipse. On a large copy of the projected orbit of either planet, locate the empty focus, point S', which is symmetrical with respect to the sun's position S. Tie a piece of string in a loop which will just extend from P to S' and back again, and place the loop around the thumb tacks (Fig. 2). Then put your pencil

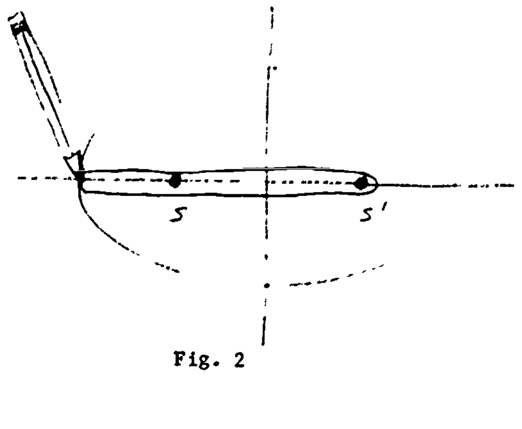


Fig. 2

point in the loop and draw the ellipse, always keeping the string taut. How well does this ellipse (drawn assuming Kepler's first law) match the observed orbit of the planet?

You may think that these are not measurements in the true sense of the word; after all, didn't the computer "know" about Kepler's laws and display the orbits accordingly? Not so—the computer "knew" (through the program we gave it) only Newton's laws of motion and the inverse-square law of gravitation. What we are measuring here is whether these laws of mechanics have as their consequence the Kepler laws which describe, but do not explain, the planetary orbits. This is exactly what Newton did, but without the aid of a computer to do the routine work. Our procedure in its essentials is the same as Newton's, and our results are as strong as his.

FILM LOOP 17 Perturbation

The word "perturbation" refers to a small force which slightly disturbs the motion of a celestial body. For example, the planet Neptune was discovered because of its gravitational pull on Uranus. The main force on Uranus is the gravitational pull of the sun, and the force exerted on it by Neptune is a perturbation which changes the orbit of Uranus very slightly. By working backward, astronomers were able to predict the position and mass of the unknown planet from its small effect on the orbit of Uranus. This spectacular "astronomy of the invisible" was rightly regarded as a triumph for the Newtonian law of universal gravitation.

A typical result of perturbations is that a planet's orbit rotates slowly, always remaining in the same plane.

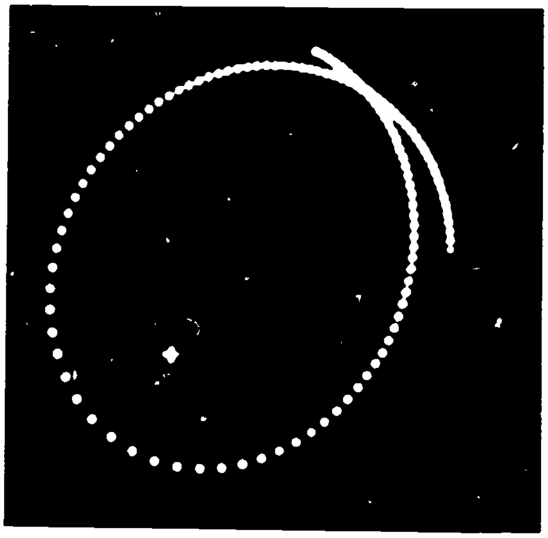


Fig. 1(a)

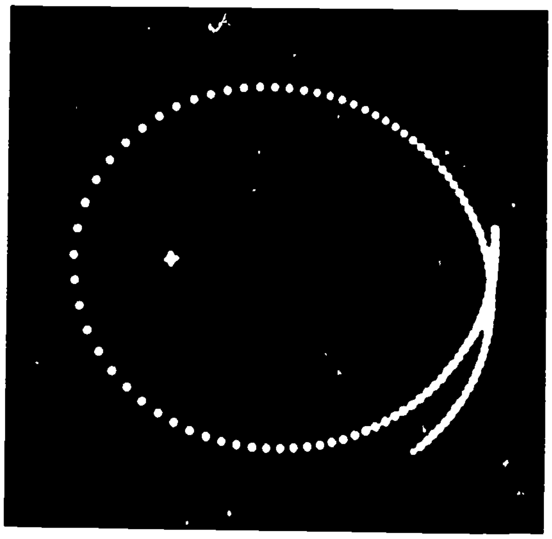


Fig. 1(b)

This effect is called the advance of perihelion, illustrated in Fig. 1. The earth is closest to the sun about Jan. 3 each year; but the perihelion point is slowly advancing. This slow rotation of the earth's orbit (not to be confused with the precession of the direction of the earth's axis) is due to the combined effect of many perturbations: small gravitational forces of the other planets (chiefly Jupiter), and the retarding force of friction due to dust in the space through which the earth moves.

Mercury's perihelion advances at the rate of more than 500 seconds of arc per century. Most of this was fully explained by perturbations due to Newtonian mechanics, the inverse-square law of gravitation and the attractions of the other planets. However, about 43 seconds per century remained unaccounted for. These 43 seconds are crucial; we cannot sweep them under the rug any more than Kepler could ignore the 8 minutes' discrepancy of the position of Mars as calculated on the circular-orbit theory (see Sec. 7.1). When Einstein re-examined the nature of space and time in developing the theory of relativity, he found a slight modification of the Newtonian gravitational theory. As discussed at the end of Sec. 8.18, relativity theory is important for bodies moving at high speeds and/or near large masses. Mercury's orbit is closest to the sun and therefore is most affected by Einstein's extension of the law of gravitation. The relativity theory was successful in explaining the extra 43 seconds per century of advance of Mercury's perihelion, but recently this "success" has again been questioned.

In this film we use a modification of the program which is described in the notes to Loop 15 (central forces). The force on the mass is still a central one, but no longer an exact inverse-square force.

The first sequence shows the advance of perihelion caused by a small force proportional to the distance R. This perturbation is added to the usual inverse-square force. The dialogue between operator and computer starts as follows (the dots at the end of machine statements represent requests for data):

The subject of the advance of Mercury's perihelion has recently been reopened, for recent evidence indicates that the perturbation due to the sun's elliptical shape is very slightly different from the previously accepted value. This would change the amount of the discrepancy by about 4 seconds, so the relativity prediction of 43 seconds would no longer be in exact agreement with the revised amount of the discrepancy. It is possible, of course, that still other components of the perturbation have larger experimental error than previously assumed.

```
(machine)

PROGRAM HAS NOW BEEN TRANSLATED INTO
MACHINE INSTRUCTIONS....

PROGRAM PRECES
SUBROUTINE GRAPH

READY....

                    (operator)

                    YES

PRECESSION PROGRAM WILL USE
   ACCEL = G/(R*R) + P*R
GIVE ME PERTURBATION P

                    P = .66666

GIVE ME INITIAL POSITION IN AU....

                    X = 2.
                    Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

                    XVEL = 0.
                    YVEL = 3.

GIVE ME CALCULATION STEP IN DAYS....
   (etc.)
```

In this dialogue the symbol * means multiplication; thus G/(R*R) is the inverse-square force, and P*R is the perturbing force, proportional to R.

As a special case, a circular orbit is possible for <u>any</u> attractive force law, for then R is constant and so is the force. But such an orbit is not stable; any deviation from an exactly circular orbit would give rise to a catastrophic orbit (unless the force is inverse-square).

In the second sequence, the inverse-square force law is replaced by an inverse-cube law. The dialogue includes the following:

```
READY....

                    YES

GIVE ME POWER OF FORCE LAW....

                    -3.
```

```
THANK YOU.  PROGRAM WILL USE
   ACCEL = G/(R*R*R)
GIVE ME INITIAL POSITION IN AU....

              X = 1.
              Y = 0.

GIVE ME INITIAL VELOCITY IN AU/YR....

              XVEL = 0.
              YVEL = 6.2832

GIVE ME CALCULATION STEP IN DAYS....
   (etc.)
```

The orbit resulting from the inverse-cube attractive force is not a closed one. The planet spirals into the sun in a "catastropnic" orbit. As the planet approaches the sun it speeds up (law of areas); for this reason the last few plotted points are separated by a large fraction of a revolution.

Our use of the computer is, indeed, experimental science; we are able to see what would happen if the force law changes, but we retain the "rules of the game" expressed by Newton's laws of motion.

Problem

An astronomer claims that he has estimated the mass of an invisible star which is one member of a binary pair.

a) What measurements would he need for such an estimation?

b) How would he have proceeded from the measurements to his estimation? Outline the steps he must have followed.

The displayed points are calculated at equal time intervals. From conservation of angular momentum the product of moment of inertia ($I = mr^2$) and angular speed ω must remain constant. Then $r^2\omega$ = constant, and $\omega \propto 1/r$.

See Student Handbook p. 57 for an example of a computer program.

Suggested Answers to Unit 2 Tests
Test A

ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY
1	C	5.4	0.71
2	A	5.6	0.98
3	C	8.7	0.74
4	C	7.1	0.60
5	E	7.8	0.67
6	A	8.7	0.50
7	D	7.4	0.86
8	A	8.5	0.26
9	E	5.7	0.75
10	B	6.4	0.77
11	B	6.8	0.67
12	E	7.5	0.79
13	D	8.3	0.76
14	E	5.7	0.64
15	A	8.6	0.68

Answers
Test A

Group I

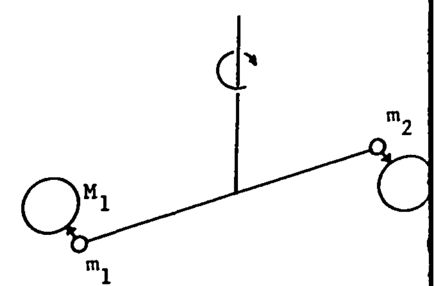
1. Sections of Unit: 5.7, 6.1-6.4

Ptolemy applied geometry in an attempt to explain the retrograde motion of planets.

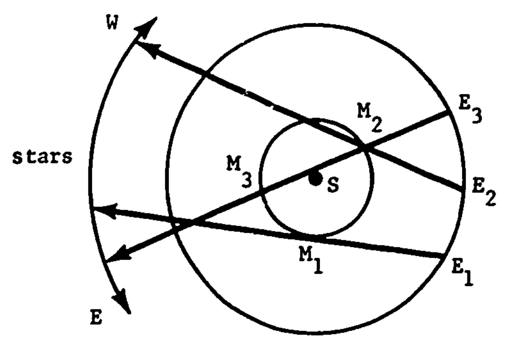
Copernicus proposed a sun-centered system in which the earth's orbit replaced the epicycles in the motions of the other planets.

2. Section of Unit: 8.15

The gravitational force of attraction between m_1 and M_1, and m_2 and M_2 caused the beam holding m_1 and m_2 to twist about the vertical axis. Cavendish calibrated the force required to twist the vertical supporting wire and thus determined F_{grav}.



3. Section of Unit: 5.6



Mercury will orbit completely in the same time as the earth covers 1/4 of its orbit.

As the earth moves from E_1 to E_2 the line of sight to Mercury goes generally westward from M_1 to M_2. But as the earth moves from E_2 to E_3 the line of sight to Mercury goes generally eastward from M_2 to M_3.

4. Sections of Unit: Ch. 5 and 6

Although nature may be validly observed and described from all frames of reference, a particular phenomenon may appear simpler when viewed from a certain frame of reference. When observed from a heliocentric frame of reference rather than a geocentric frame of reference the motions of planets do appear more simple.

5. Section of Unit: 5.1

Each day the sun rises above the horizon on the eastern side of the sky and sets on the western side. At noon the sun is highest above the horizon. From July through November the noon height of the sun above the horizon decreases. Near December 22 the sun's height at noon is at a minimum. From January into June the sun's height at noon slowly increases. About June 21 the sun's height at noon is at a maximum.

Group II

6 . Sections of Unit: 8.6-8.9

The word "falling" is used in describing the motion of the moon relative to the earth in the same sense as it is used to describe the motion of an object, for example a ball, falling freely near the earth's surface.

The explanation of this statement should involve discussion of at least the following points:

a) the acceleration of the moon towards the earth as compared to the acceleration of an object falling freely near the earth's surface,
b) the nature of the force accelerating both the moon and the freely falling object towards the earth.

7 . Sections of Unit: 8.3, 8.5

Given: $F \propto \frac{1}{R^2}$, $D = \frac{1}{2} at^2$

Derive: $\frac{T_p^{\,2}}{T_e^{\,2}} = \frac{R_p^{\,3}}{R_e^{\,3}}$

From geometry

$$\frac{R_p}{R_e} = \frac{D_p}{D_e}$$

$$\frac{R_p}{R_e} = \frac{\frac{1}{2}a_p T_p^{\,2}}{\frac{1}{2}a_e T_e^{\,2}} = \frac{a_p T_p^{\,2}}{a_e T_e^{\,2}}$$

But since $F \propto \frac{1}{R^2}$ and $F = ma$, then $a \propto \frac{1}{R^2}$

$$\text{So } \frac{R_p}{R_e} = \frac{\frac{1}{R_p^{\,2}} \cdot T_p^{\,2}}{\frac{1}{R_e^{\,2}} \cdot T_e^{\,2}} \text{ or } \frac{R_p^{\,3}}{R_e^{\,3}} = \frac{T_p^{\,2}}{T_e^{\,2}} .$$

Suggested Answers to Unit 2 Tests
Test B

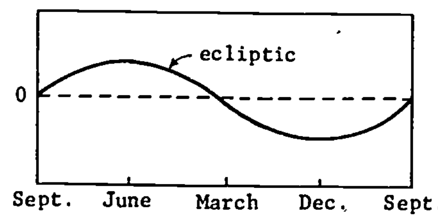
ITEM	ANSWER	SECTION OF UNIT
1	A	7.4
2	B	7.1
3	D	6.8
4	D	5.7
5	C	7.1
6	A	6.6-6.8
7	E	7.8
8	C	5.7
9	C	8.7
10	A	8.5
11	E	8.6
12	B	8.19
13	C	7.5
14	C	Unit I
15	B	7.8

Group I

1. Section of Unit: 5.1 and astronomy laboratory

The sun moves eastward by about 1° per day relative to the stars. It completes one revolution through the stars in one year. There is an observable north-south oscillation completed in one year also. The path among the stars is called the ecliptic.

OR a star chart



2. Sections of Unit: 5.6, 5.7, 6.4, 6.6

If the earth is at rest, one would not expect a stellar parallax, and indeed it was not observed until well into the nineteenth century. If the earth moves around the sun, then one should observe a parallax unless the stars are too far away. Opponents of Copernicus used the absence of a parallax as an argument against his theory, and were not convinced that the stars could be far enough to account for the absence of a parallactic shift.

3. Section of Unit: 8.7

Let m_1 be the mass of the earth, m_2 the mass of a falling body.

Then $F = m_2 a = \dfrac{Gm_1m_2}{R^2}$

$$a = \frac{Gm_1}{R^2}$$

Therefore a does not depend upon m_2, the mass of the falling body.

4. Sections of Unit: 7.10, 7.11

The answer could be yes, no, or somewhere between, depending upon the point of view taken. There were some rational criteria used. For example, the absence of stellar parallax is a reasonable argument against the heliocentric theory. However, religious and philosophical beliefs played a large role, and many people refused even to consider new theories on their own merits. One possible scientific criterion of the time was agreement with observations. For the most part the scholastics would not even consider the observations, but at that time scientific criteria were nebulous if not inadequate. There was no conclusive way to judge the relative merits of the competing theories at the time.

5. Sections of Unit: 8.5 and 8.6

a) $F_g \propto$ mass of planet/ r^2

Since the mass of planet A equals the mass of planet B, and since the radius of planet A is greater than the radius of planet B, then F_g on the surface of B is greater than F_g on the surface of A.

b) $$\frac{F_{g_A}}{F_{g_B}} = \frac{\frac{M_A}{{r_A}^2}}{\frac{M_B}{{r_B}^2}} = \frac{{r_B}^2}{{r_A}^2} = \frac{1}{4}$$

Group II

6. General

Galileo

a) first of modern scientists—experimentation and thought experiments
b) gave kinematic description of freely falling bodies
c) used telescope to gather evidence in support of heliocentric system
d) helped popularize the Copernican point of view through writings

Kepler

a) three laws gave a kinematic description of the solar system
b) helped to make heliocentric theory description more simple than geocentric model
c) first to break away from Plato's uniform circular motion
d) three laws were used by Newton in his search for a force law of gravitation

Newton

a) three laws of motion
b) synthesized terrestrial and celestial theories of motion
c) developed gravitational force law
d) universal law of gravitation
e) Principia and "Rules of Reasoning in Philosophy"

7. Sections of Unit: 5.7, 6.3, 6.4

a) Yes and no. It could have been simpler if it had not used combinations of circular motions.
It was in fair agreement with observations.

b) Agreement with observations was about the same for both theories. The Copernican theory eliminated the major epicycle of the Ptolemaic theory, but it was not clear that this was a simplification. Actually, the Copernican theory seemed in its components to be very complicated because it upset very fundamental beliefs without resolving these problems satisfactorily. Some of Copernicus' contemporaries regarded religious knowledge through teachings or the Bible as facts which must be accounted for in a satisfactory theory, whereas we would not regard Biblical quotations as evidence.

Suggested Answers to Unit 2 Tests
Test C

ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY	ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY
1	A	5.6	0.93	21	D	8.18	0.86
2	E	5.1	0.70	22	A	8.5	0.46
3	E	6.4	0.77	23	D	5.7	0.37
4	B	7.8	0.76	24	B	5.5	0.59
5	C	5.3	0.79	25	C	8.7	0.63
6	B	8.6	0.74	26	D	8.5	0.36
7	A	6.4	0.54	27	E	6.3	0.58
8	B	6.7	0.78	28	D	5.3	0.57
9	B	8.13	0.84	29	C	5.7	0.68
10	B	7.4	0.63	30	C	7.5	0.44
11	B	6.3	0.67	31	C	7.5	0.33
12	E	8.14	0.58	32	B	8.19	0.91
13	D	6.7	0.88	33	D	8.5	0.51
14	E	5.4	0.75	34	D	8.0	0.90
15	D	7.4	0.92	35	B	8.6	0.65
16	E	8.4	0.80	36	C	8.3	0.64
17	A	8.6	0.73	37	E	7.5	0.69
18	A	7.5	0.85	38	B	8.4	0.57
19	C	7.5	0.64	39	C	8.7	0.65
20	A	6.1	0.74	40	C	7.6	0.54

Suggested Answers to Unit 2 Tests
Test D

1. Sections of Unit: 7.2-7.5, 8.1-8.9

Kepler produced a new geometrical theory based on three empirical laws to explain Tycho Brahe's astronomical observations.

Newton produced a theory of universal gravitation that explained the dynamics of planetary motion by uniting Kepler's laws and terrestrial gravitation.

2. Sections of Unit: 8.2, 8.3, 8.5, 8.6

His synthesis brought together the work of Galileo and Kepler along with his own to build a system of dynamics which described the motion of all bodies. It synthesized the laws of motions of both terrestrial and celestial objects.

3. Section of Unit: 7.1

Ptolemy and Copernicus both used Plato's assumption of uniform circular motion. Kepler was the first to abandon this assumption. He constructed the orbits directly and determined that the orbits were ellipses.

4. Section of Unit: 7.5

$$T^2 = ka^3$$

$$\frac{T_s^{\ 2}}{T_e^{\ 2}} = \frac{a_s^{\ 3}}{a_e^{\ 3}}$$

$$\frac{T_s^{\ 2}}{1^2} = \frac{9^3}{1^3}$$

$$\underline{T_s = 9^{3/2} = 27 \text{ yr.}}$$

or

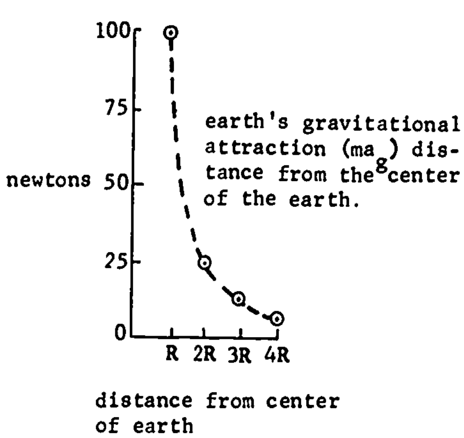
$$T_s^{\ 2} = ka_s^{\ 3}$$

$$k = 1 \frac{(\text{year})^2}{(\text{astronomical unit})^3}$$

$$T_s^{\ 2} = 729 \ (\text{yr})^2$$

$$\underline{T_s = 27 \text{ yr.}}$$

5.

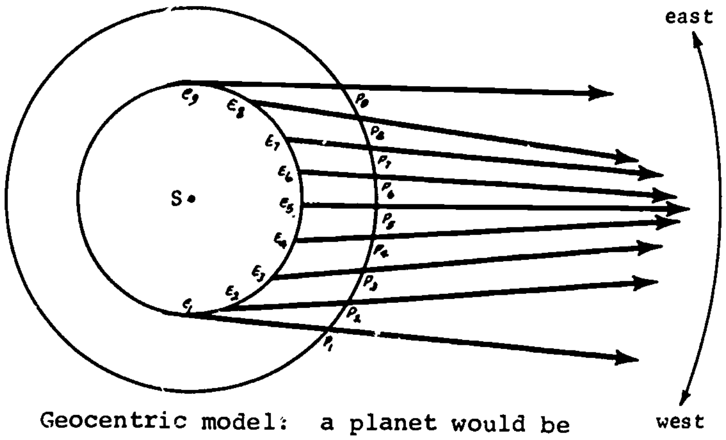


earth's gravitational attraction (ma$_g$) vs distance from the center of the earth.

6. Chapter 5

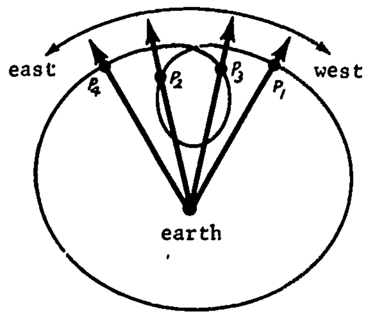
a) The motion of a planet appears sometimes to reverse its original direction with respect to the fixed stars.

b) heliocentric model: a planet more distant from the sun than the earth will have a smaller period. As the earth laps the planet, the apparent motion of the planet changes direction.



Generally eastward from E_1 to E_4 and westward from E_4 to E_8, then eastward again.
See p. 17 of Unit II.

Geocentric model: a planet would be traveling along an epicycle. Therefore an orbit around the earth might look something like the diagram at right.



From P_1 to P_2 and P_3 to P_4 the planet tends to move eastward across the sky. From P_2 to P_3 it appears to move westward in retrograde.

7. Section of Unit: 8.7

a) Newton chose B

b) If $M_p \to 0$ as a limit, F is non-zero in case A and infinite in case C, both of which are not sensible.

Another argument

The force on two planets stuck together should be the sum of the forces on each planet. $F = M_s + M_p$ would not predict this:

$$F_1 + F_2 \propto (M_s + M_p) + (M_s + M_p)$$

$$F_{12} \propto M_s + (2M_p)$$

$$F_1 + F_2 \neq F_{12}$$

Similarly for $F \propto \frac{M_s}{M_p}$.

8. Section of Unit: 5.7

Yes and no.

The theory was not as simple as the one we use today, but given the philosophical conditions of the time, it was as simple as possible. (e.g., earth at center and uniform circular motion). Even modern theory rests on firmly held beliefs. The theory agreed pretty well with most observations. One exception was that the moon, according to Ptolemy, should change apparent size by a factor of two.

An alternative approach would be that it did not satisfy the criteria because it was a reconciliation of observation with the religious-scholastic concern of the importance of man and his earth. At the time Ptolemy was too burdened with the belief that the purpose of all motion was the divinity of man.

Index to Unit 2 Teacher Guide